# Quantum dynamics in Josephson junction circuits


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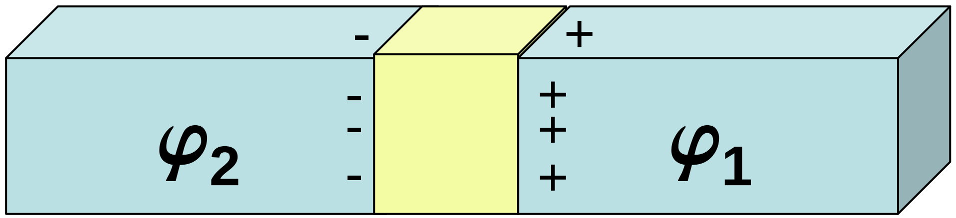
**Journée Information quantique, Monday 15 June 2009**

# Outline

- **Quantum dynamics in a coupled qubit circuit**
- **Josephson junction arrays: Towards the realization of a topologically protected qubit ?**

# Ultrasmall Josephson junction




**$R_T$ C**

$$E_J = \frac{R_Q}{R_T}\frac{\Delta}{2}$$

$$R_T \sim \frac{1}{S}$$

**S = Surface**

$$C \sim S$$

$$E_C = \frac{e^2}{2C}$$

**Josephson effect:**

$$I(\varphi) = \frac{2e}{\hbar} E_J \sin(\varphi)$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\hat{\varphi}$$

**Charging effects**

**Coulomb blockade:**
**I=0 for V<e/C**

$$\Delta\varphi \Delta N \geq 1/2$$

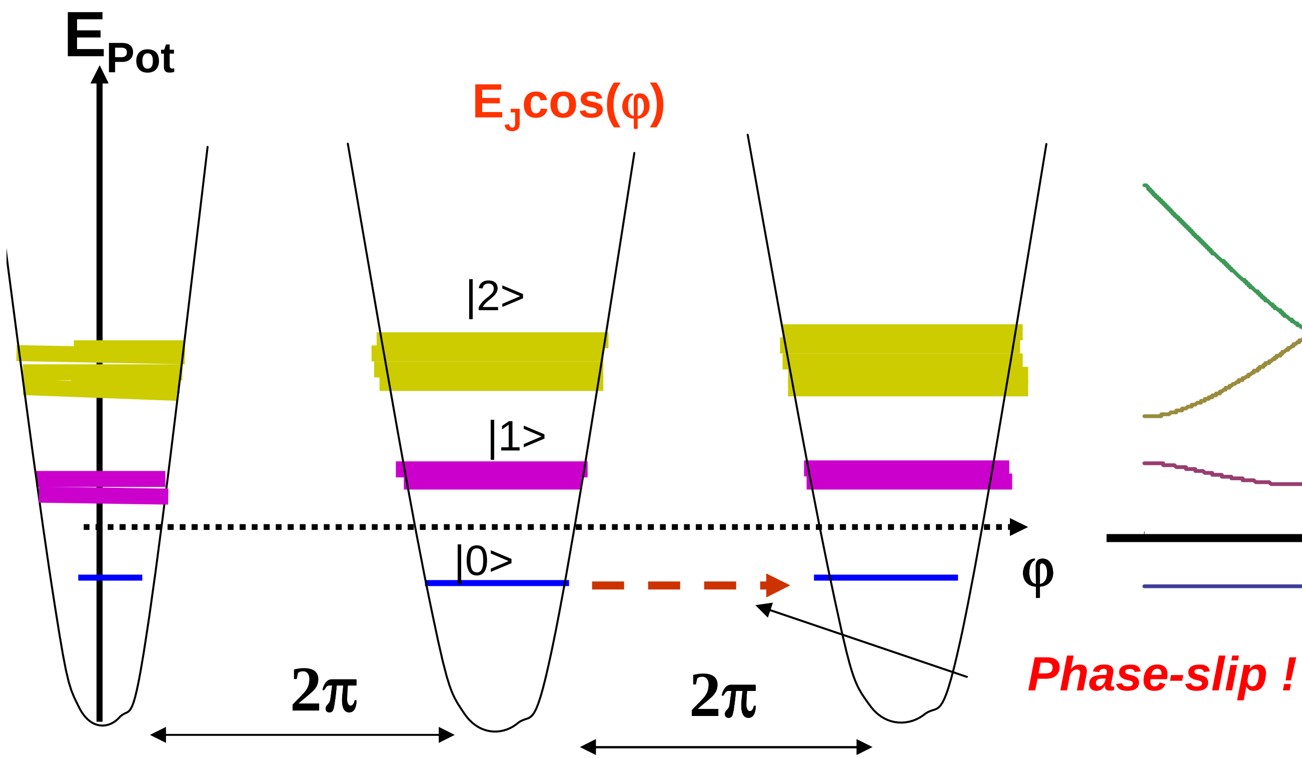
## Bloch Bands

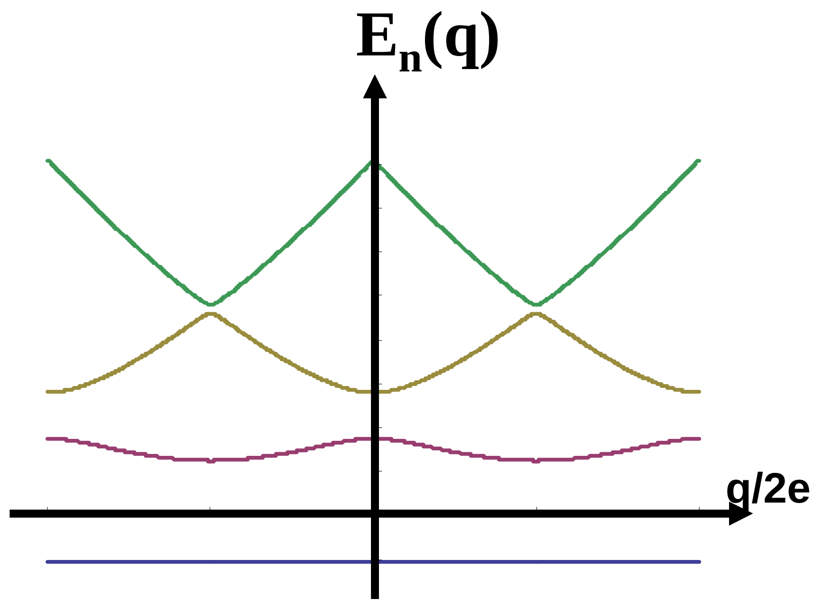
$$\frac{d^2\psi}{d\varphi^2}+\left(\frac{E}{E_c}+\frac{E_J}{E_C}\cos\varphi\right)\psi=0$$

$$H=-E_c\frac{d^2}{d\varphi^2}-E_J\cos\varphi$$

$E_J \gg E_C$ → **Tight Binding Model**

**Movement of a particle in a periodic potential**

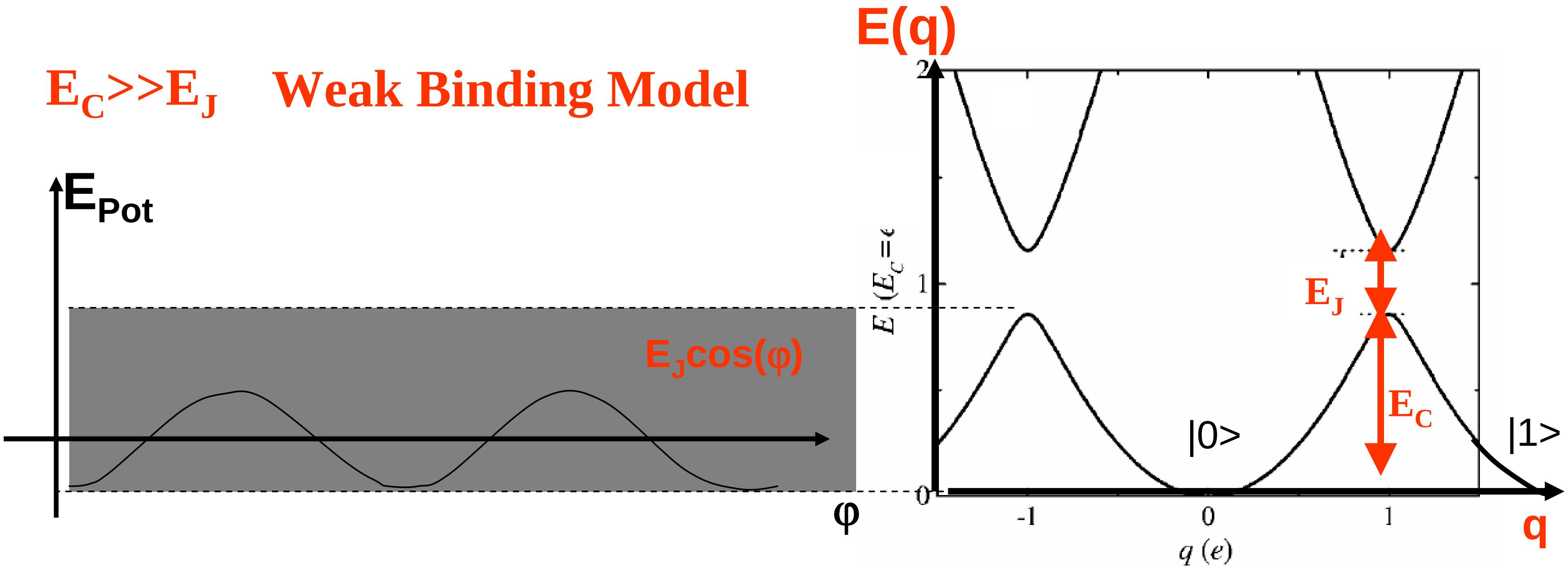



**Tunnelling amplitude for phase slip:**

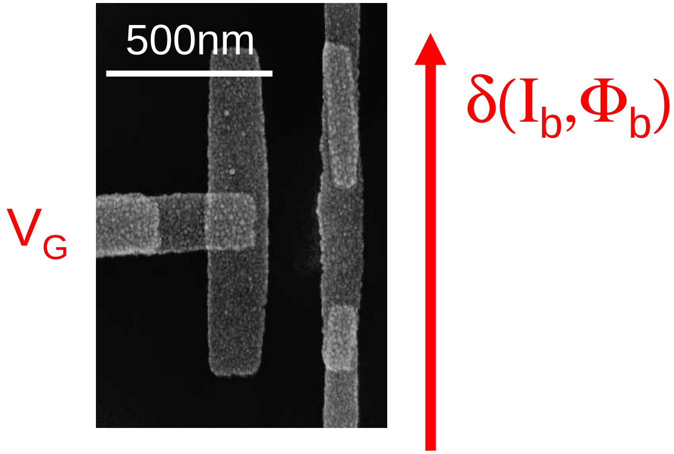
$$\nu\approx\left(E_J^3E_C\right)^{1/4}\exp\left(-\sqrt{8E_J/E_C}\right)$$

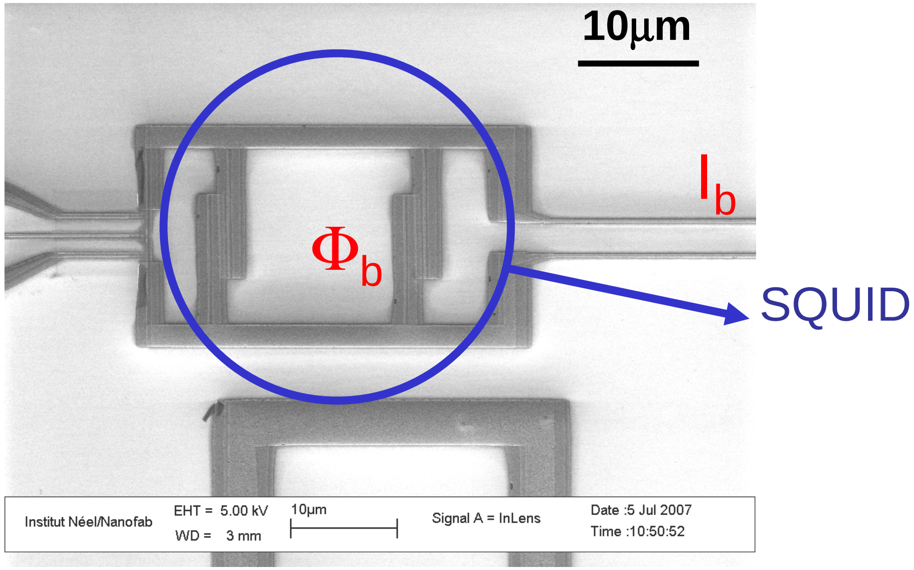
# $E_C >> E_J$ Weak Binding Model



# The coupled circuit

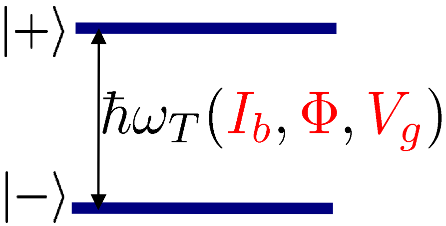
Asymmetric Cooper pair transistor




$$\hat{H}_T = \frac{(2e)^2(\hat{n}-n_g)^2}{2C_\Sigma} - \Sigma e_j \cos(\delta/2)\cos(\hat{\Theta}_d) - \Delta e_j \sin(\delta/2)\sin(\hat{\Theta}_d)$$

$$\hat{H}_S = \frac{1}{2}\hbar\omega_p(\hat{P}^2 + \hat{X}^2) - \sigma\hbar\omega_P \hat{X}^3$$

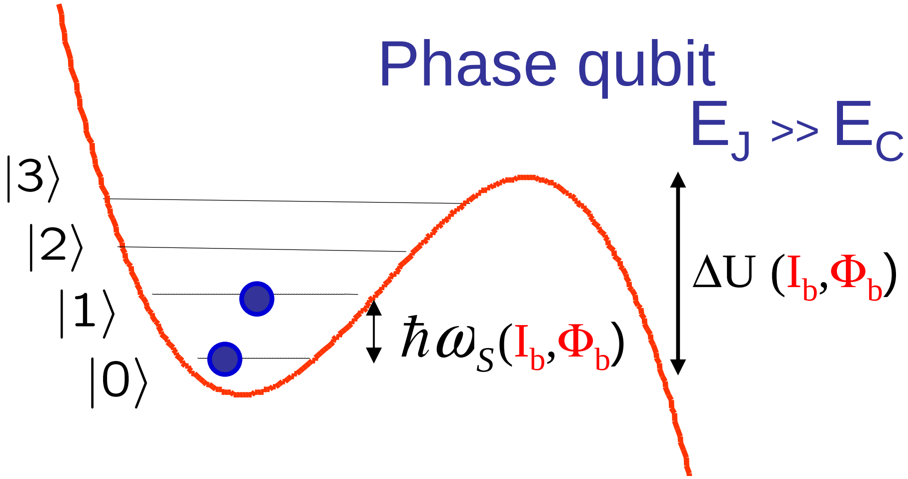



Charge qubit

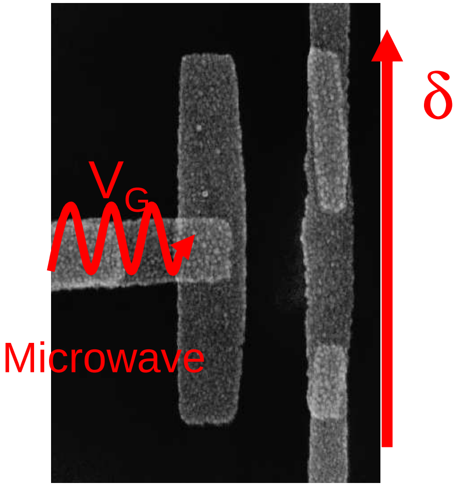
$E_J \sim E_C$

Phase qubit

$E_J >> E_C$



# Asymmetric Cooper pair transistor: charge qubit

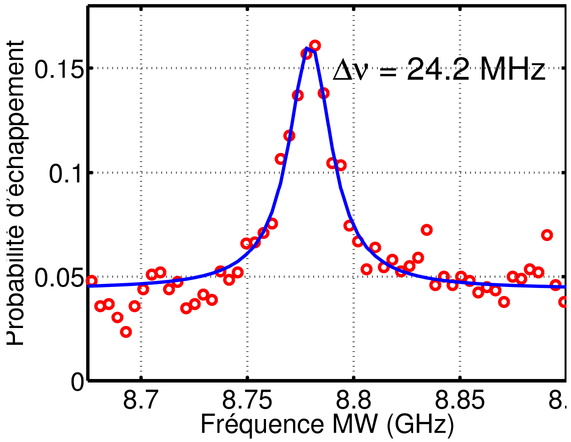


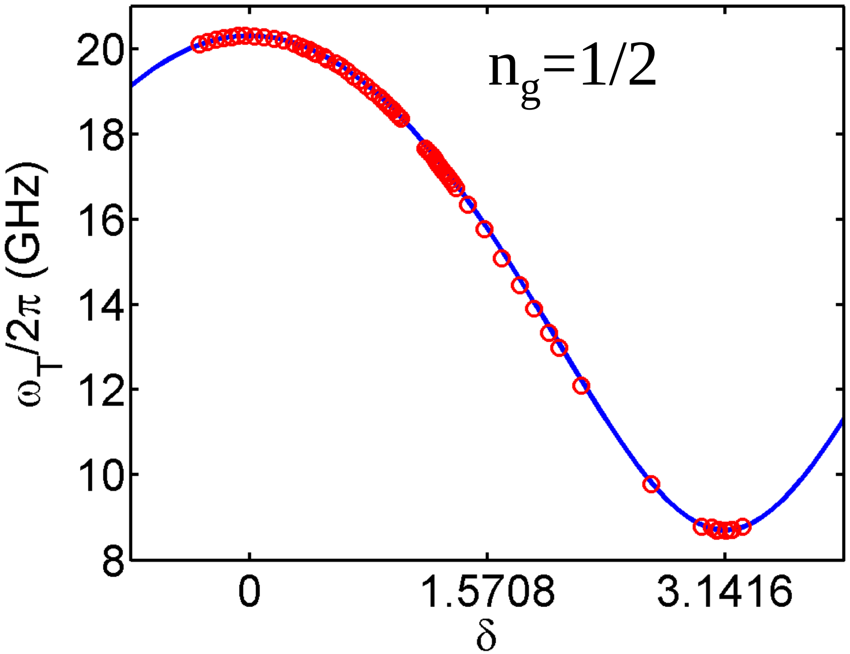

Charging energy — Josephson energy: symmetric, asymmetric

$$\hat{\mathrm{H}}_T = \frac{(2e)^2(\hat{\mathbf{n}} - n_g)^2}{2C_\Sigma} - 2e_j\cos(\delta/2)\cos(\widehat{\mathbf{\Theta}}_\mathbf{d}) - 2\Delta e_j\sin(\delta/2)\sin(\widehat{\mathbf{\Theta}}_\mathbf{d})$$

with $\left[\hat{n}, \hat{\Theta}_d\right] = i$

A charge qubit: $|+\rangle$, $|-\rangle$, $\hbar\omega_T(I_b, \Phi, V_g)$




Josephson energy

$$2e_j \sim 21.8\ GHz$$

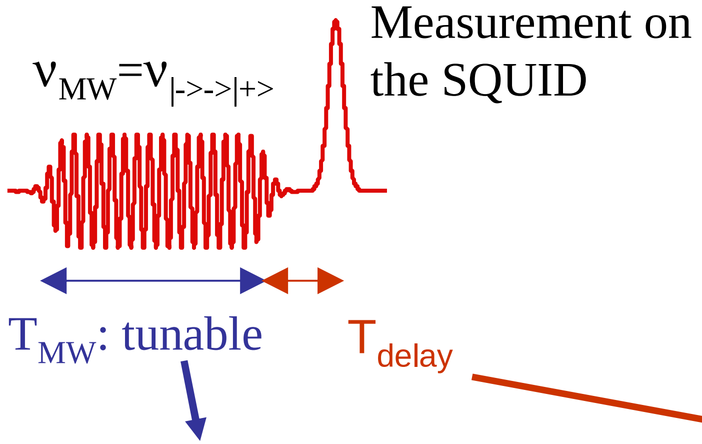
$$2\Delta e_j \sim 8.8\ GHz$$

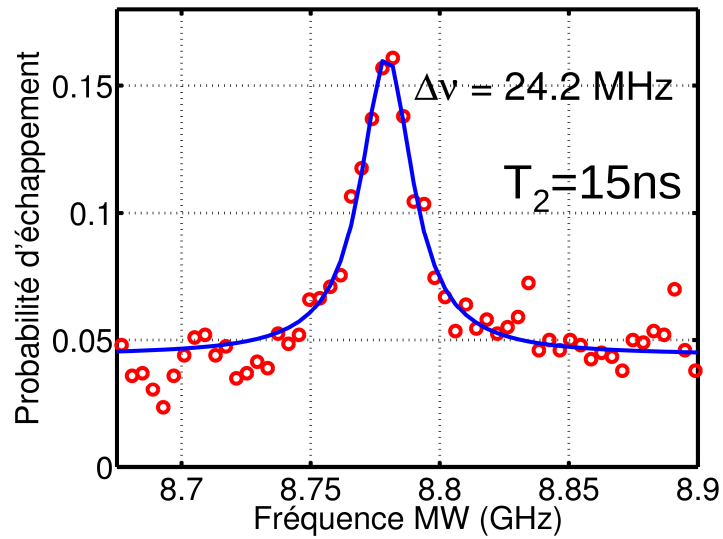
Transistor asymmetry ~ 40 %

Charge energy

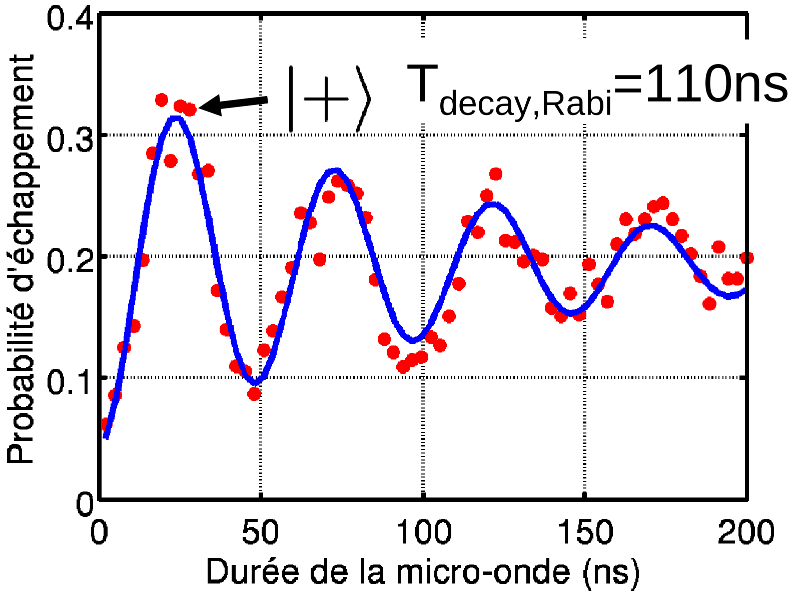
$$\frac{(2e)^2}{2C_\Sigma} \sim 19.3\ GHz$$

