école normale
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Quantum optics for electrons propagating along a chiral edge channel

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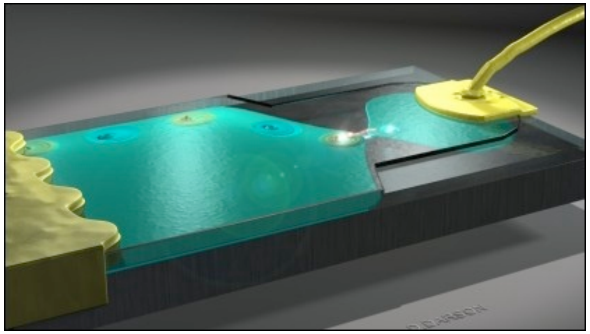
Journée information quantique, Grenoble, june 2009.

Plan

- Introduction and motivation
- An approach to electron quantum optics
- A model for linear detectors
- Detector induced relaxation and decoherence
- Conclusion & perspectives

Quantum optics devices

For photons	For electrons
Photon beams	Edge channels
Beam splitter	Quantum Point Contact
Mirrors	Sample edges
Light source	Voltage source
Single photon source	On demand single electron source



G. Fève *et al*,
Science **316**, 1169 (2007)

Photons vs electrons: little differences

Photons	Electrons
Bosons	Fermions
“True” vacuum	Fermi sea
Non interacting	Coulomb interactions

Photon quantum optics

Coherence within the QED framework

Glauber, Phys. Rev. Lett. **10**, 84 (1963)
Phys. Rev. **130**, 2529 (1963)
Phys. Rev. **131**, 2766 (1963)

What is the equivalent for electron quantum optics ?

To what extent do these “little differences” matter ?

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Quantum optics coherence (Glauber)

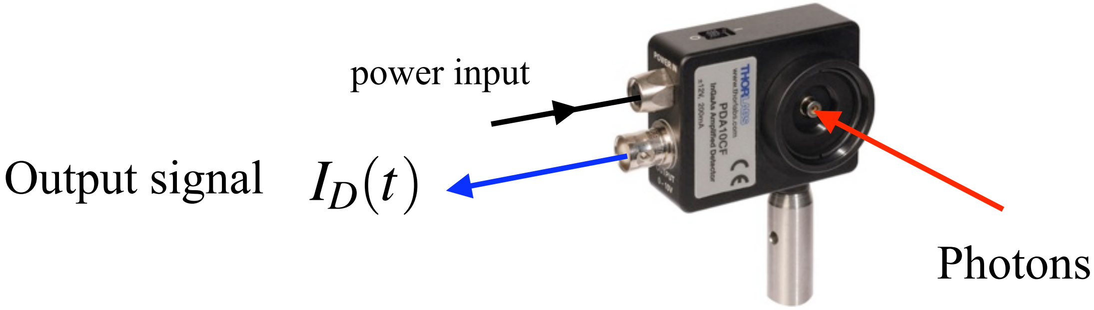
Glauber's correlators

$$\mathcal{G}^{(n)}_{\rho_0}(\underline{x}|\underline{x}') = \mathrm{Tr}\left(\prod_{j=n}^{1} E^{+}(x_j,t_j).\rho_0.\prod_{j=1}^{n} E^{-}(x'_j,t_j)\right) \qquad \underline{x} = (x_j,t_j)_{j=1\ldots n}$$

destruction operators

creation operators

Photodetection signals



$$I_D(t) = \int_0^t \boxed{\mathcal{G}^{(1)}_{\rho_0}(x_D,\tau|x_D,\tau')}\,\boxed{K_D(\tau-\tau')}\,d\tau d\tau'$$

Single photon coherence

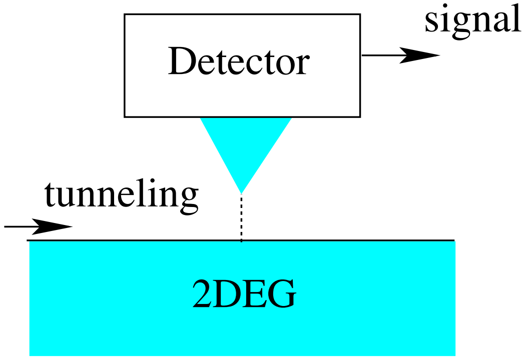
Detector properties:
spectral width, efficiency etc...