# Manipulation of the qubit at the optimal point δ=π

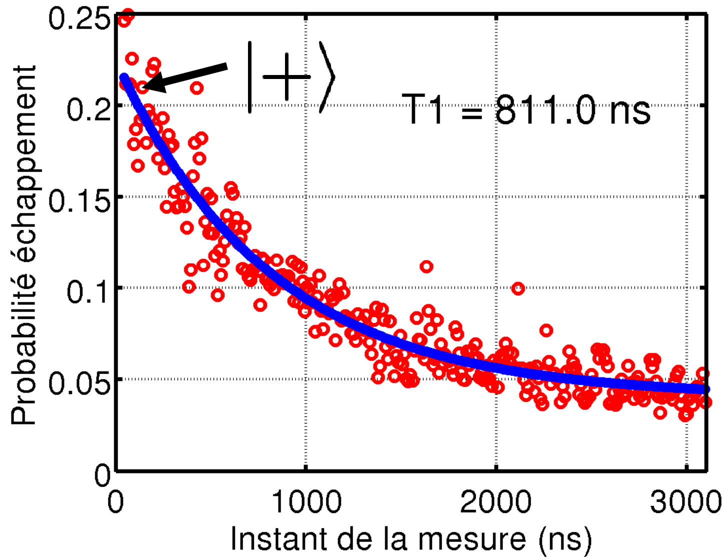


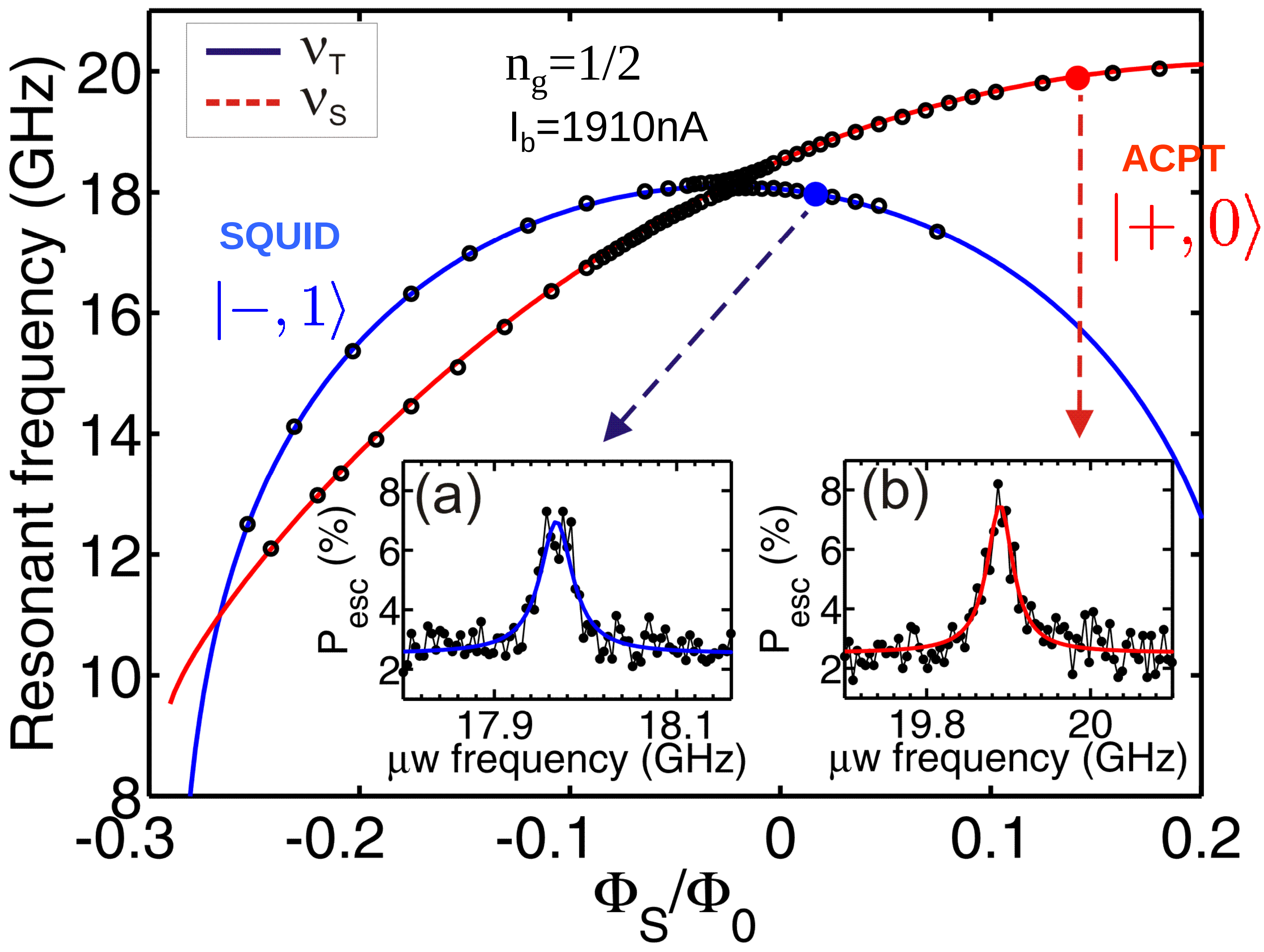

Rabi oscillations :




Energy relaxation

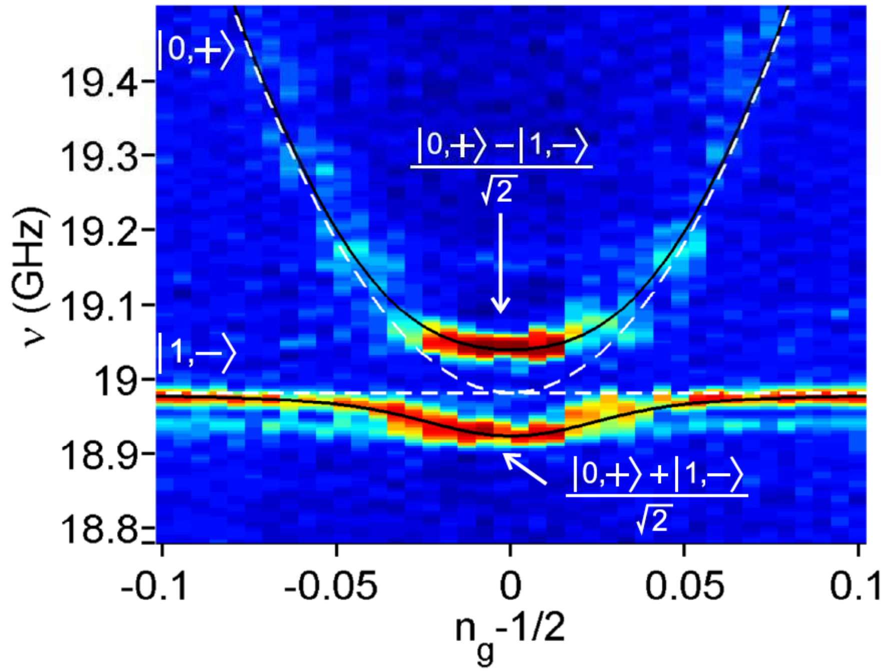


# Spectroscopy measurement of the two quantum systems

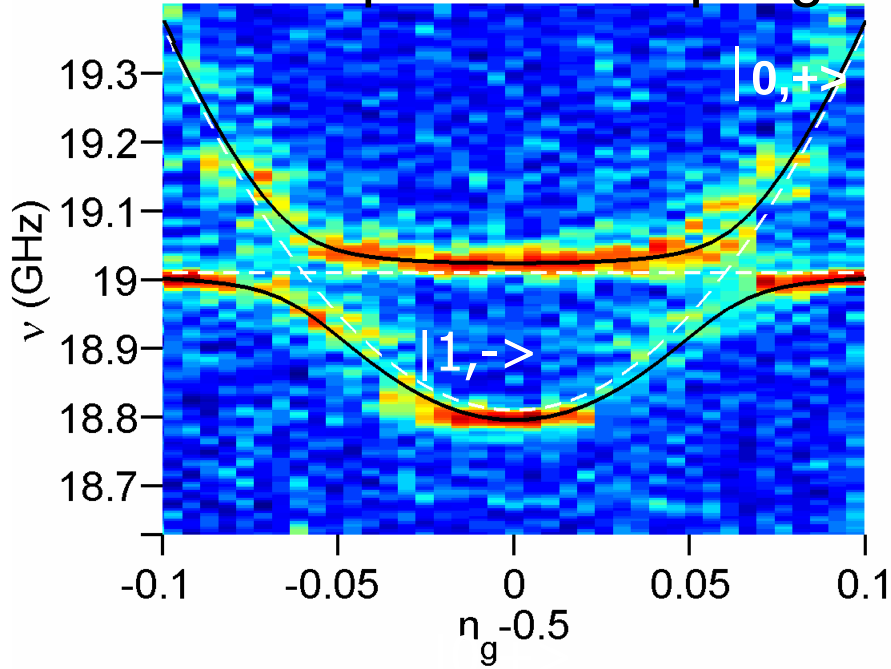


# Spectroscopy versus $V_g$

## Resonant coupling



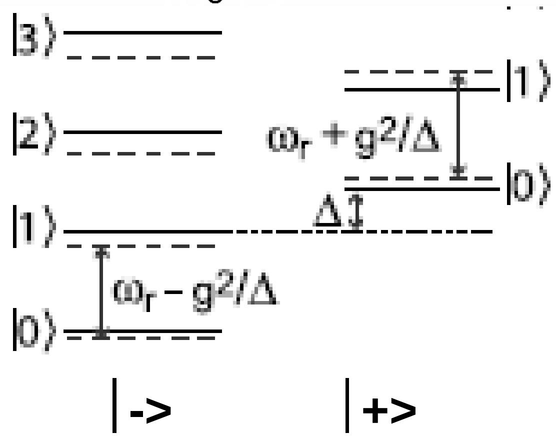

## Dispersive coupling




$g \sim 110$MHz

$\delta\nu_T \sim 40$MHz (charge noise limitation)

$\delta\nu_S \sim 20$MHz (fluctuator and flux noise limitation)

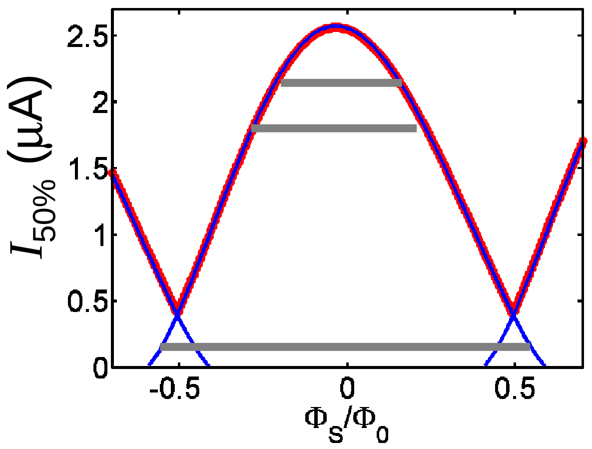


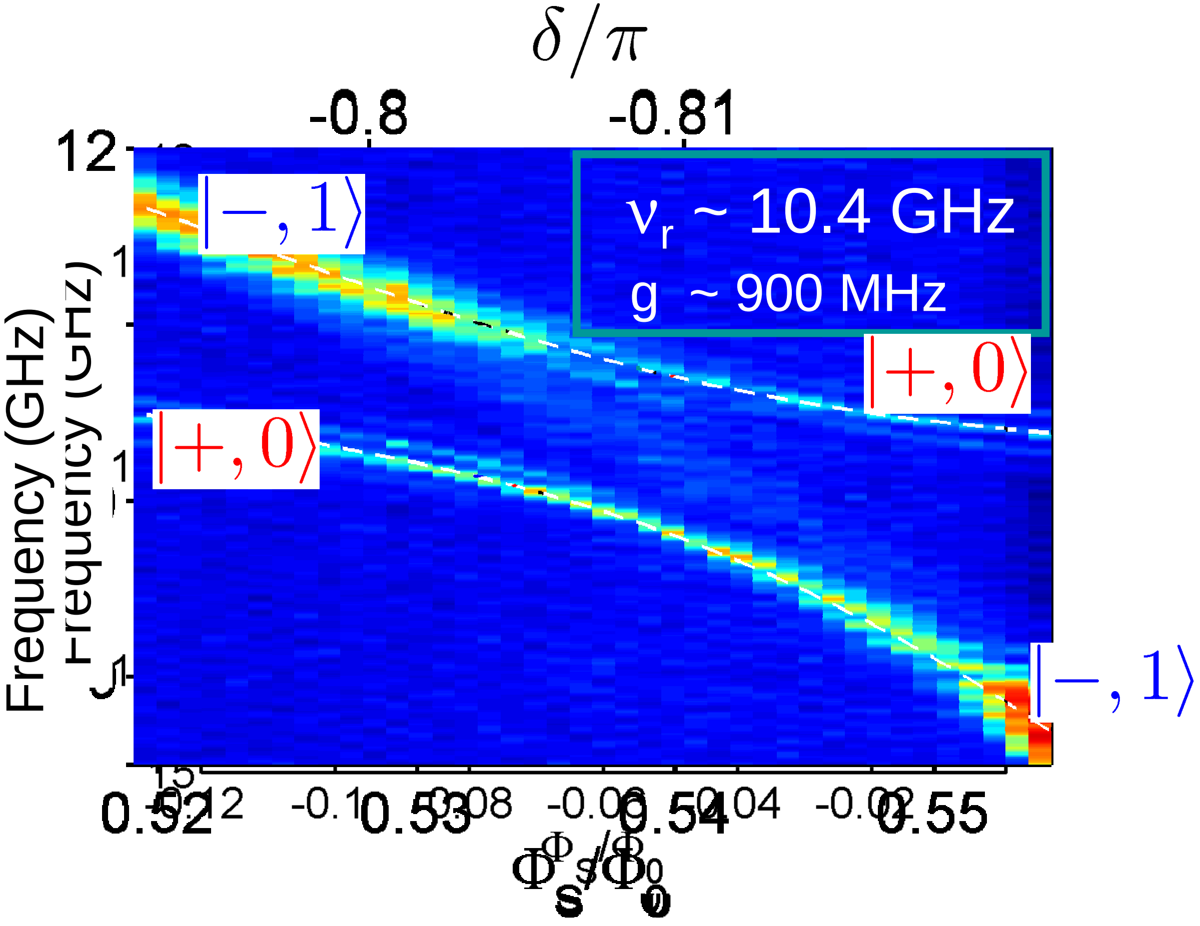

$\Delta \sim 200$MHz

$\chi \sim g^2/4\Delta \sim 10$MHz

# Spectroscopy versus flux

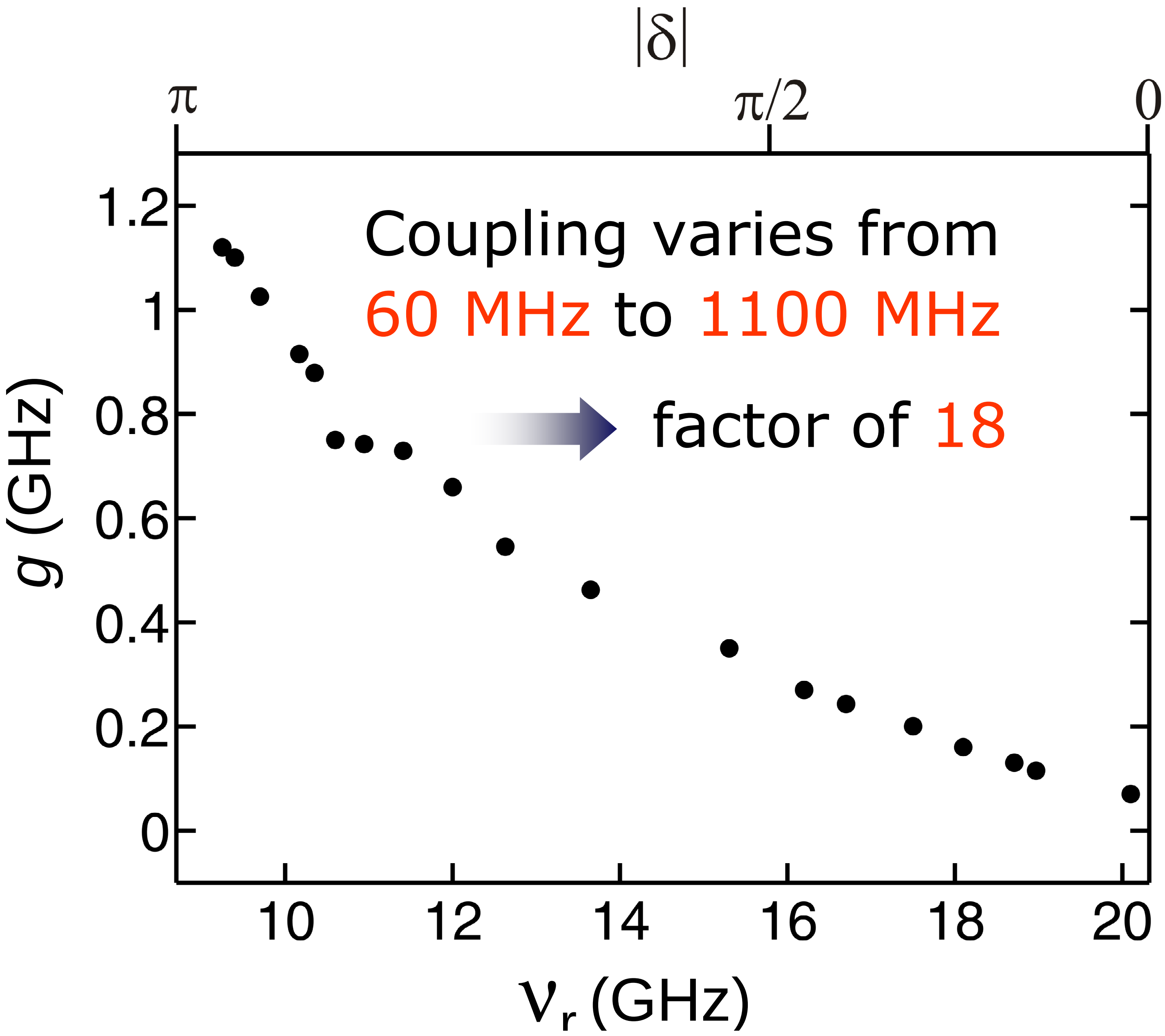
## Spectroscopy at $I_{bias}$ = 2840 nAA



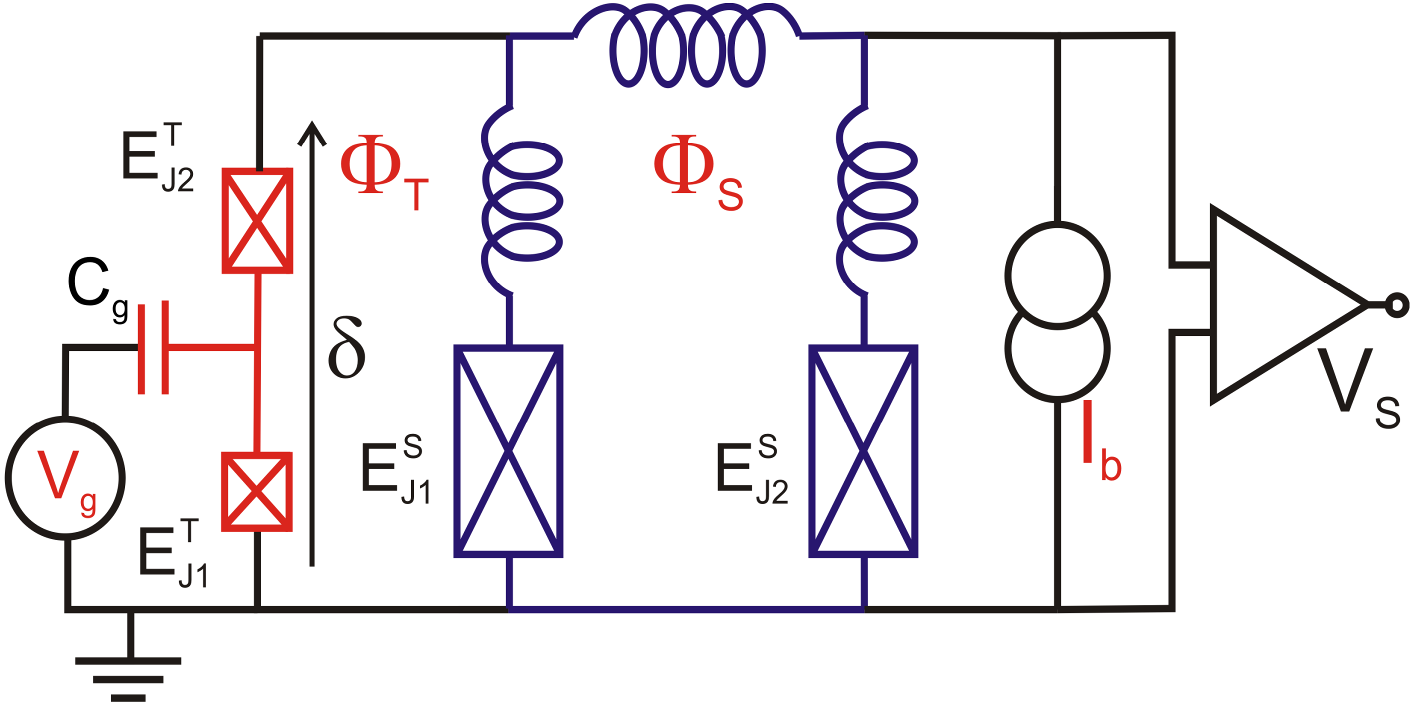




- Two qubits can be in resonance from 9 GHz to 20 GHz.
- Strong variation of the coupling strength.

Resonant coupling



# Electrical schematic of the circuit




$$\hat{H} = \hat{H}_{ACPT} + \hat{H}_{SQUID} + \hat{H}_{COUPL}$$

# Coupling in resonance

$$\hat{H}_{COUPL} = +\frac{1}{2}hg\left(\hat{\sigma}_S^+\hat{\sigma}_T^- + \hat{\sigma}_S^-\hat{\sigma}_T^+\right)$$

$$hg = E_{c,c}/2 - E_{c,j}\cos(\delta/2 + \mu\tan(\delta))$$

**Capacitive coupling**

$$E_{c,c} = (1-\lambda)\sqrt{\frac{E_C^S}{h\nu_p}}h\nu_p$$

Capacitance asymmetry

$$\lambda = (C_1^T - C_2^T)/(C_1^T + C_2^T)$$

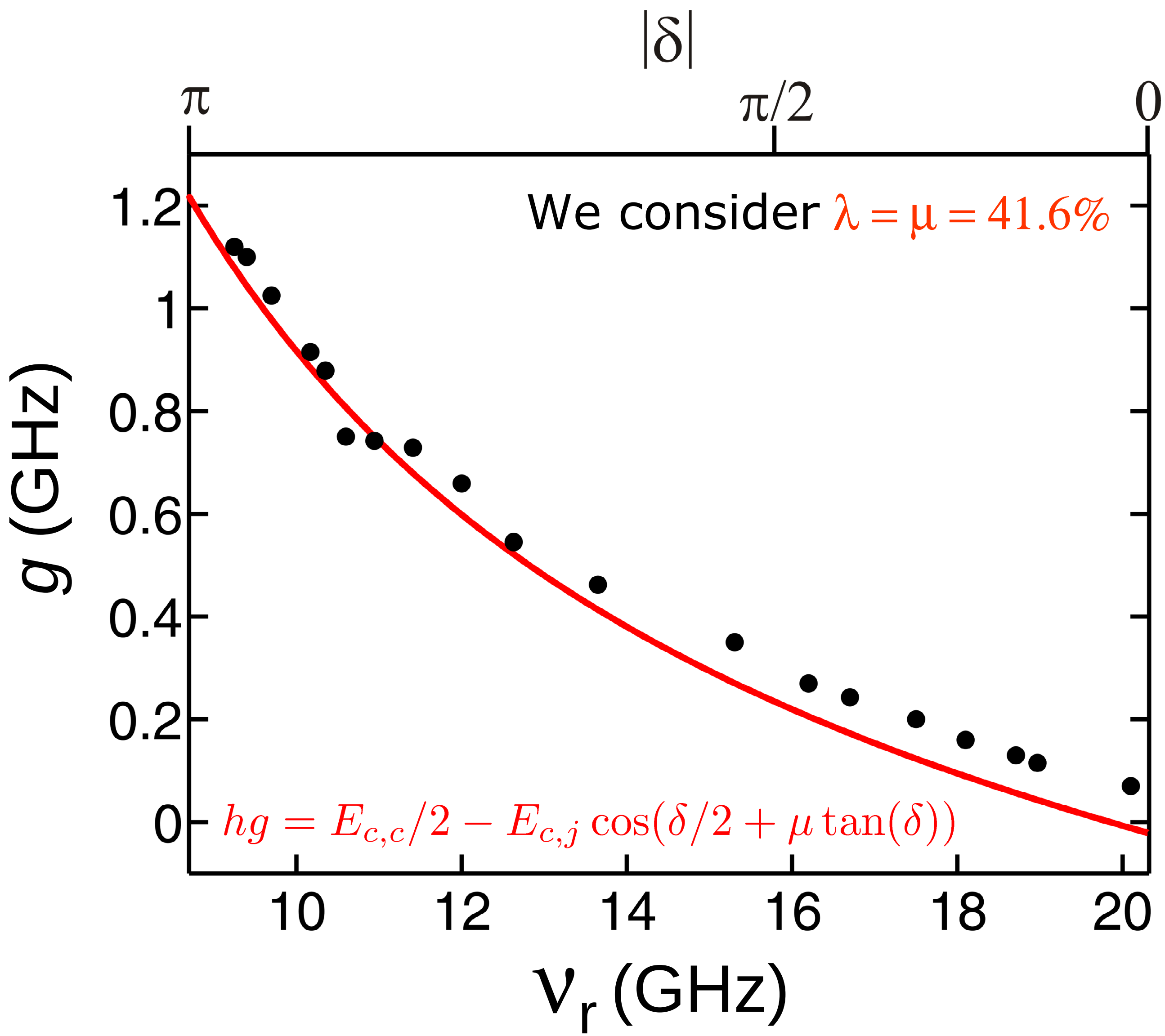
**Josephson coupling**

$$E_{c,j} = (1-\mu)\sqrt{\frac{E_C^S}{h\nu_p}}E_J^T/2$$

Josephson energy asymmetry

$$\mu = (E_{J,1}^T - E_{J,2}^T)/E_J^T$$

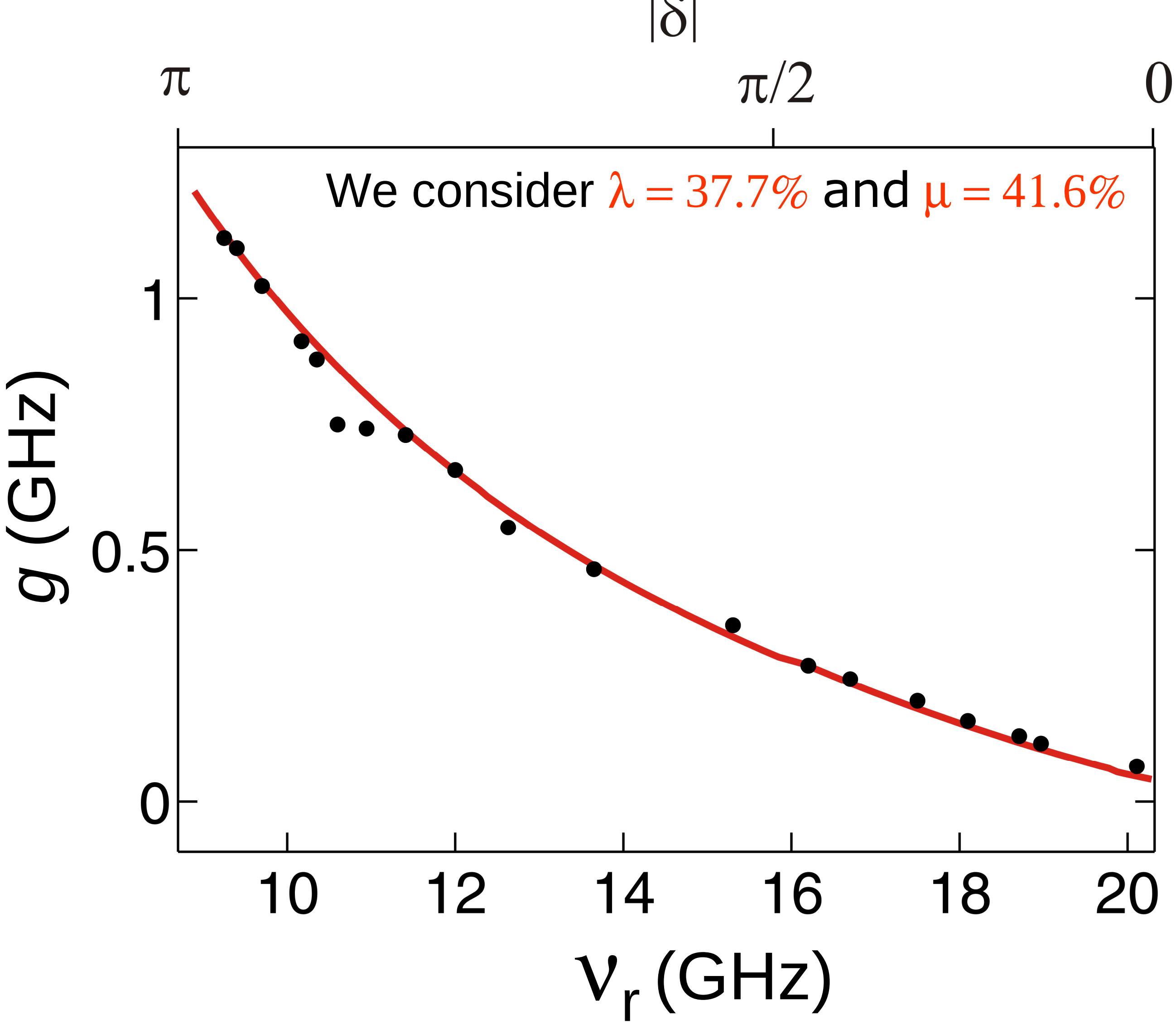
Resonant coupling    A. Fay *et al.*, PRL 100, 187003 (2008)




If transistor was symmetric ($\lambda$=$\mu$=0) coupling would be zero

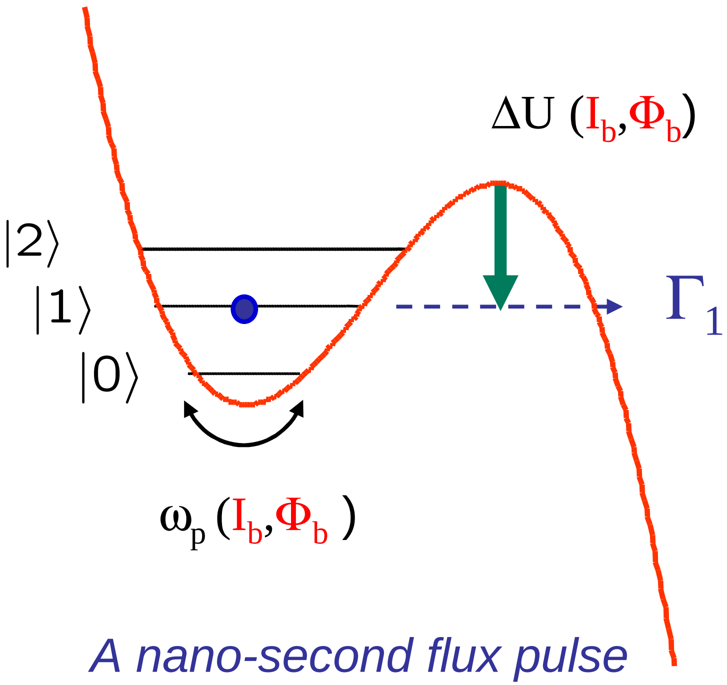
# Resonant coupling

A. Fay *et al.*, PRL 100, 187003 (2008)

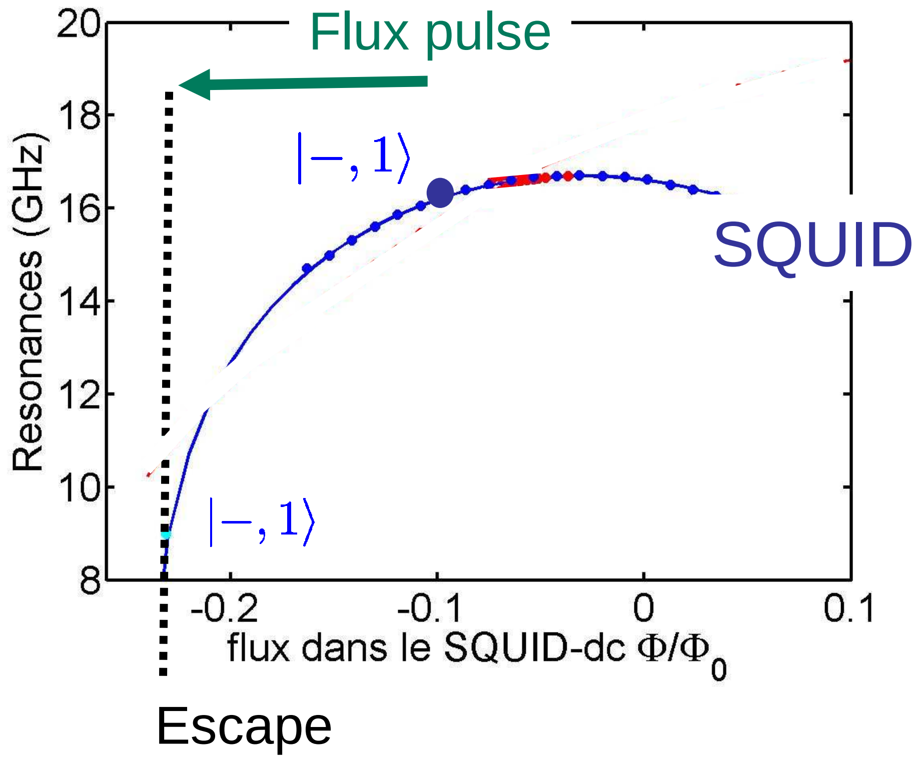


# Two qubits read-out

*Aurelien Fay Thesis*



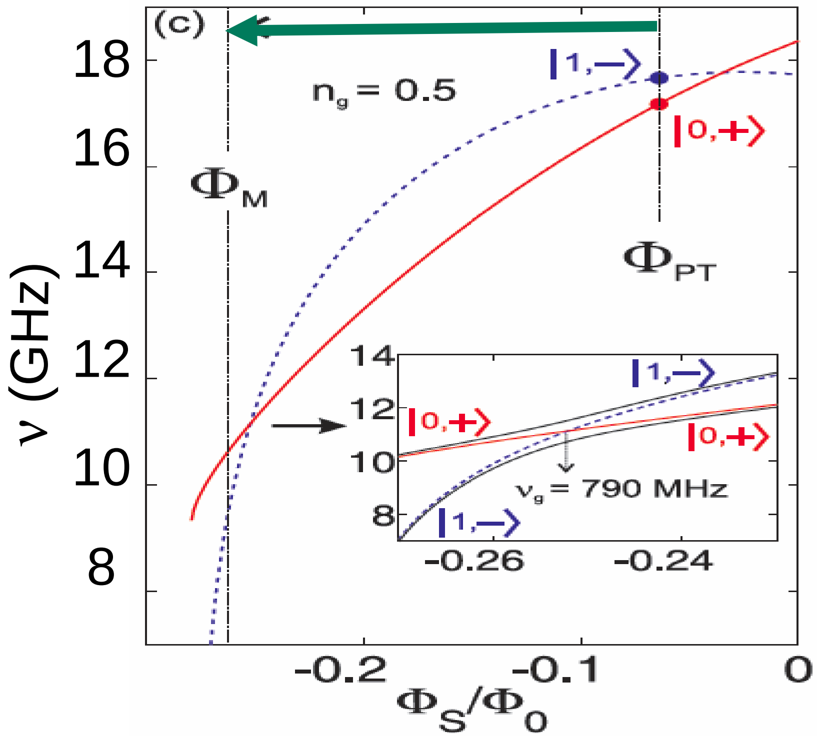

*A nano-second flux pulse reduces the barrier*

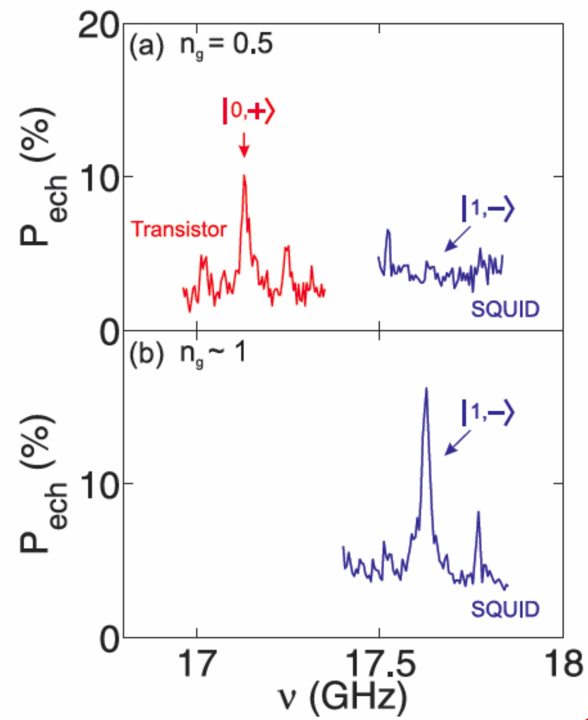



Quantronium read-out : classical Josephson junction $\omega_S << \omega_T$

In our case: $\omega_S \approx \omega_T$ !!!

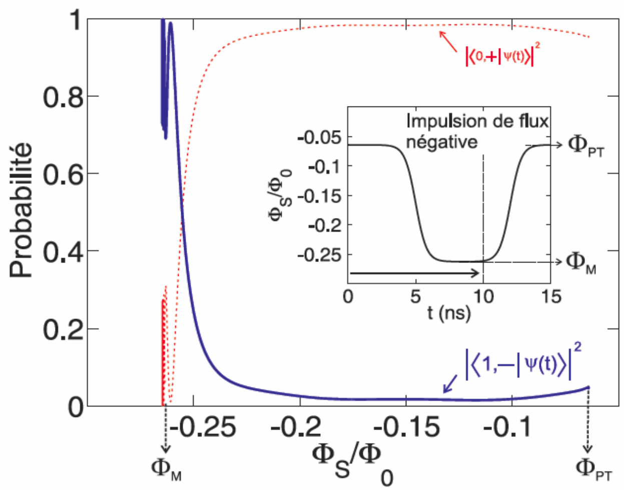
# Presence of an anti-level crossing




With anti-level crossing

SQUID alone




$\dot{\epsilon} \sim 2.9\ \mathrm{GHz/ns}$

$$P_{LZ} = e^{-2\pi \frac{g^2}{\hbar \dot{\epsilon}}} \approx 0\%$$

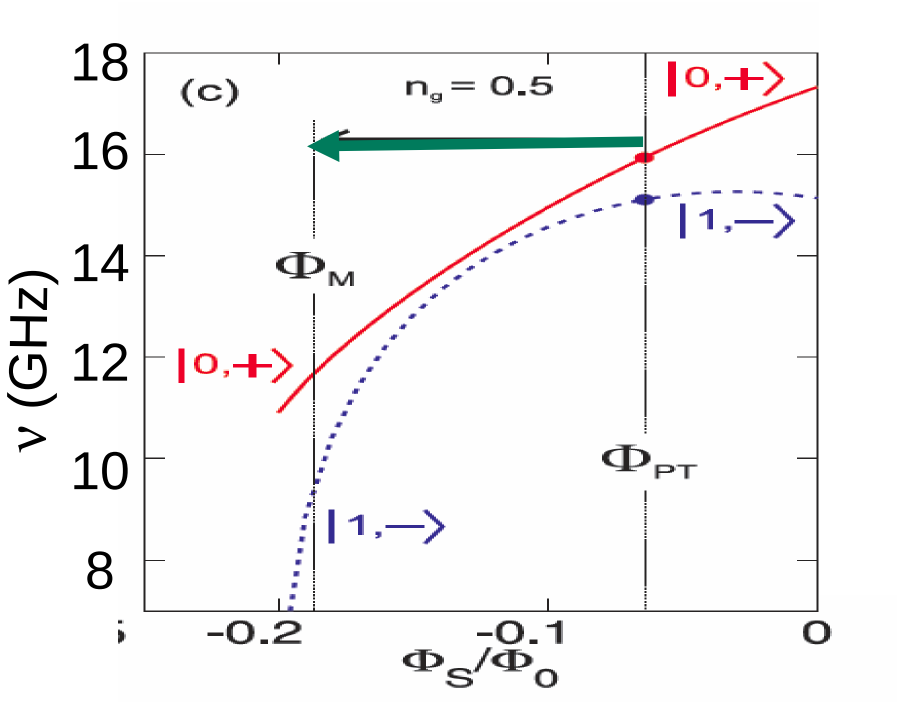
Very weak Landau-Zener transition

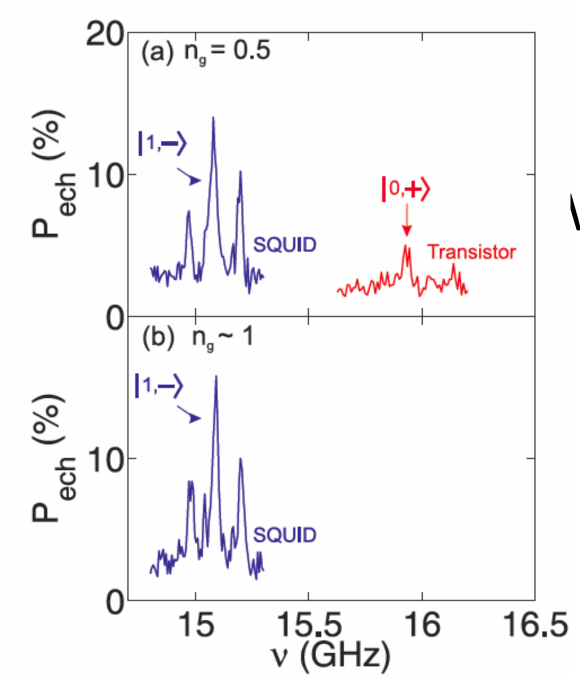
Adiabatic quantum transfer!