Electron coherence

n-particle reduced density operator

$$\mathcal{G}^{(e)}_{\rho_0}(\underline{x}|\underline{x}') = \mathrm{Tr}\left(\prod_{j=n}^{1}\psi(x_j,t_j).\rho_0.\prod_{j=1}^{n}\psi^\dagger(x'_j,t'_j)\right)$$

Electrodetection signals



Tunneling from the conductor into the detector:

$$H_\mathrm{t} = \hbar(\psi^\dagger(x_D)\,O + O^\dagger\psi(x_D))$$

Current flow into the detector:

$$I_D(t) = \int_0^t \boxed{\mathcal{G}^{(e)}_{\rho_0}(x_D,\tau|x_D,\tau')}\boxed{K_D(\tau-\tau')}\,d\tau d\tau'$$

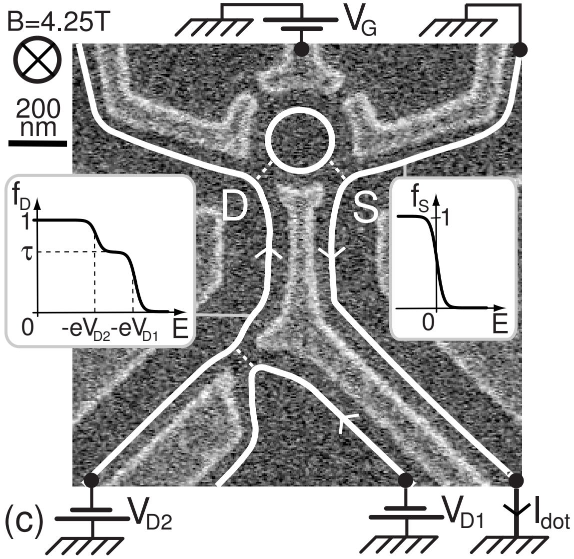
Single electron coherence

Detector properties:
spectral width, efficiency etc...

Electrodetection using a dot

Detector's properties

$$K_D(\tau) = v_F \int g_d(\Omega) e^{i\Omega\tau} \frac{d\Omega}{2\pi}$$



Courtesy F. Pierre

Single electron excitations

Deviation to the Fermi distribution $\delta n(\Omega)$

Sum rule: $$\int \delta n(\Omega) d\Omega = 1$$

Electrodetection signal

Injection rate ν_0

$$\overline{I} = e \int (n_S - n_D)(\omega) g_d(\omega) \frac{d\omega}{2\pi} + e\nu_0 \int \delta n(\omega) g_d(\omega)\, d\omega$$

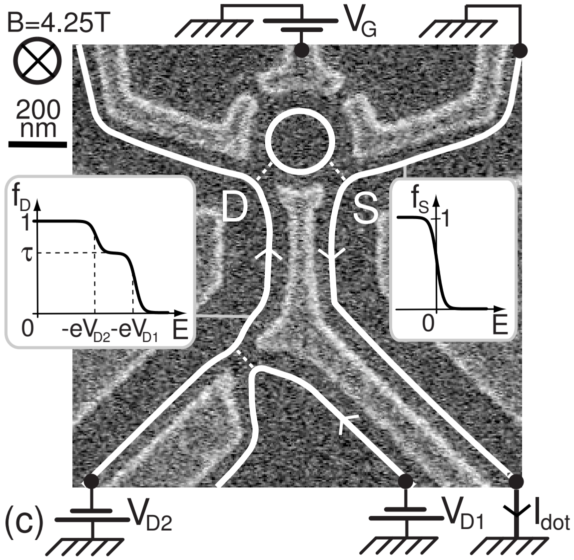
Electrodetection bandwidth

Narrow band detection

Use of a quantum dot as an energy filter

Directly probes $\delta n(\omega)$ and gives access to energy relaxation!