$$|+,0\rangle \longrightarrow |-,1\rangle$$

$$|-,1\rangle \longrightarrow |+,0\rangle$$

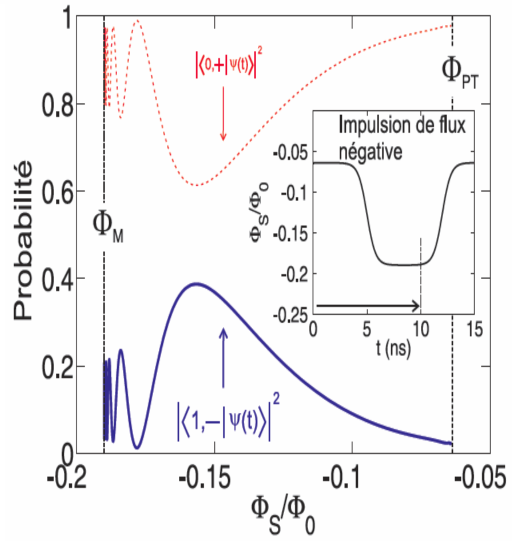
# Absence of an anti-level crossing




Without anti-level crossing

Squid alone




Very weak Landau-Zener transition

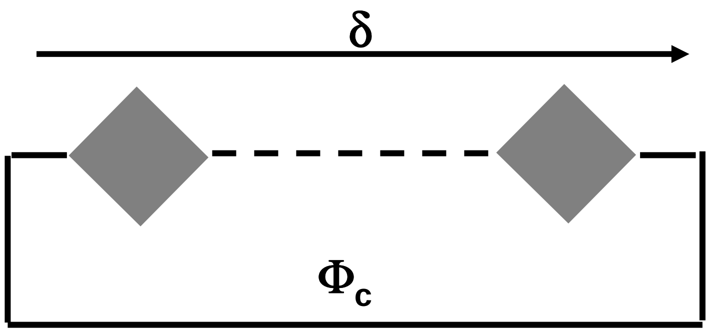
$$|+,0\rangle \longrightarrow |+,0\rangle$$

$$|-,1\rangle \longrightarrow |-,1\rangle$$

# Quantum dynamics in Josephson junction arrays

→ Candidate for the realisation of a topologically protected qubit

→ Dual of Shapiro steps in a Josephson junction array

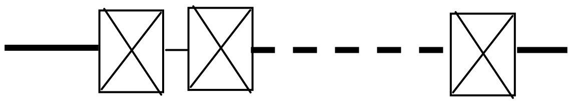



Phase bias and frustration:

$$f = \frac{\Phi_R}{2\pi\Phi_0},$$

$$\delta = \frac{\Phi_c}{\Phi_0}$$

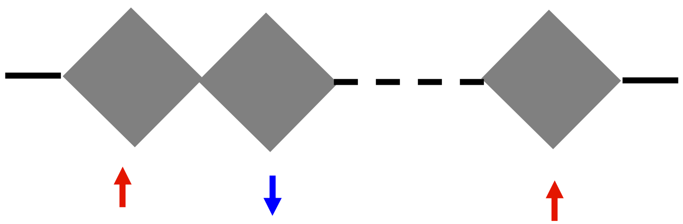
**f=0**



$$E_J^{eff}, E_C^{eff}$$

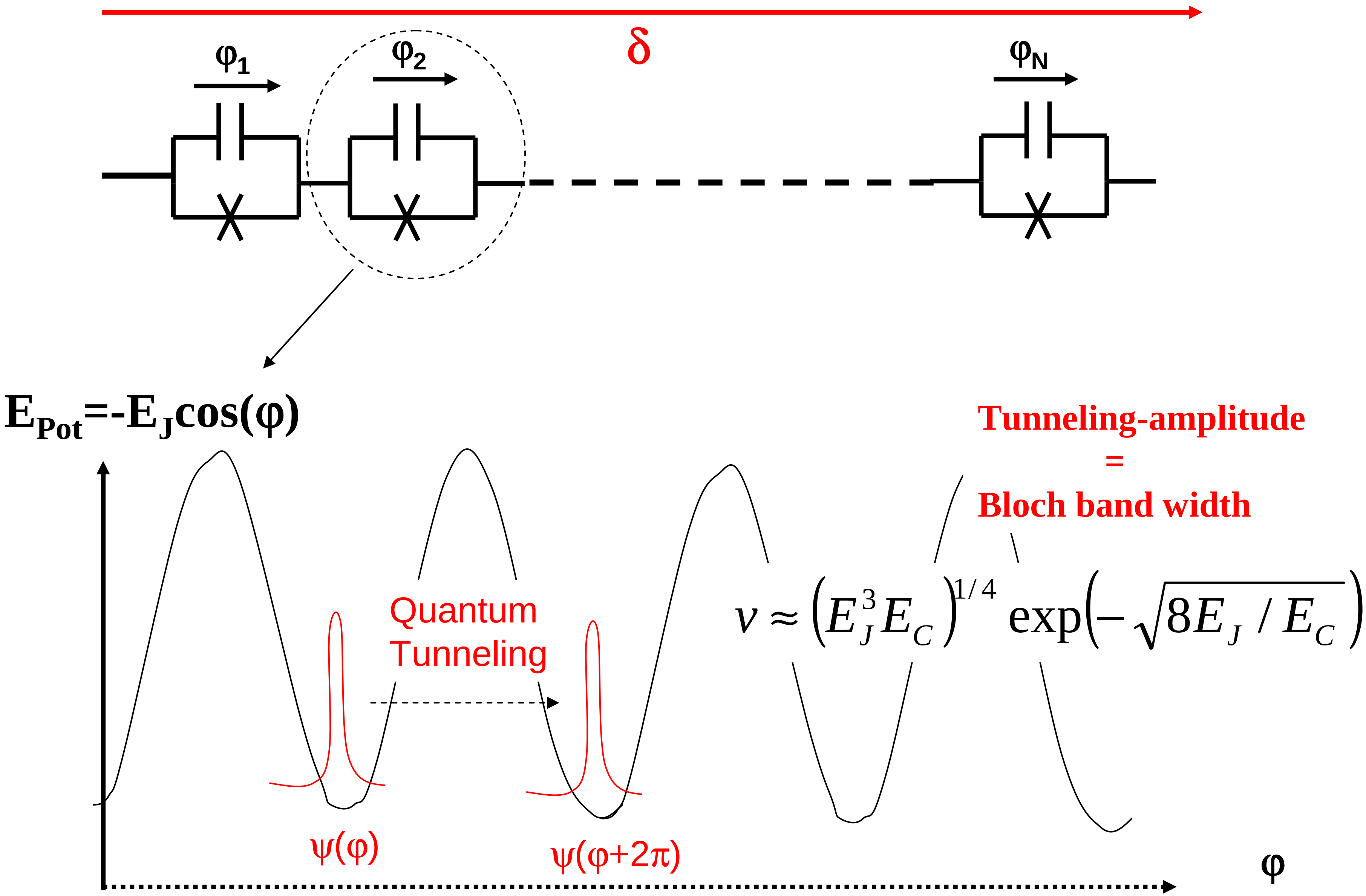
Chain of Josephson junctions with effective $E_J$ and $E_C$

**f=0.5**



Chain of N spins
$2^N$ possible states

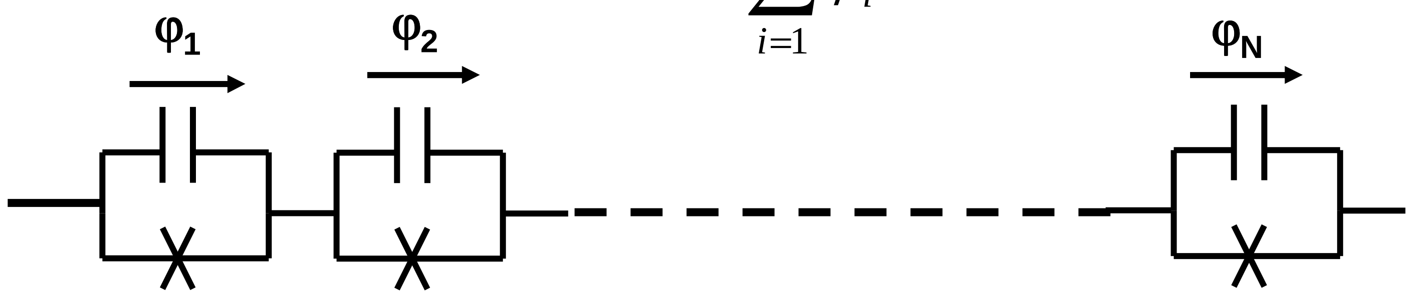
# Phase biased Josephson junction array



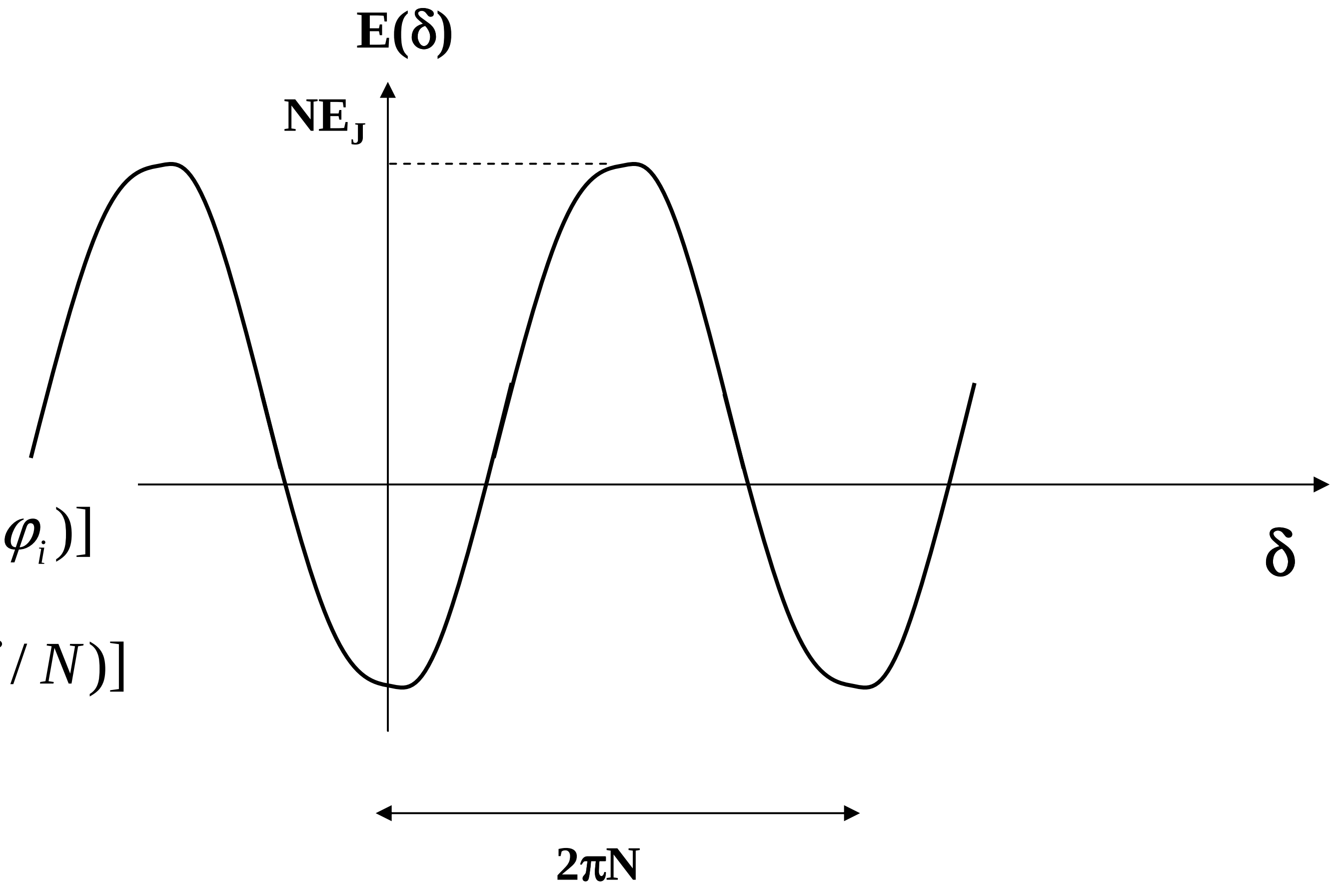

$$\nu \approx \left(E_J^3 E_C\right)^{1/4} \exp\left(-\sqrt{8E_J / E_C}\right)$$

# Chain of single Josephson junctions: classical regime $E_J >> E_C$

$$\delta = \sum_{i=1}^{N} \varphi_i$$




$$\varphi_i = \frac{\delta}{N}$$



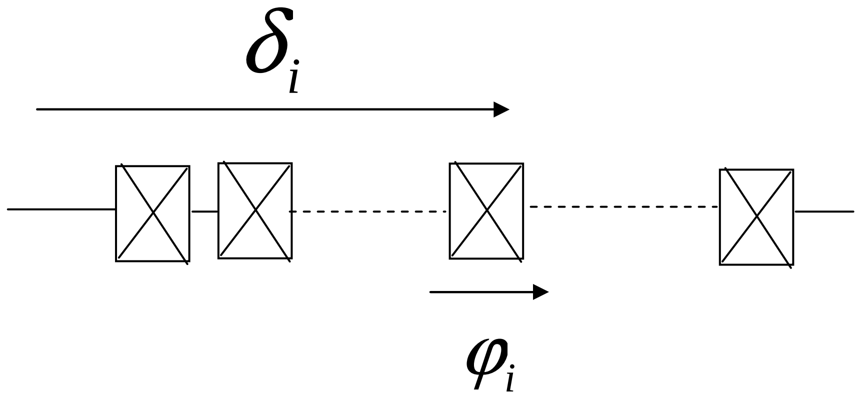

$$E_{pot} = \sum_i E_J[1-\cos(\varphi_i)]$$

$$= NE_J[1-\cos(\delta / N)]$$

$$= \frac{\mathrm{E_J}}{\mathrm{2N}}\delta^2$$

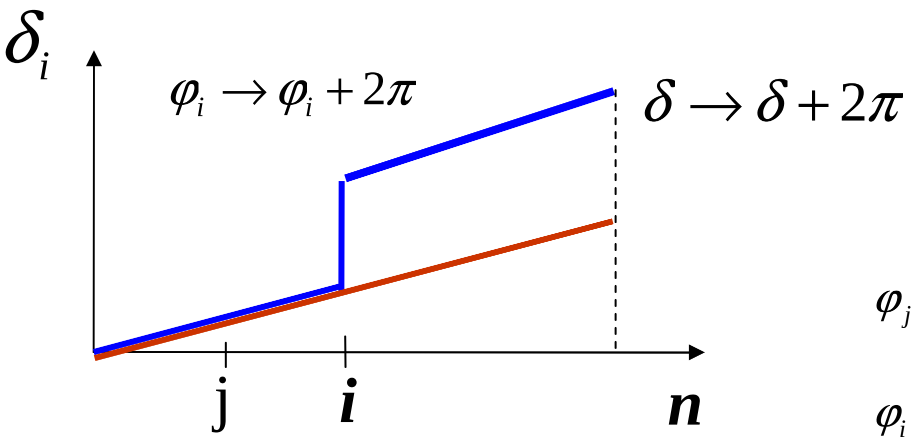
# Effect of phase slip in phase biased chain