See work in progress by F. Pierre *et al* (LPN Marcoussis)



Courtesy F. Pierre

Broad band detection

In a chiral system: $I(x,t) = ev_F n(x,t)$

$$\mathcal{G}^{(e)}_{\rho_0}(x_D,t+0^+|x_D,t) - \mathcal{G}^{(e)}_{F}(x_D,t+0^+|x_D,t) = \langle n(x_D,t)\rangle$$

Average current: 1st order coherence
Noise of the current: 2nd order coherence

Basic questions

Decoherence

Behavior of $\mathcal{G}^{(e)}_{\rho_0}(x,t|y,t)$ as a function of $x-y$?

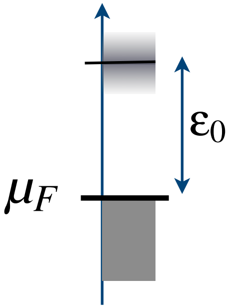
Coherence length at time t? $l_c(t)$

Evolution of $l_c(t)$ in time ?

Energy relaxation

Fourier transform of $\mathcal{G}^{(e)}_{\rho_0}(x,t|x,0)$ with respect to time.

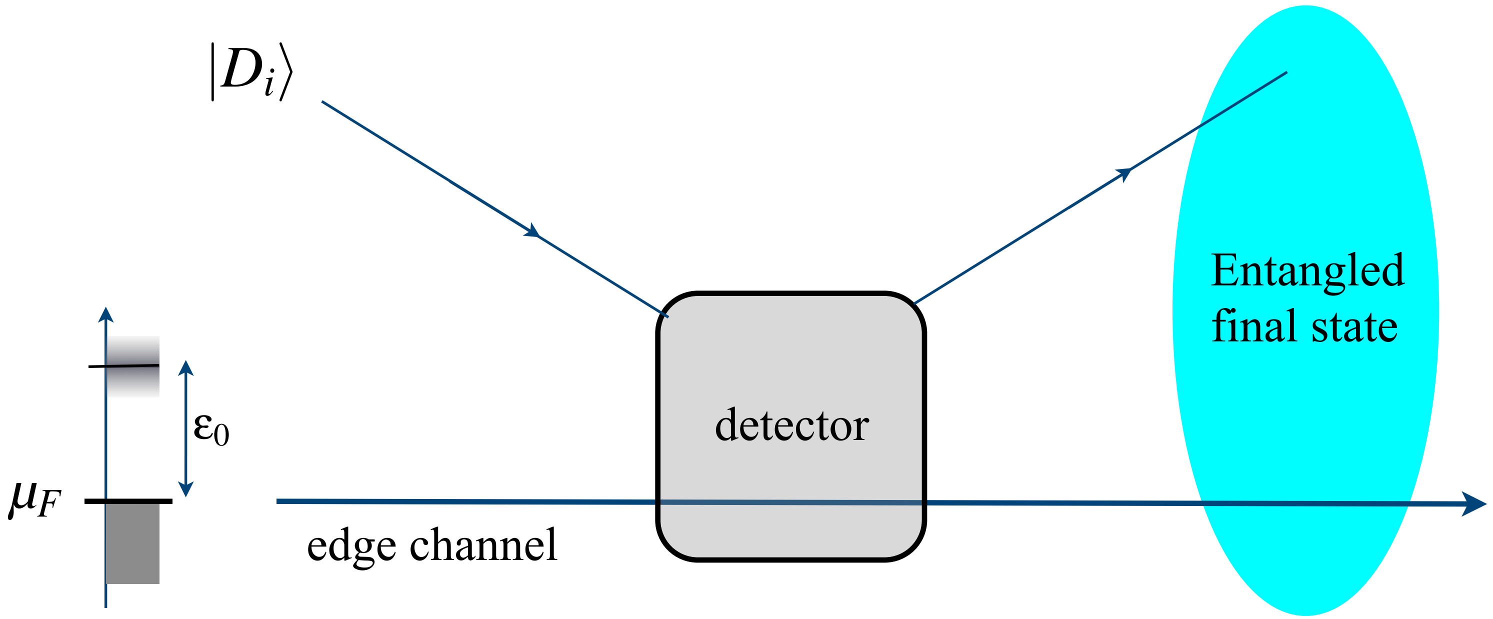
Real time evolution of an energy resolved single electron excitation above the Fermi sea ?