**Phase biased chain:** $\delta = \sum_{i=1}^{N} \varphi_i$ **where** $\varphi_i = \frac{\delta}{N}$



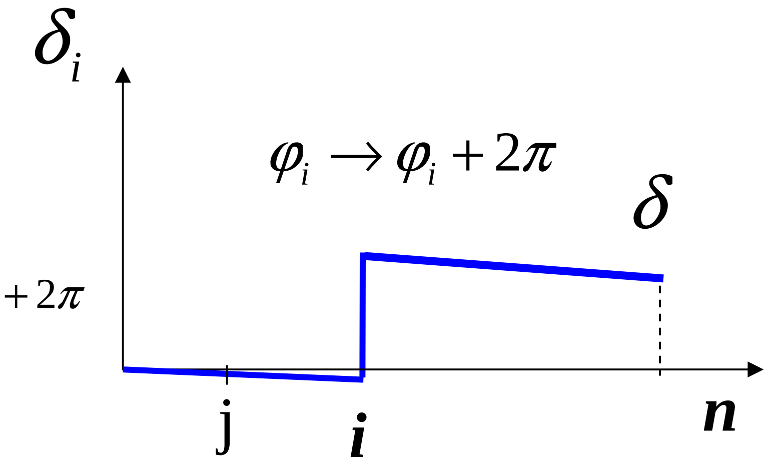

**Phase slip: energy unchanged but contraint violated !**

**Phase slip combined with small adjustments: constraint satisfied !**

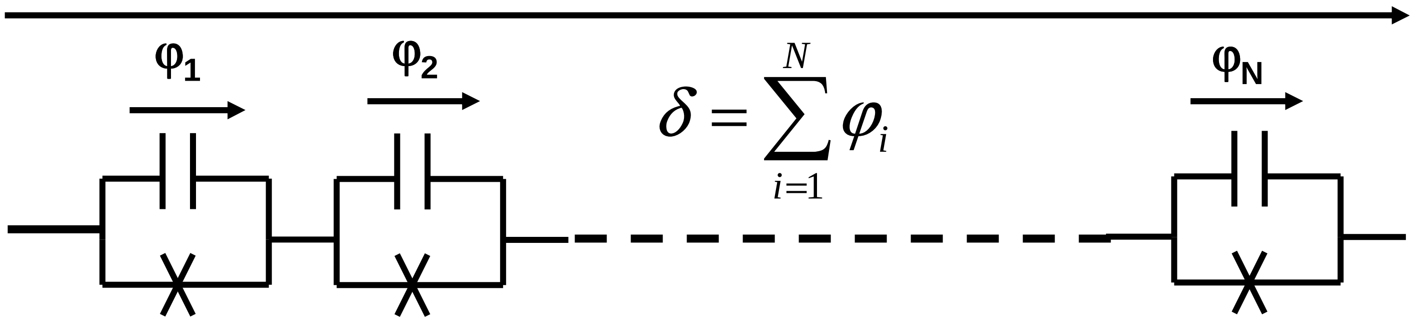



$$\varphi_j = \frac{\delta - 2\pi}{N} + 2\pi$$

$$\varphi_i = \frac{\delta - 2\pi}{N}$$



# Chain of single Josephson junctions: classical regime $E_J >> E_C$




$$\delta = \sum_{i=1}^{N} \varphi_i$$

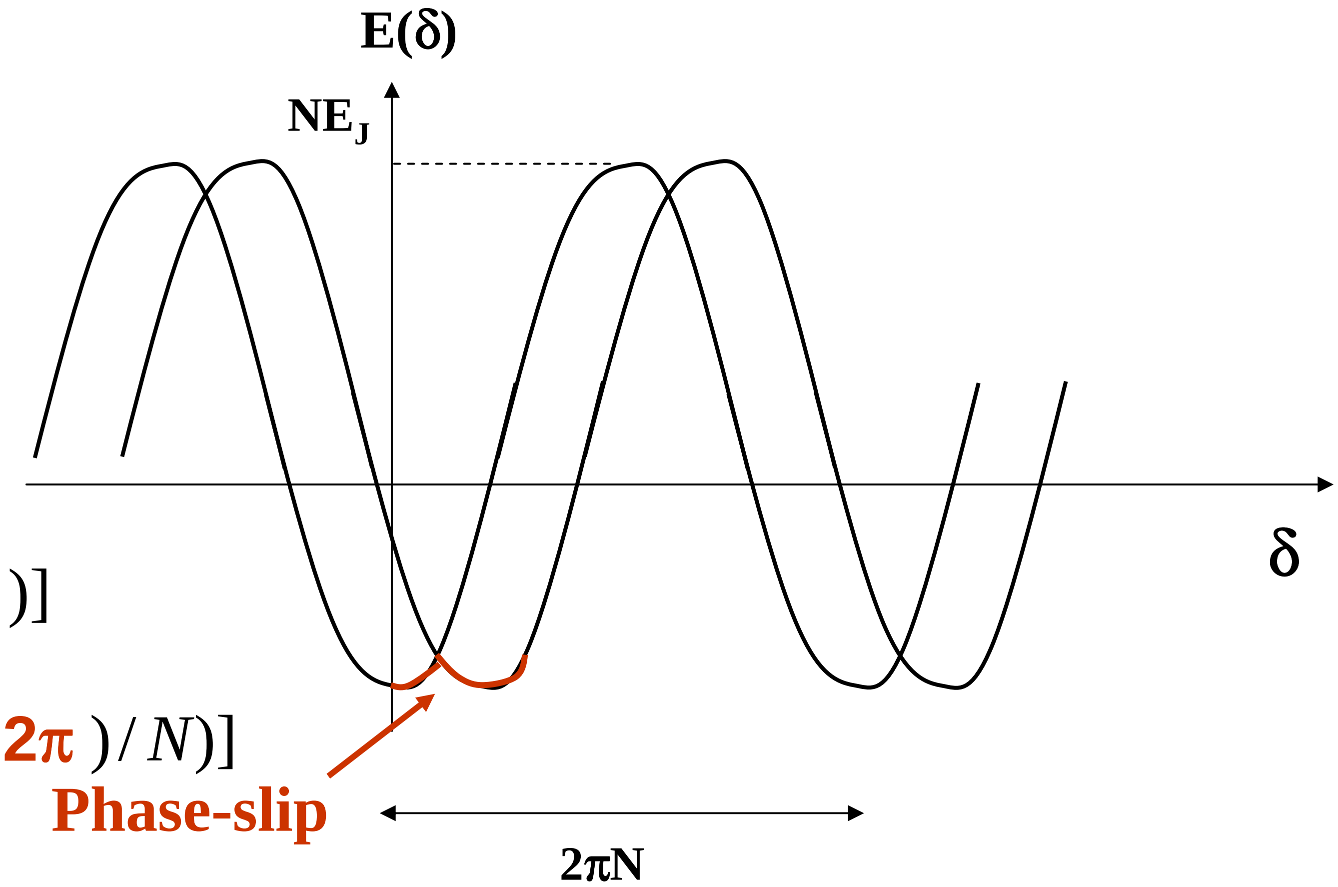
$$\varphi_i = \frac{\delta}{N}$$

$$\varphi_j = \frac{\delta - 2\pi}{N} + 2\pi$$

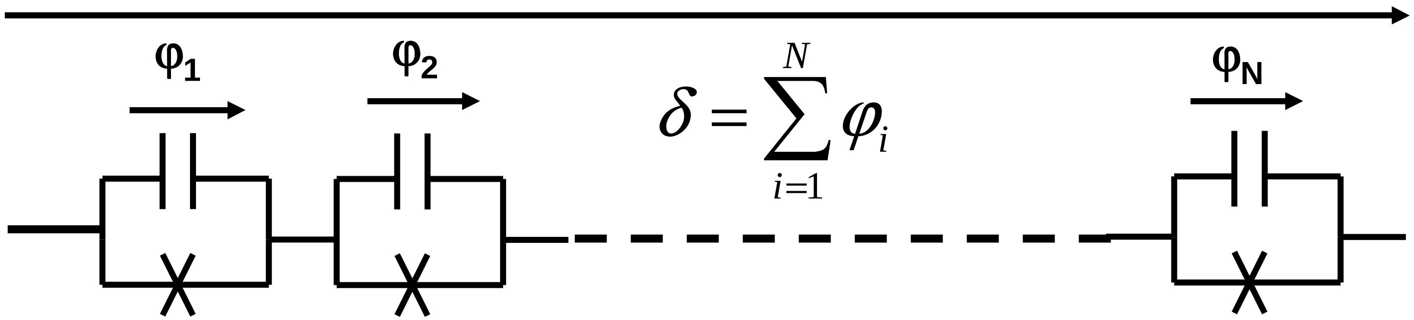
$$\varphi_i = \frac{\delta - 2\pi}{N}$$

$$E_{pot} = \sum_i E_J[1 - \cos(\varphi_i)]$$

$$= NE_J[1 - \cos((\delta \mathbf{-2\pi})/N)]$$

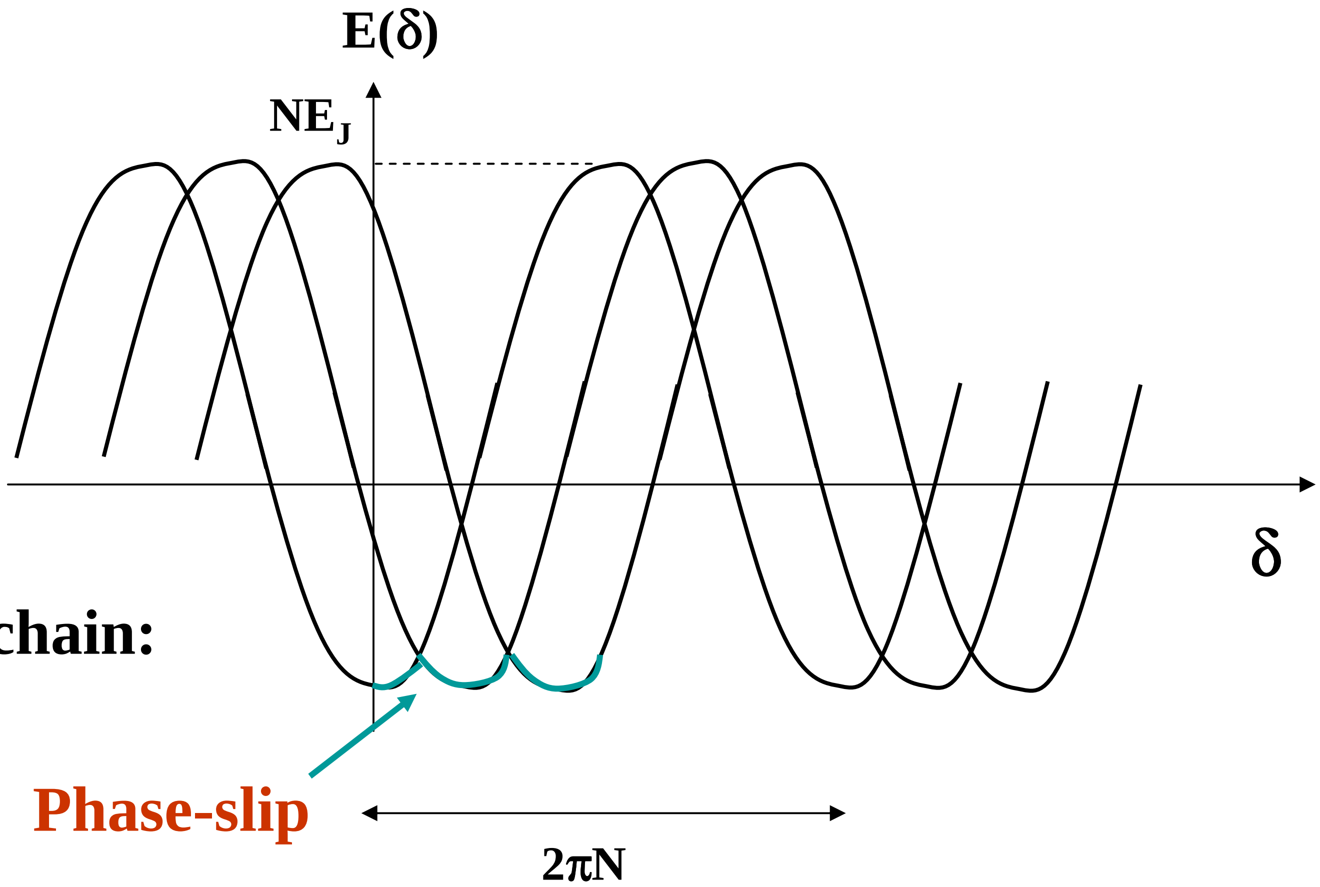


# Chain of single Josephson junctions: classical regime $E_J >> E_C$




$$\delta = \sum_{i=1}^{N} \varphi_i$$

$$\varphi_i = \frac{\delta}{N}$$



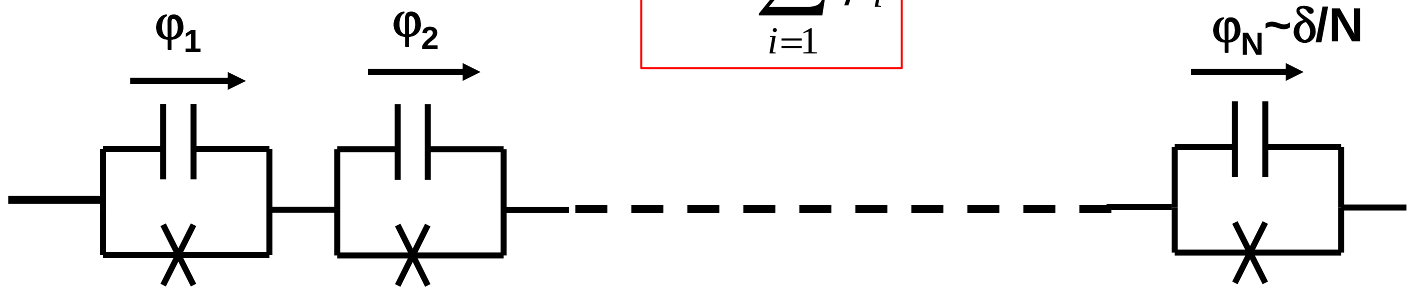

$$E_m \approx \frac{E_J}{2N}(\delta + 2\pi m)^2$$

**Critical current of chain:**

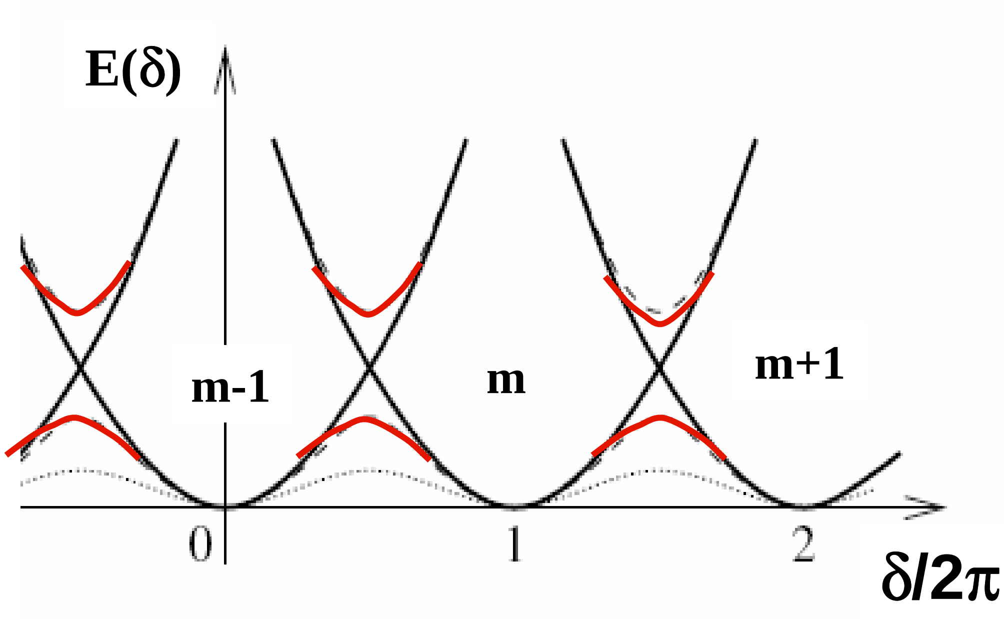
$$I_c \approx \frac{\pi I_c}{N}$$

# Phase biased Josephson junction array

$$\delta = \sum_{i=1}^{N} \varphi_i$$




$$H\psi_m = E_m \psi_m - N\nu(\psi_{m-1} + \psi_{m+1})$$




*Matveev et al (2002)*

$$E_m = \frac{E_J}{2N}\left(2\pi\hat{m} - \delta\right)^2$$

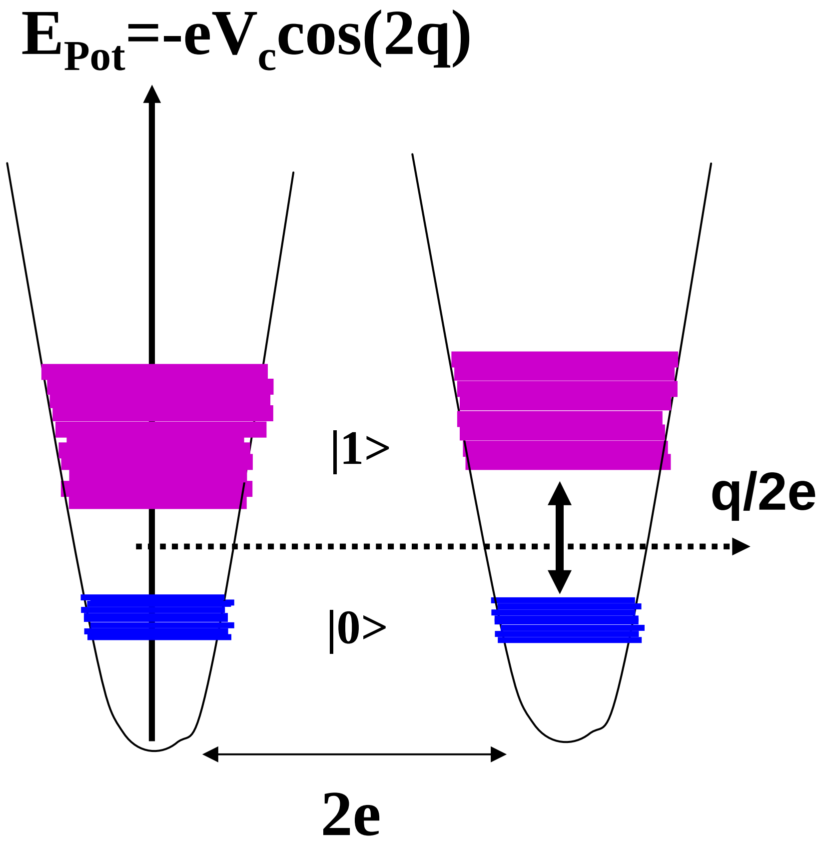
**Quantum variable**

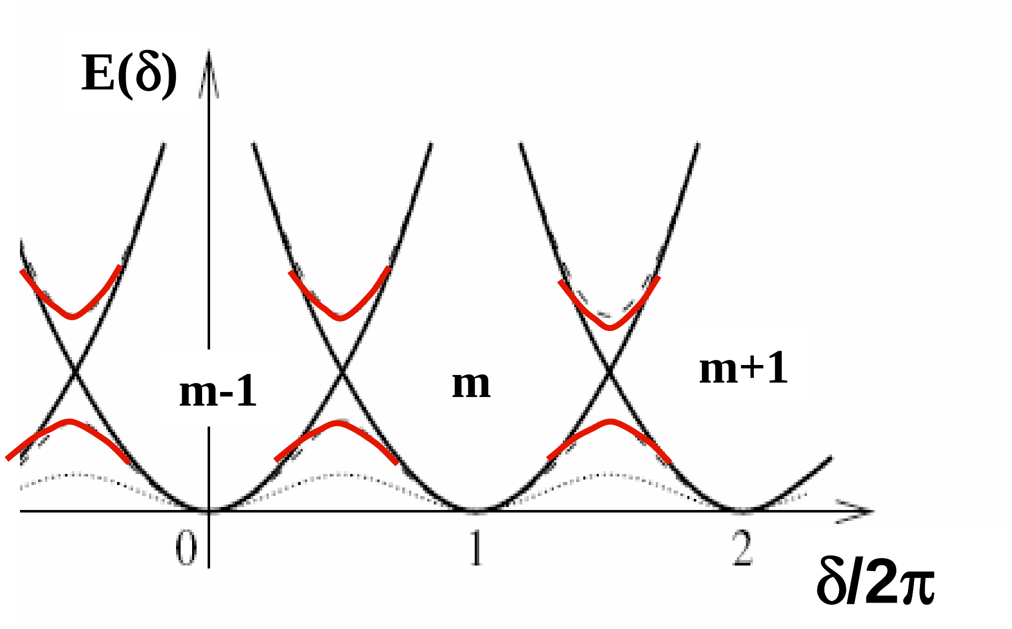
$$E_L = \frac{\pi^2}{2L}$$

$$L = \frac{N}{E_J}\left(\frac{\hbar}{2e}\right)^2$$

# Mathieu equation for Josephson array

$$\frac{d^2\psi(q)}{dq^2}+\left(\frac{E}{E_L}+\frac{2N\nu}{E_L}\cos 2q\right)\psi(q)=0$$



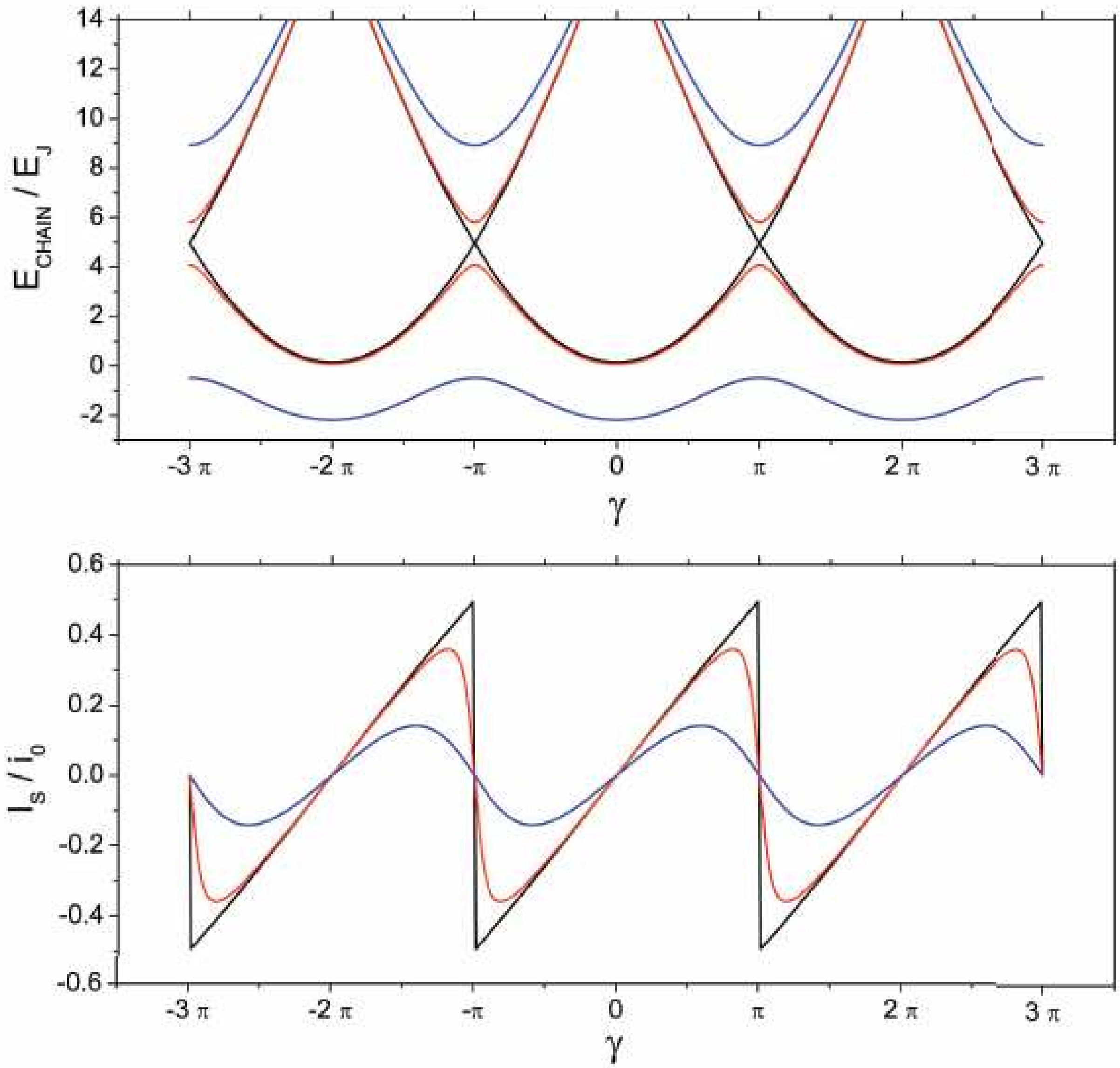

$$L\ddot{q}+R\dot{q}+V_c\sin(2\pi q/(2e))=V_{bias}$$

# Energy spectrum and current-phase relation of chain

$E_J/E_c$=20

$E_J/E_c$=3

$E_J/E_c$=1.3



# Sample set-up

**Sample caracteristics:**

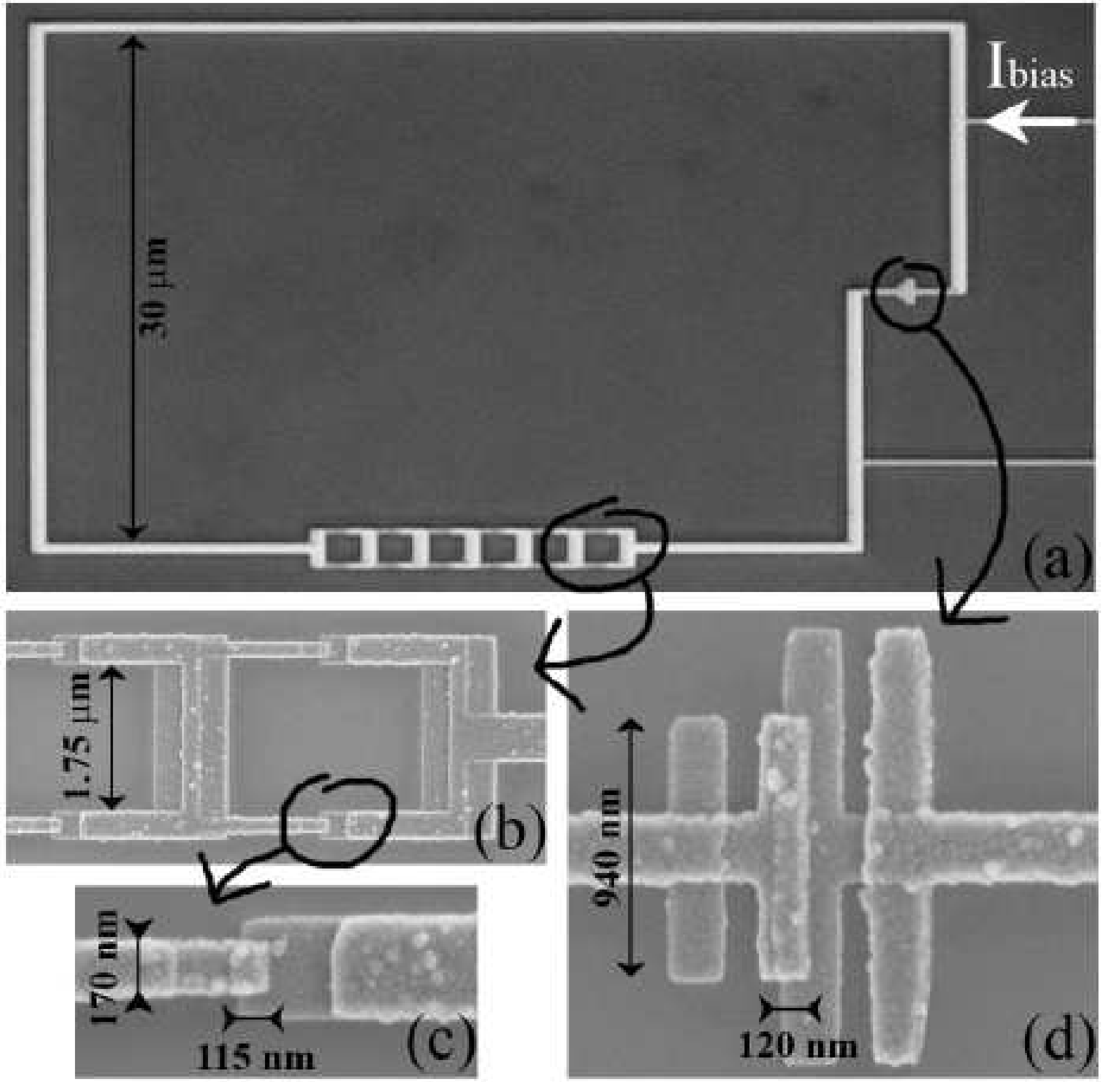
**Single junction in chain**

**$R_J$=6kΩ**
**$C_J$ =1.2fF**

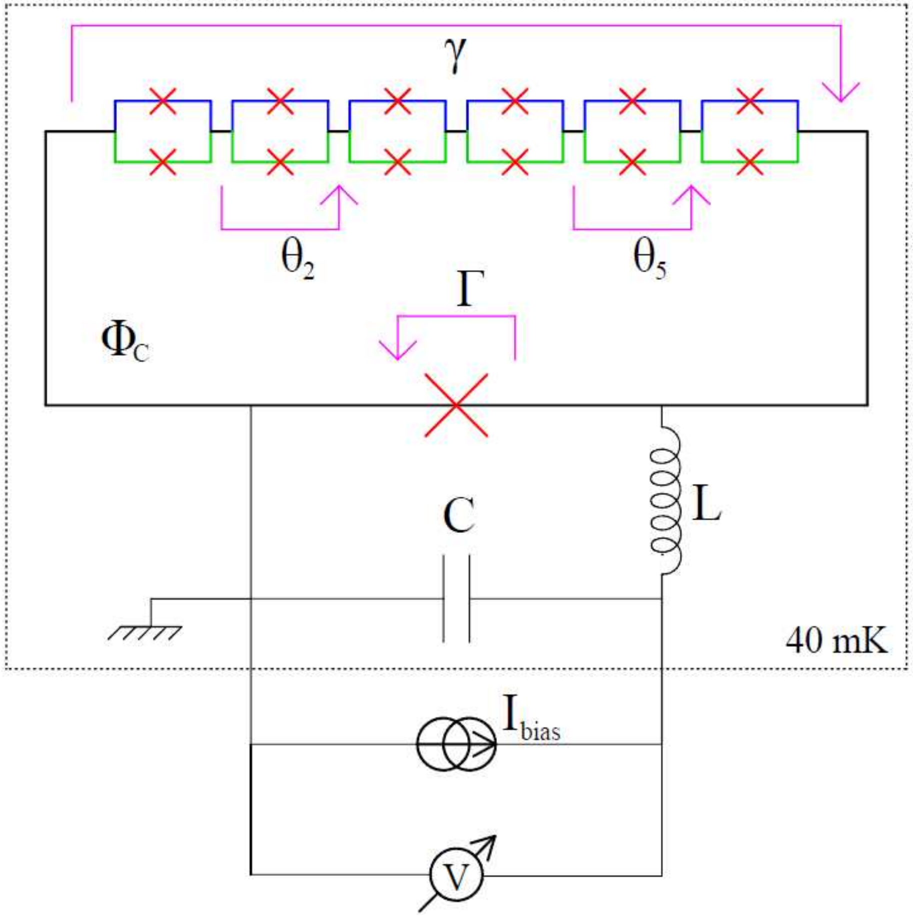
**Read-out Josephson junction:**

**R=1kΩ**

**Area ratio between big loop and SQUID loop: 285**



# Measurement of the current-phase relation I




$$I_{bias} = I_{chain} + I_J$$

$$I_{bias} = I_{chain}(\gamma) + I_C \sin(\Gamma)$$

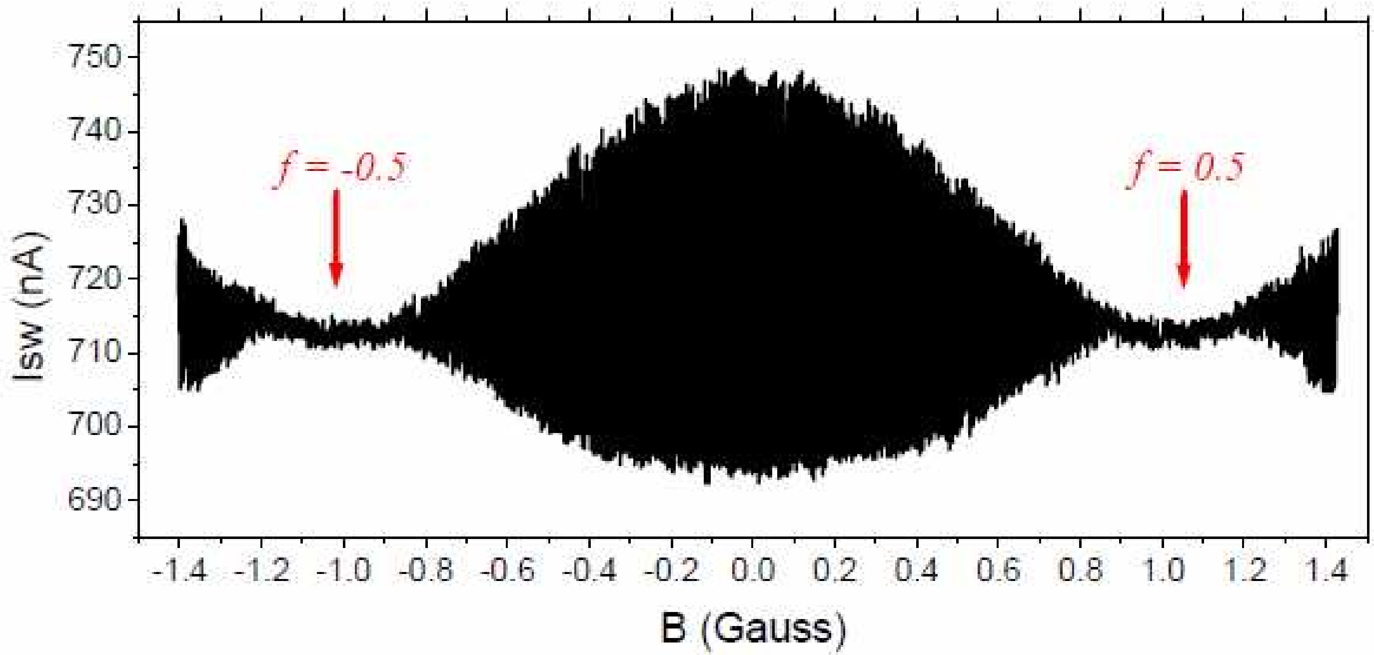
$$I_c^{Chain} << I_c^J$$

$$I_{bias} = I_{chain}\left(2\pi\frac{\Phi}{\Phi_0} - \frac{\pi}{2}\right) + I_c \sin\left(\frac{\pi}{2}\right)$$

**Current-phase relation yields information on the ground state**

$$I_S(\gamma) = \frac{\partial E_0}{\partial \gamma}$$

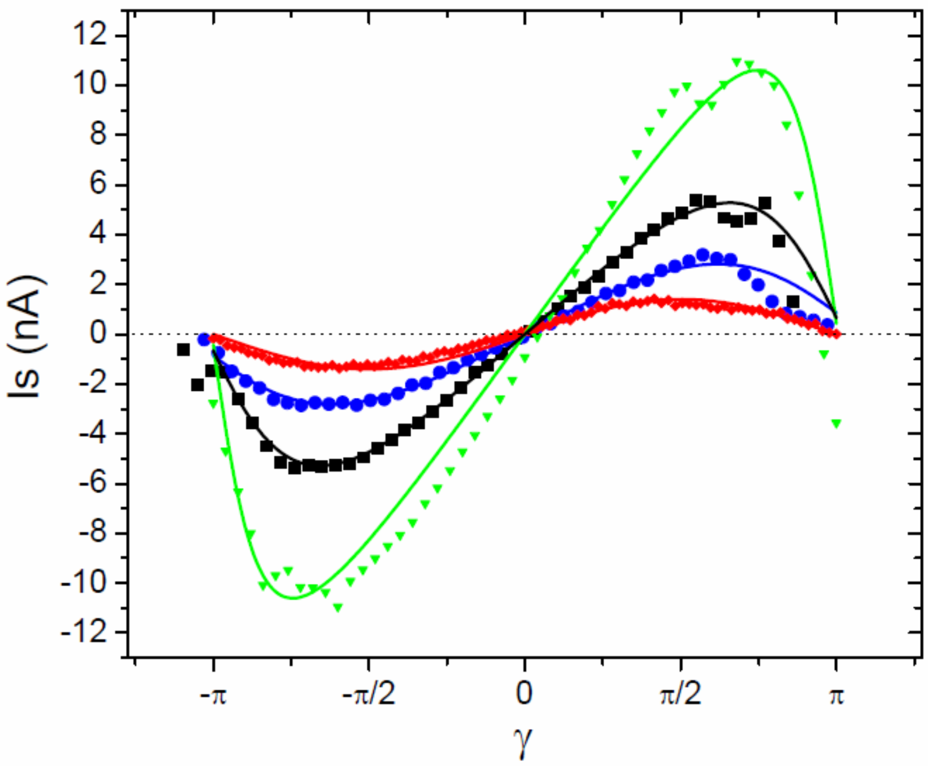
# Measurement of the current-phase relation II

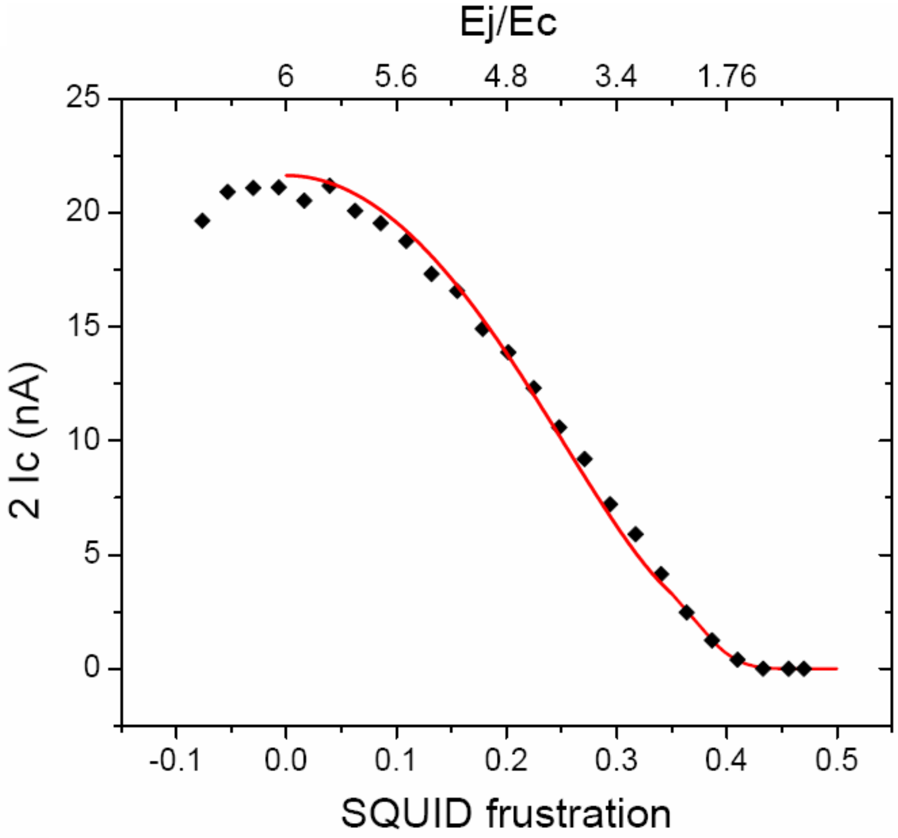



Large Josephson junction: $I_{sw}$=710nA

*PhD: Ioan Pop, to be published*

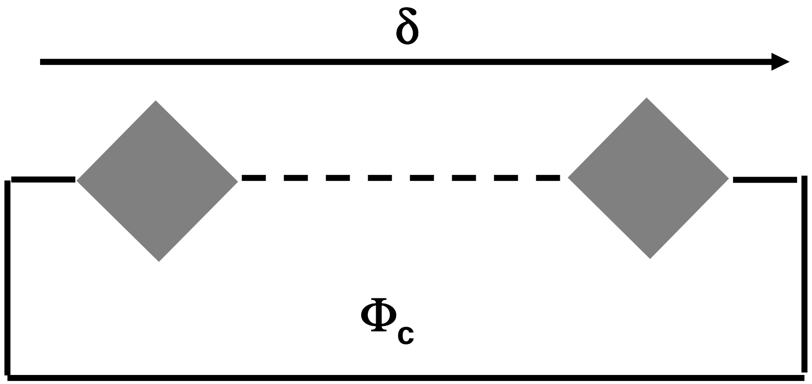
# Measurement of the current-phase relation III




*PhD: Ioan Pop, to be published*

# Josephson junction rhombi chain

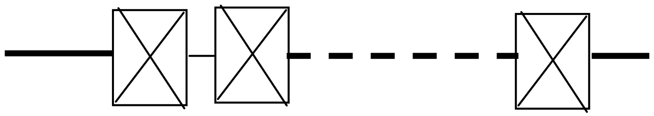



Phase bias and frustration:

$$f = \frac{\Phi_R}{2\pi\Phi_0},$$

$$\delta = \frac{\Phi_c}{\Phi_0}$$

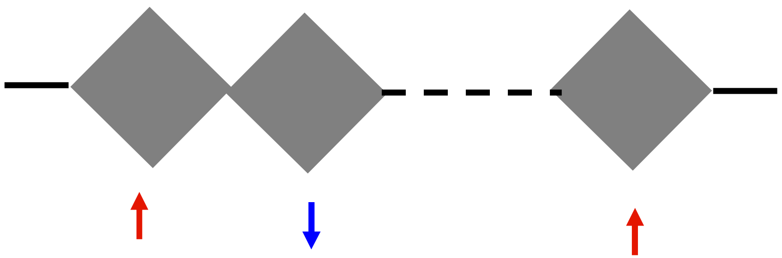
**f=0**



$$E_J^{eff}, E_C^{eff}$$

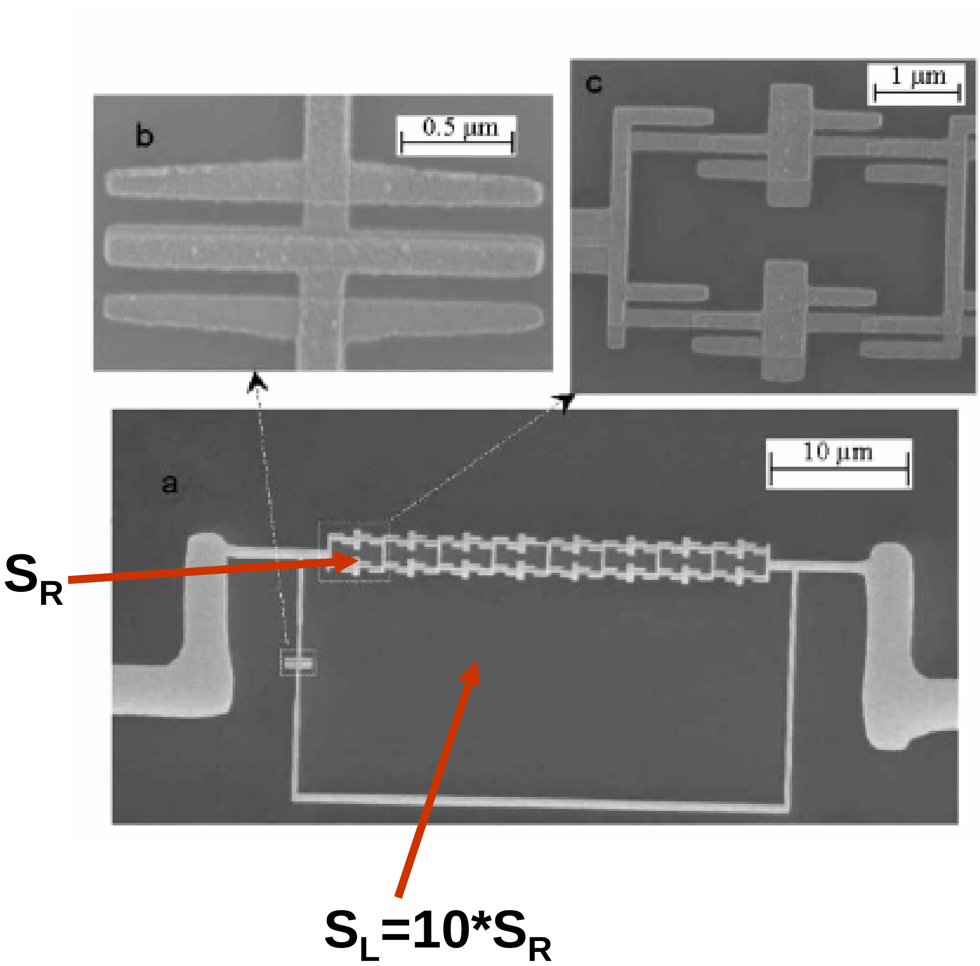
Chain of Josephson junctions
with effective $E_J$ and $E_C$

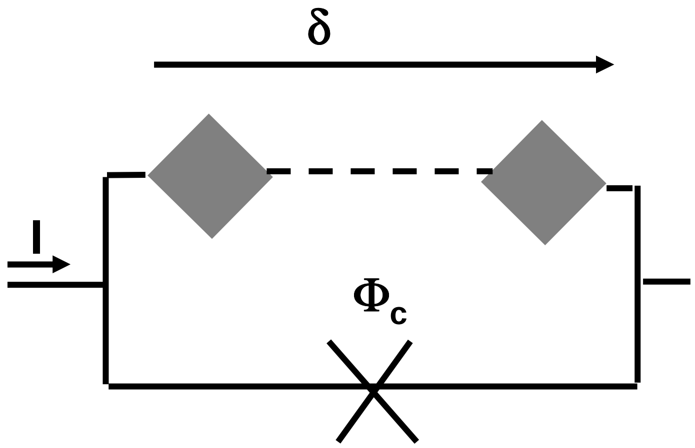
**f=0.5**



Chain of N spins
$2^N$ possible states

# Measurement of the current-phase relation




$$I_c^{Chain} << I_c^{J}$$

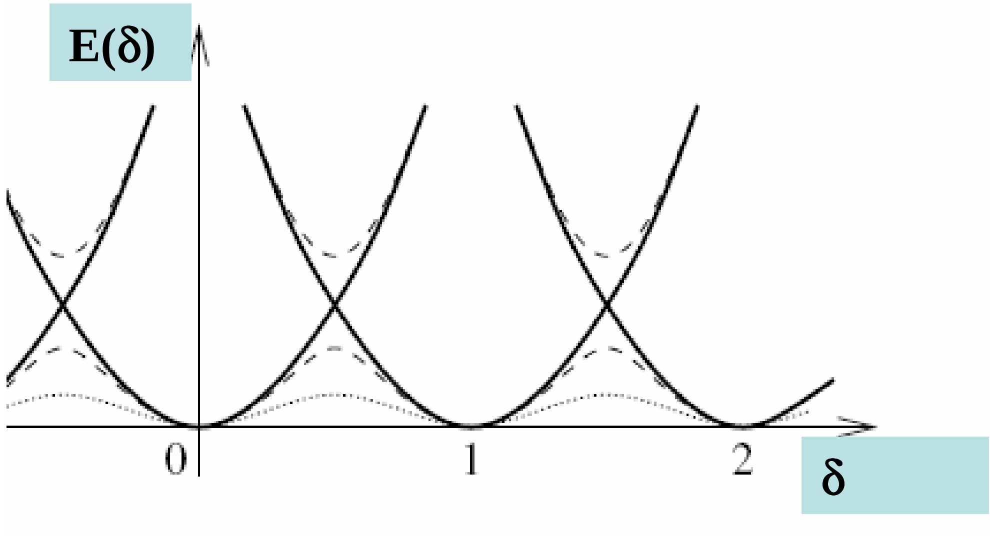
Current-phase relation yields information on the ground state

$$I_S = \frac{\partial E_0}{\partial \delta}$$

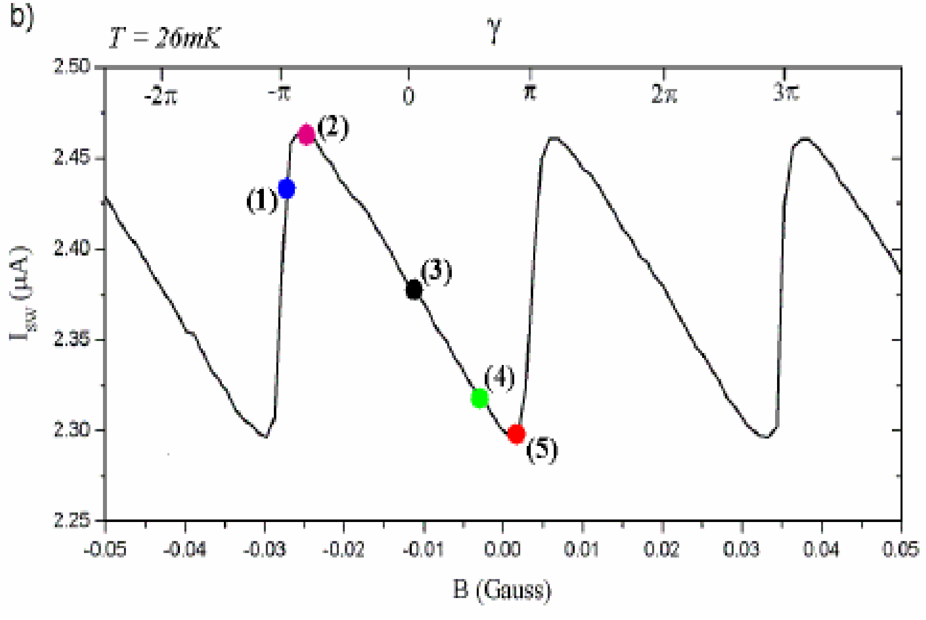
*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