$$\int_{-\infty}^{+\infty} \varphi_0(x)\,\psi^\dagger(x)\,|F\rangle$$

Influence of Coulomb interactions within the conductor ? (intrinsic effects)
Influence of the electromagnetic environment? (extrinsic effects)

Detector induced decoherence



Initial state

Final state

Single electron coherent wave packet above the Fermi sea.

Single electron coherence ?
Energy relaxation ?

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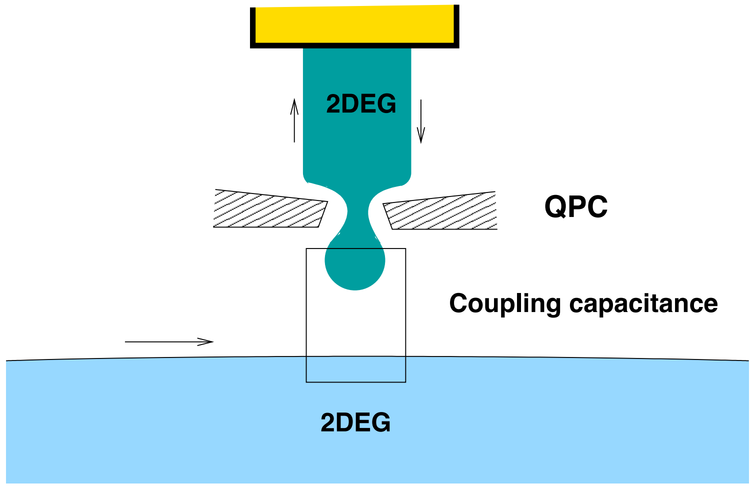
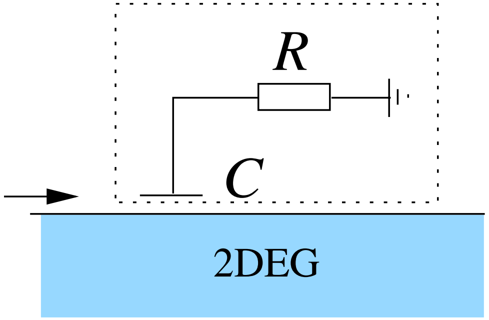
Linear capacitive detectors

Mesoscopic “detector”



Equivalent circuit



Effective description

R and C are effective parameters

Relaxation resistance: $R = R_K/2$ for a coherent single channel capacitor.

Electrochemical capacitance: $C^{-1} = C_g^{-1} + C_D^{-1}$

Prêtre, Thomas and Büttiker, Phys. Rev. B **54**, 8130 (1996).
Nigg and Büttiker, Phys. Rev. B **77**, 085312 (2008).
Gabelli *et al*, Science **313**, 5786 (2006).

Method

Step 1: solve the edge + detector dynamics exactly

Within the bosonization framework, the equations of motion are linear.

The solution is encoded into the edge plasmon / detector's mode scattering.

Step 2: compute the exact many body state for the edge

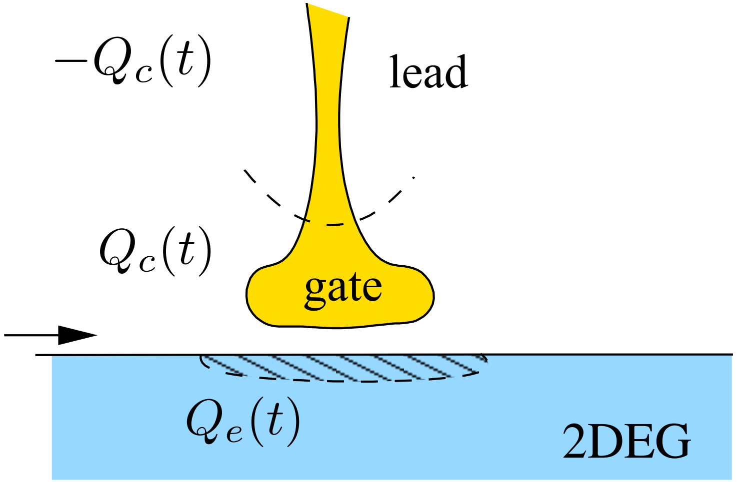
Bosonization says that a localized electron is a coherent state for the edge plasmonic modes.