# Current-phase relation at f=0: classical regime

$E_J/E_C \sim 20$



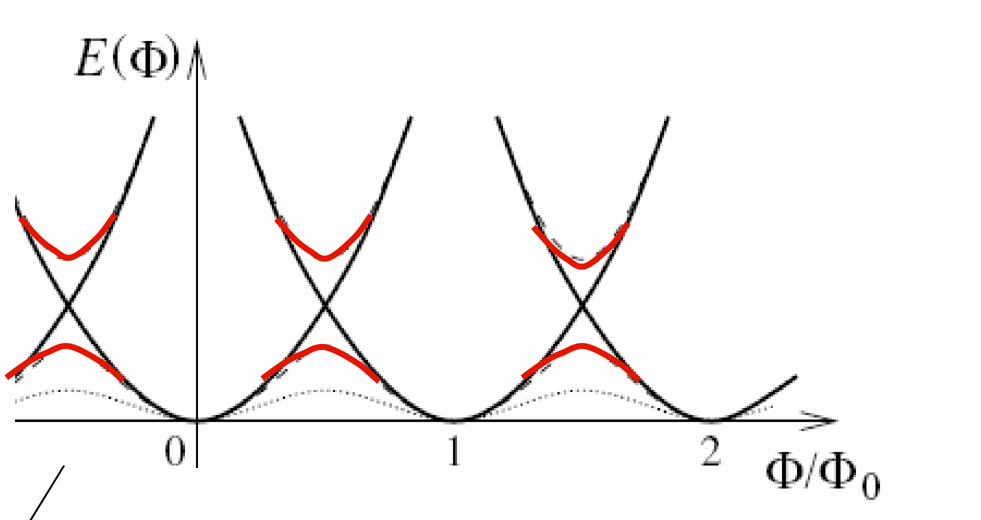

$$I_S = \frac{\partial E_0}{\partial \delta}$$



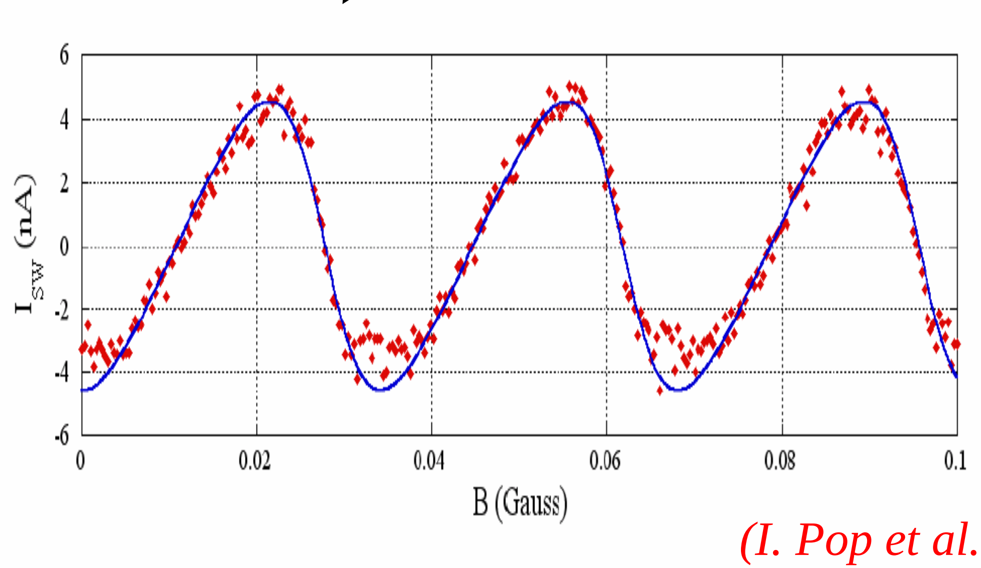

*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

# Current-phase relation at f=0: quantum regime

$E_J/E_C \sim 2$




$$I_S = \frac{\partial E_0}{\partial \delta}$$




$$H\psi_m = E_m\psi_m - N\upsilon(\psi_{m-1} + \psi_{m+1})$$

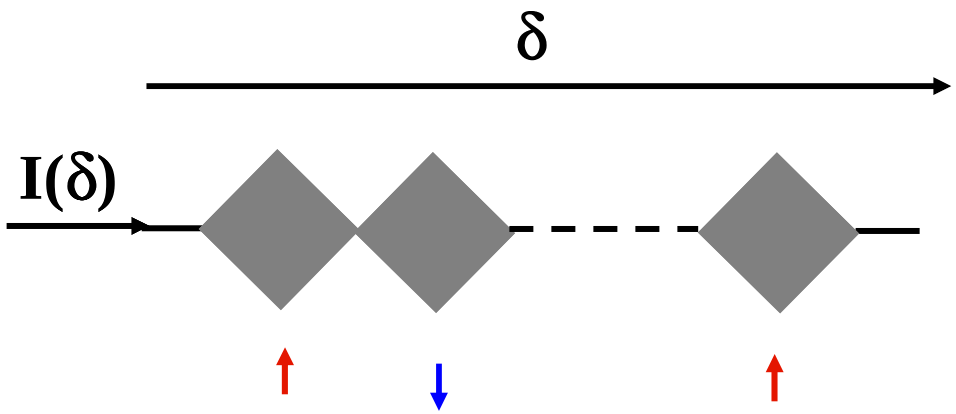
$$\upsilon_{\text{Rhombus}} = A\exp(-S_0)$$

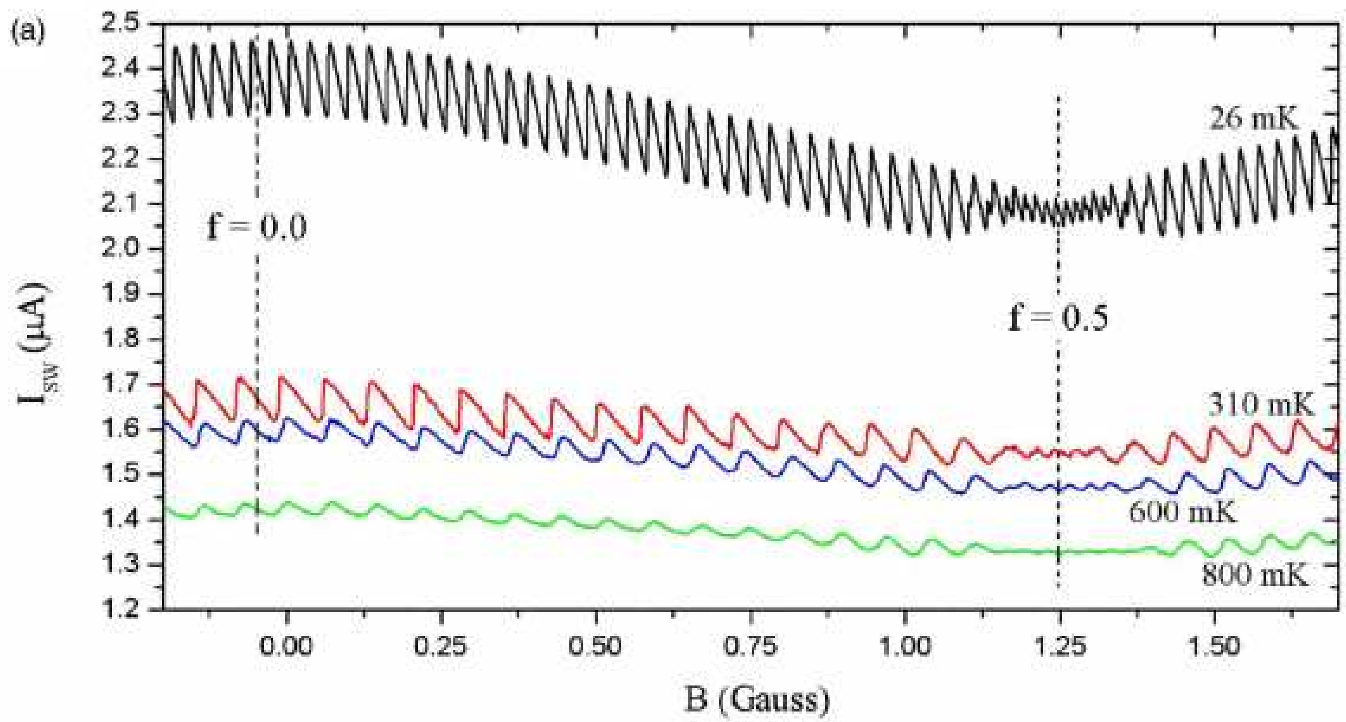
$$S_0 = 2\sqrt{\frac{8E_J}{E_c}}$$

$$A \approx 4.50(E_J^3 E_C)^{1/4}$$

*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

# Current phase relation at f=0.5

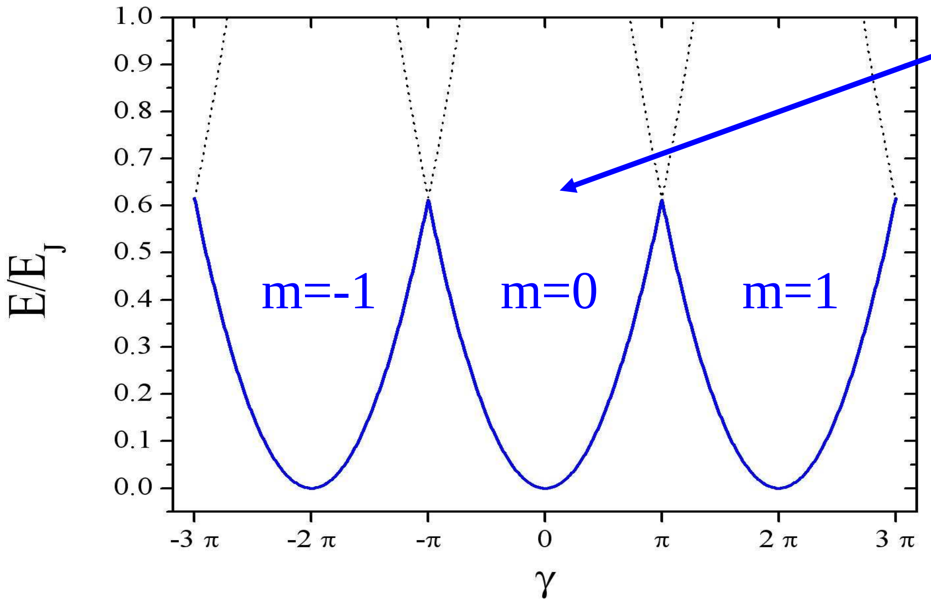


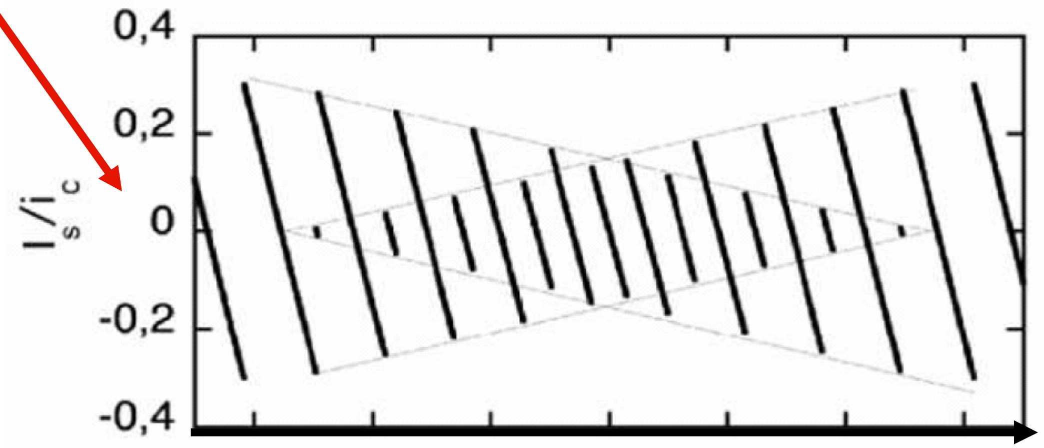
# Measurement of the ground state energy in the classical regime (1)

f=0

| ↓↓↓↓↓↓↓↓>




Phase slips across Rhombi




f

*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

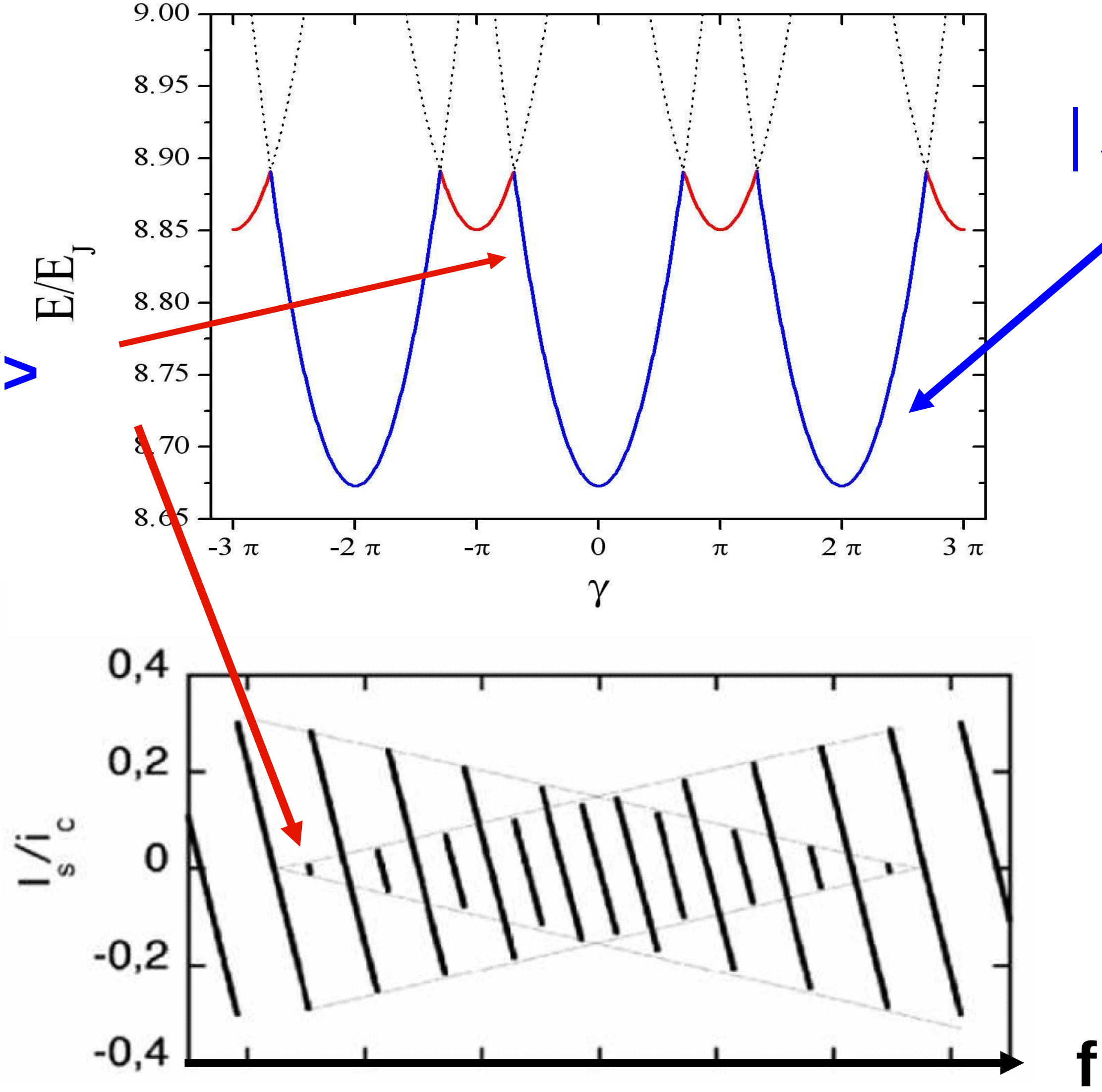
**0.5**

# Measurement of the ground state energy in the classical regime (2)

f=0.48

|↓↓↓↓↓↓↓↓>

|↓↓↓↓↓↓↓↑>

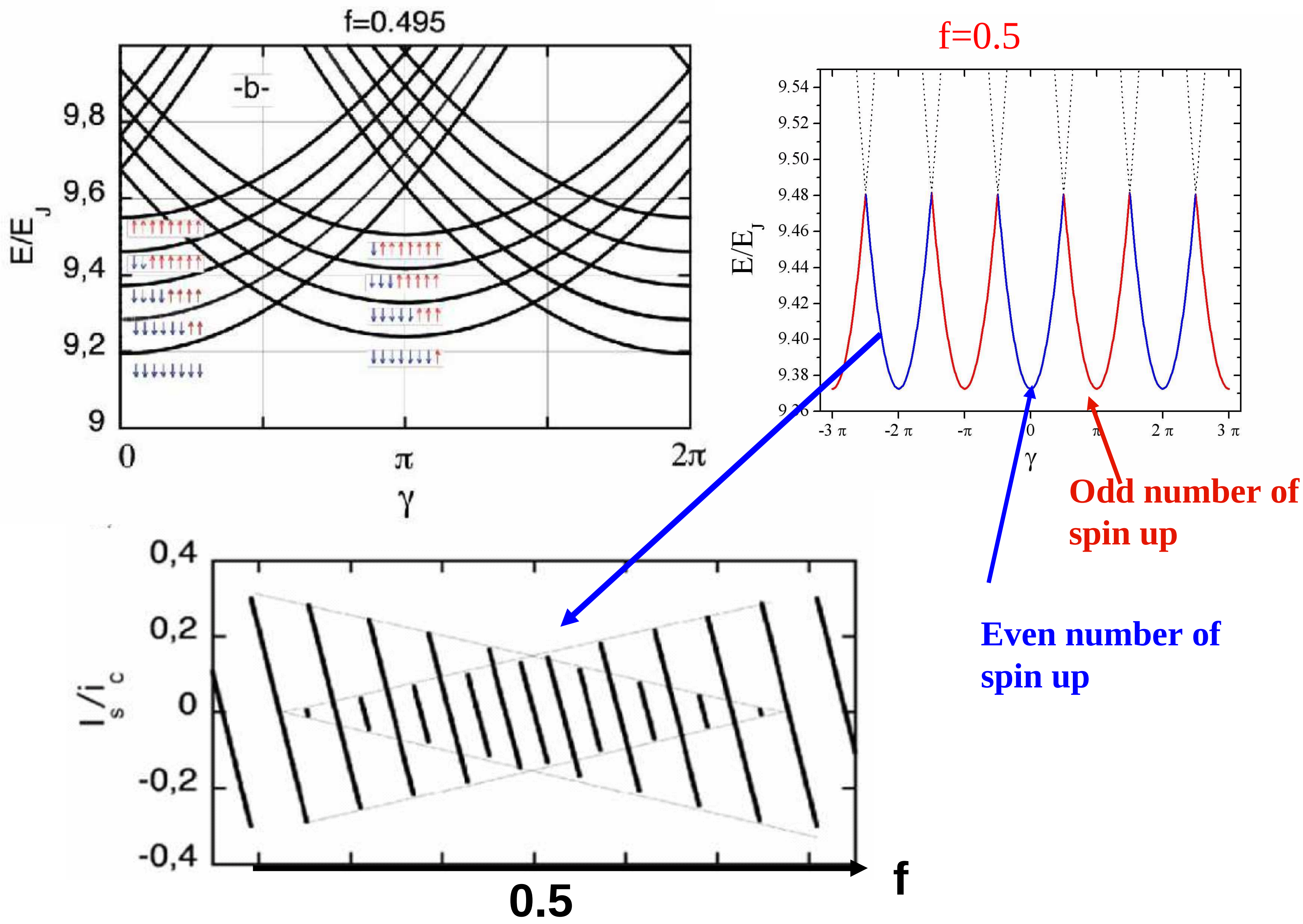



*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

**0.5**

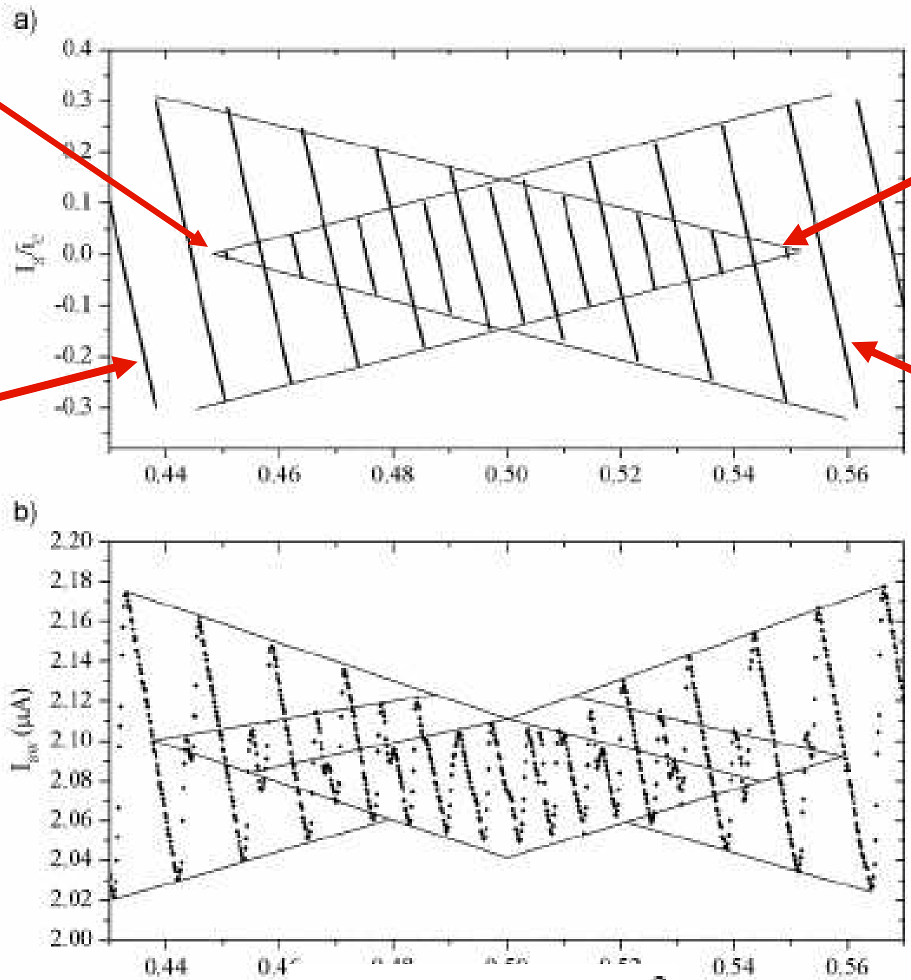
# Measurement of the ground state energy in the classical regime (3)

*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*



# Measurement of the ground state energy in the classical regime (4)

*(I. Pop et al.,Phys. Rev. B, 78, 104504 (2008))*

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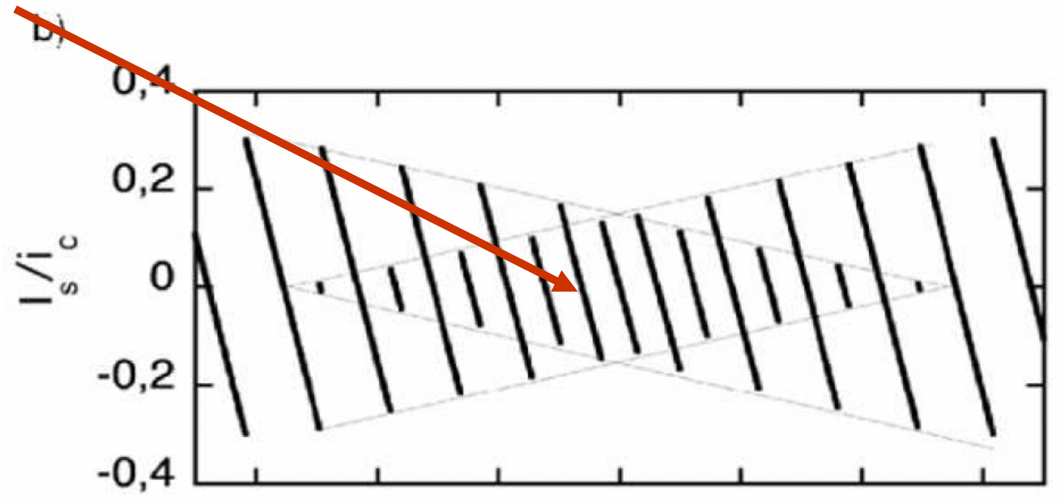
| ↓↓↓↓↓↓↓↓>

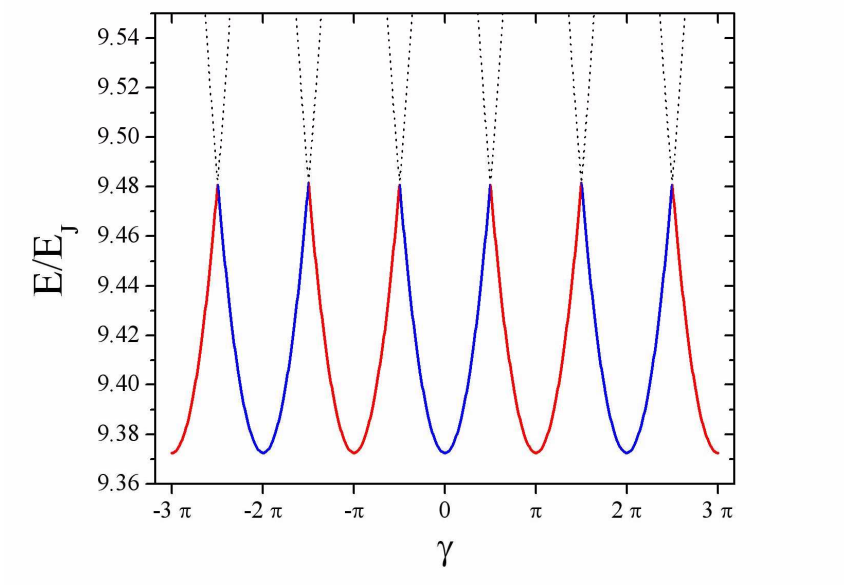
| ↑↑↑↑↑↑↑↑>

$$1 - \tan\frac{\pi f}{2} < \frac{\pi^2}{8N}$$

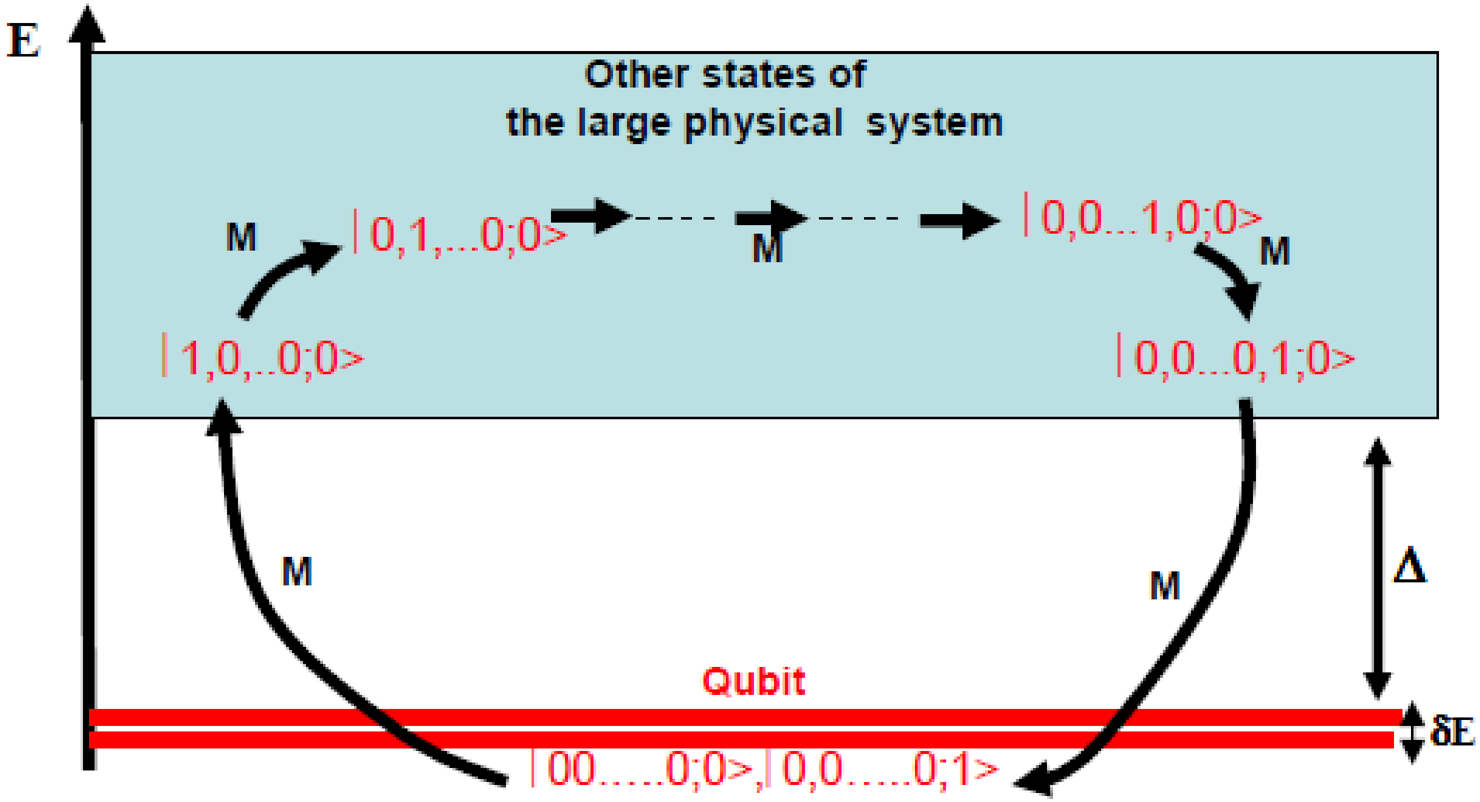
# Towards a topologically protected qubit ?

**Add quantum fluctuations at f=0.5 in order to lift the degeneracy of the states**



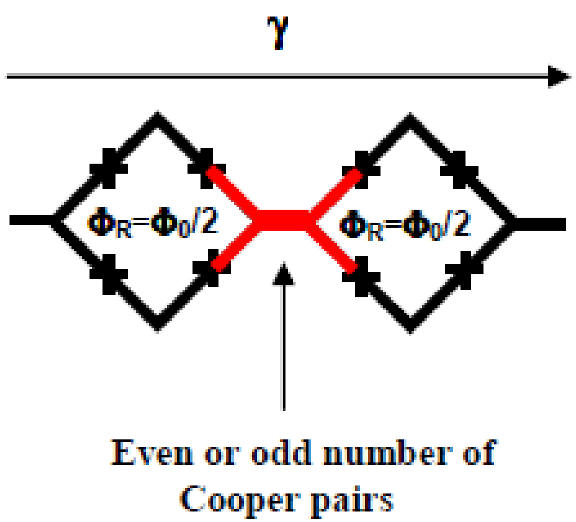
# Idea of topologically protected qubit

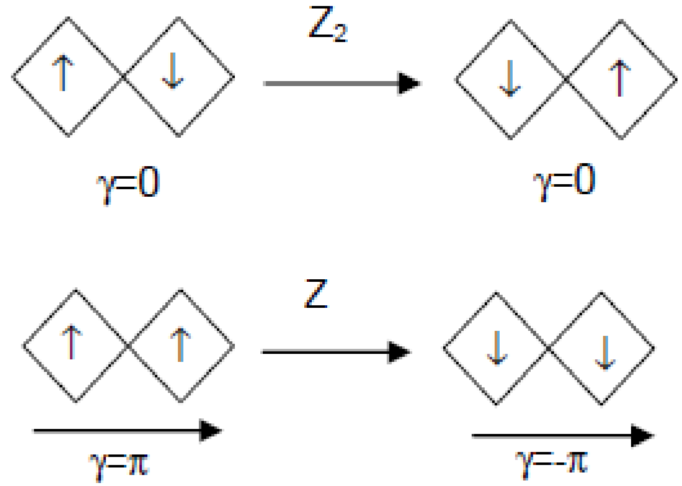


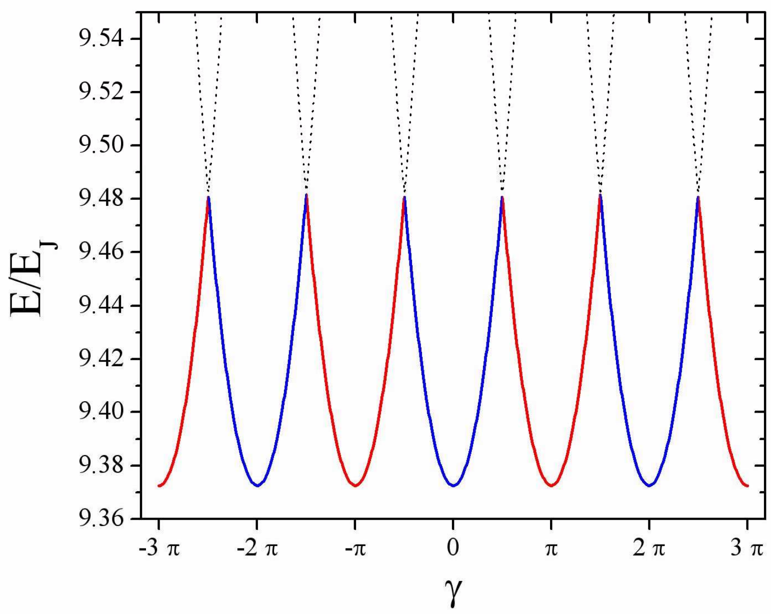

$$\delta E \approx \frac{[M]^N}{\Delta^{N-1}}$$

*Kitaev et al (2003)*

# Energy spectrum of a rhombi chain at half flux frustration I



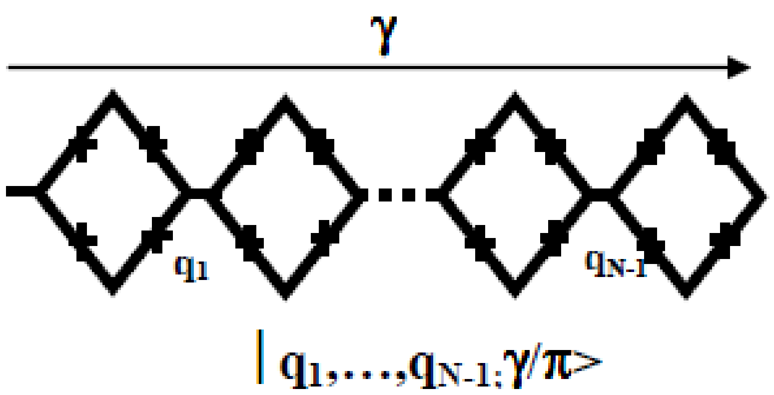

$$\left|\uparrow\uparrow\right\rangle + \left|\downarrow\downarrow\right\rangle \longrightarrow \left|0;\gamma=0\right\rangle$$

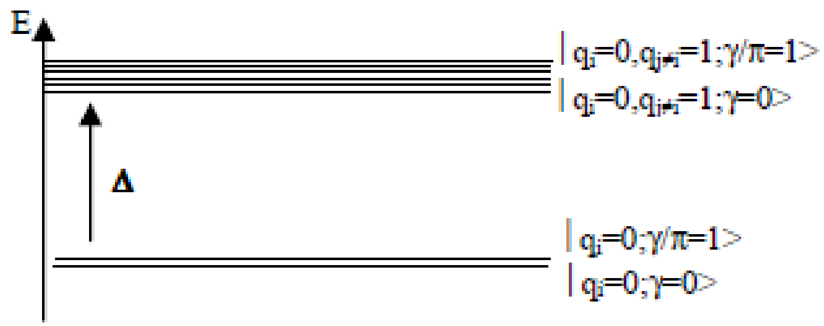
$$\left|\uparrow\uparrow\right\rangle - \left|\downarrow\downarrow\right\rangle \longrightarrow \left|1;\gamma=0\right\rangle$$

$$\left|\uparrow\downarrow\right\rangle + \left|\uparrow\downarrow\right\rangle \longrightarrow \left|0;\gamma=\pi\right\rangle$$

$$\left|\uparrow\downarrow\right\rangle - \left|\uparrow\downarrow\right\rangle \longrightarrow \left|1;\gamma=\pi\right\rangle$$

# Energy spectrum of a rhombi chain at half flux frustration II



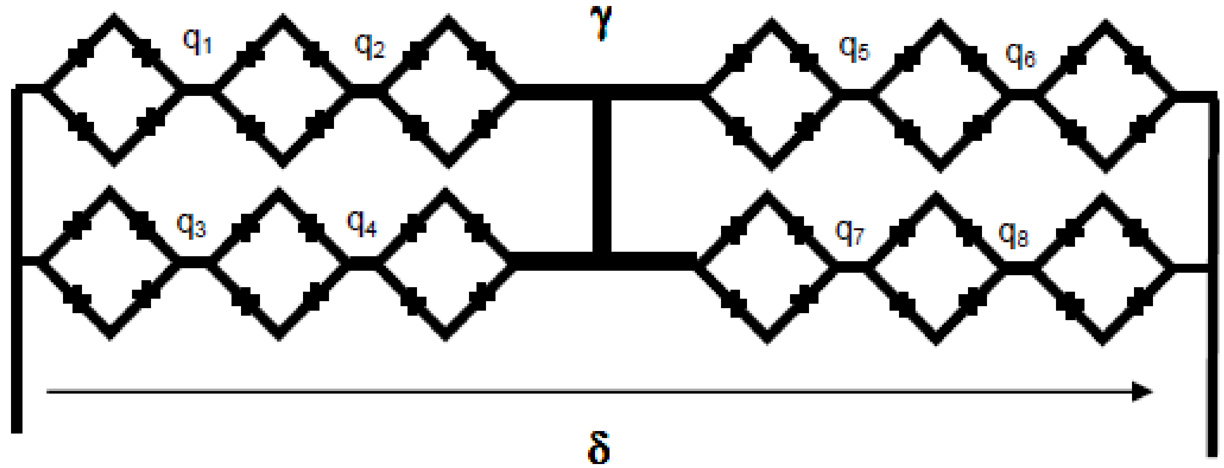



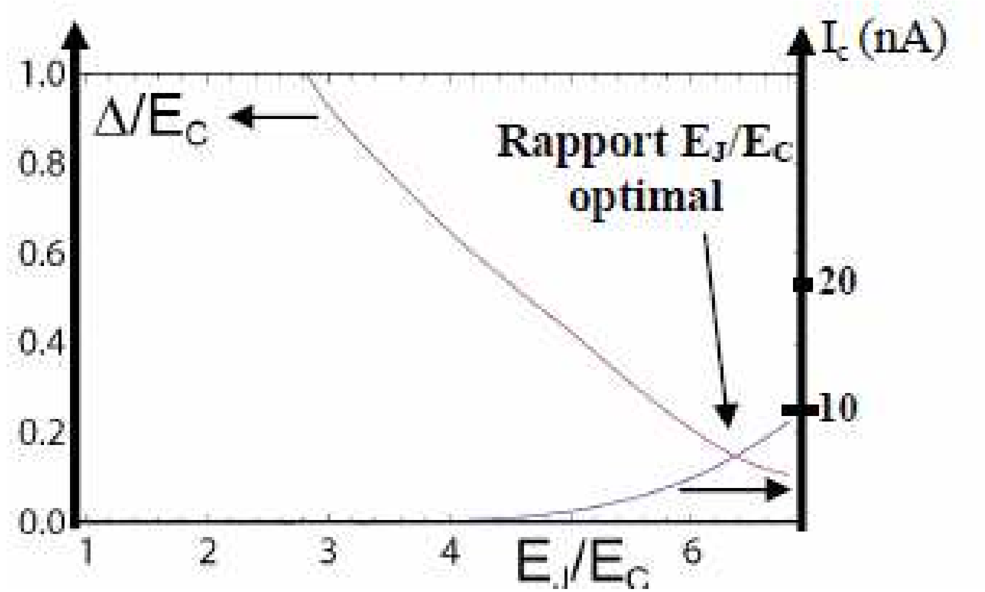

$$\Psi_{qubit} = \Psi_0 e^{i(\varphi_0 + \delta\varphi)t}$$

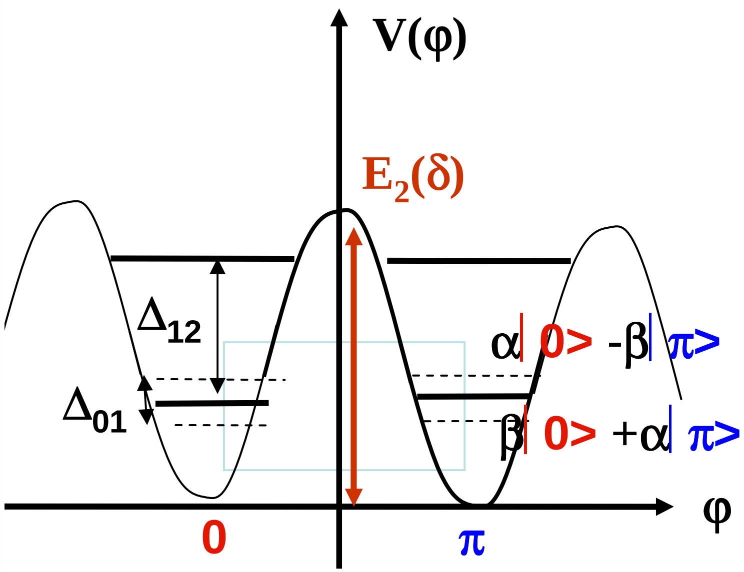
$$\delta\varphi \approx \frac{\left[\left(\frac{\delta\Phi}{\Phi_0}\right)E_J\right]^N}{\Delta^{N-1}}$$

*Lev Ioffe and Benoit Doucot unpublished*

# Proposed sample



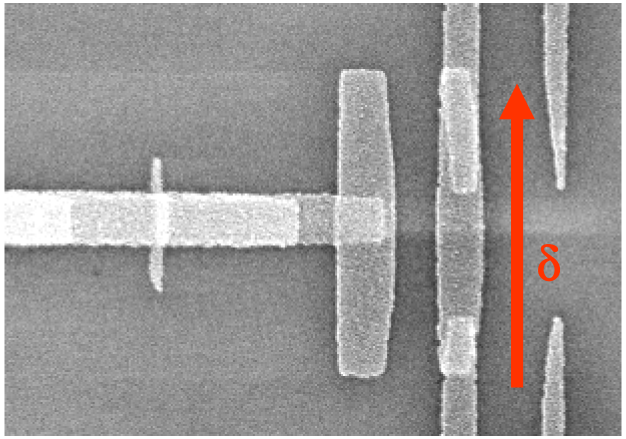




*Lev Ioffe and Benoit Doucot unpublished*

## Phase-charge qubit:
## Asymmetric Cooper pair transistor

$$H_{ACPT} = \frac{(Q_\psi + C_g V_g)^2}{2C_\psi} - 2e_j \cos(\Psi - \delta/2)\cos(\delta/2) - 2\Delta e_j \sin(\Psi - \delta/2)\sin(\delta/2)$$

**Charging energy** **Josephson energy**




**$E_J/E_C$=1**