Phys. Rev. B **62**, 10706 (2000)

Step 3: compute the single electron coherence

Once the edge channel many body state is known, decompose excitations into single electron excitations and electron/hole pairs.

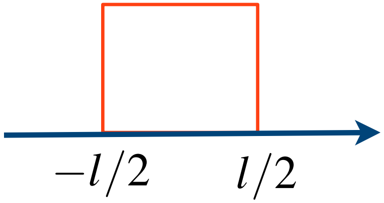
The RC-circuit model



$$Q_e(t) = -e \int f(x) n(x,t)\, dx$$

$f(x) = 0$ for $|x| > l/2$

Here for ex:



Capacitor charge: $Q_c(t) = -Q_e(t)$

Voltage drop: $C(U(t) - V_c(t)) = Q_c(t)$

Voltage seen by edge electrons: $V(x,t) = f(x)U(t)$

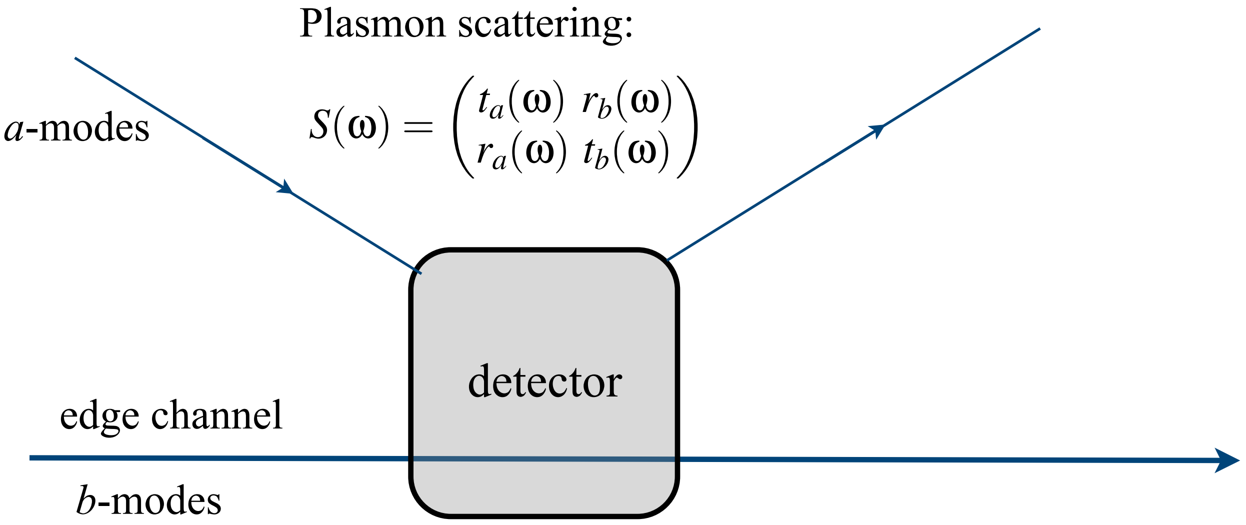
Equation of motion for the edge: $(\partial_t + v_F \partial_x)\phi(x,t) = \frac{e\sqrt{\pi}}{h} V(x,t)$

Dynamics of the RC circuit: $-Q_g(t) = Q_{\text{in}}(0,t) + Q_{\text{out}}(0,t)$

See also Blanter, Hekking, Büttiker, Phys. Rev. Lett. **81**, 1925 (1998)

The RC circuit model: solution

Solving the model leads to plasmon scattering:



Plasmon scattering:

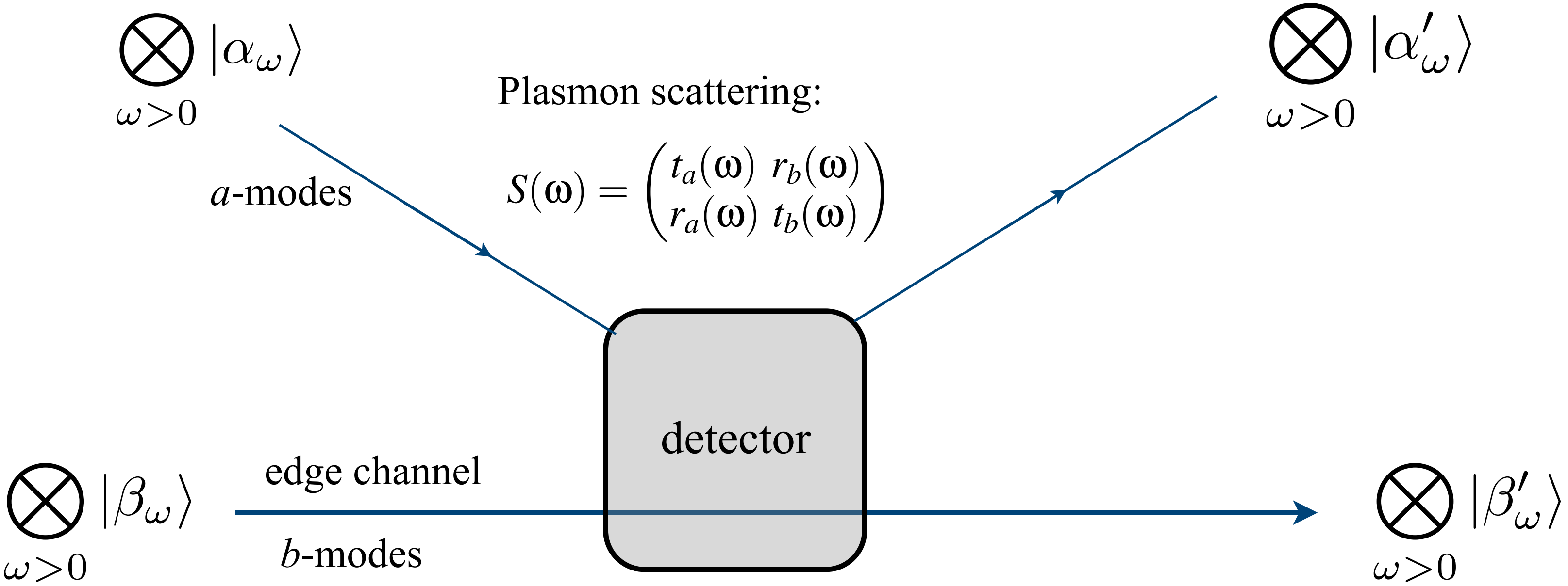
$$S(\omega) = \begin{pmatrix} t_a(\omega) & r_b(\omega) \\ r_a(\omega) & t_b(\omega) \end{pmatrix}$$

Elastic scattering: linearity of the coupling + passive system

Unitarity: $S(\omega)^{\dagger} = S(\omega)^{-1}$ energy conservation

Scattering of coherent plasmons

$T = 0\ \mathrm{K}$



Plasmon scattering:

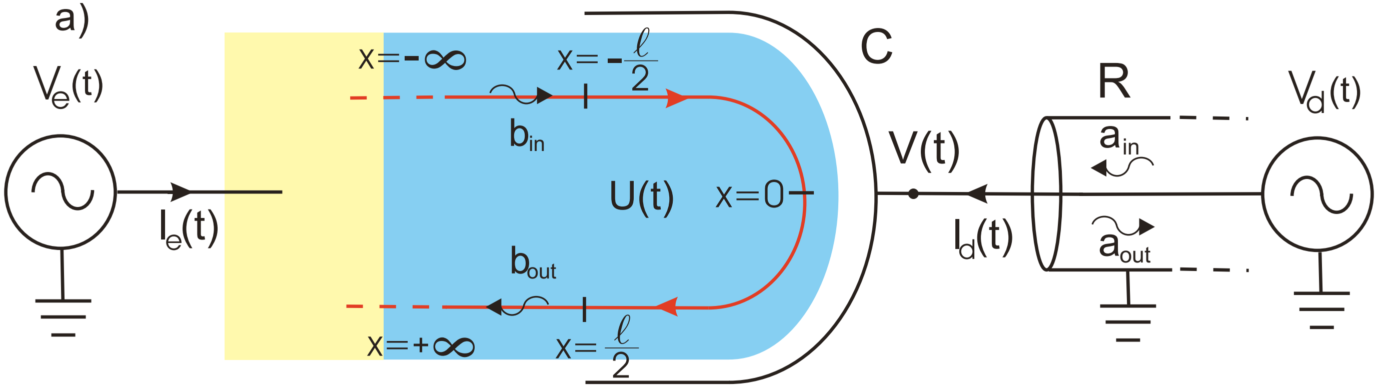
$$S(\omega) = \begin{pmatrix} t_a(\omega) & r_b(\omega) \\ r_a(\omega) & t_b(\omega) \end{pmatrix}$$

Factorized final state with:

$$\begin{pmatrix} \alpha'_\omega \\ \beta'_\omega \end{pmatrix} = S(\omega).\begin{pmatrix} \alpha_\omega \\ \beta_\omega \end{pmatrix}$$

Scattering of coherent plasmons

Voltage drives generate coherent plasmon states (at $T = 0$ K)



$$\beta_\omega = \langle b_{\mathrm{in}}(\omega)\rangle \qquad \alpha_\omega = \langle a_{\mathrm{in}}(\omega)\rangle$$

$$\beta_\omega = -\frac{e}{h}\,\frac{V_e(\omega)}{\sqrt{\omega}}\,e^{i\omega l/2v_F} \qquad \alpha_\omega = -\frac{e}{h}\,\frac{iV_c(\omega)}{\sqrt{\omega}}\sqrt{\frac{R_K}{2R}}$$

I. Safi, EPJD **12**, 451 (1999)

Scattering of coherent plasmons

Finite frequency admittances

Definition: $$I_\alpha(\omega) = \sum_\beta g_{\alpha\beta}(\omega)\, V_\beta(\omega)$$

Charge conservation:

$$\sum_\alpha g_{\alpha\beta}(\omega) = 0$$

Gauge invariance:

$$\sum_\beta g_{\alpha\beta}(\omega) = 0$$

Büttiker et al, Phys. Rev. Lett. **70**, 4114 (1993) & Phys. Rev. **B** 54, 8130 (1996)

Relation to plasmon scattering

$$\boxed{g_{ee}(\omega) = \frac{e^2}{h}(1 - t_b(\omega)\, e^{i\omega l/v_F})}$$

Büttiker model with total screening

$$Q_\alpha = \sum_\beta C_{\alpha\beta} V_\beta$$

$$C_{ee} = -C_{ec} = C$$
$$C_{cc} = -C_{ce} = C$$

$\Longrightarrow$ gauge invariance & charge conservation

To summarize

Explicit model used for illustration

Büttiker like model: RC circuit capacitively coupled to an edge channel

Valid up to $\omega l/v_F \lesssim 2\pi$

More realistic models could be used!

Plasmon scattering is equivalent to finite frequency admittances

$$\phi_\alpha^{(\mathrm{out})}(\omega) = \sum_\beta \mathcal{S}_{\alpha,\beta}(\omega)\,\phi_\beta^{(\mathrm{in})}(\omega)$$

$$g_{\alpha\beta}(\omega) = \frac{e^2}{h}(\delta_{\alpha,\beta} - \mathcal{S}_{\alpha,\beta}(\omega))$$

$$\mathcal{S}_{ee}(\omega) = e^{i\omega l/v_F}\,t_b(\omega)$$

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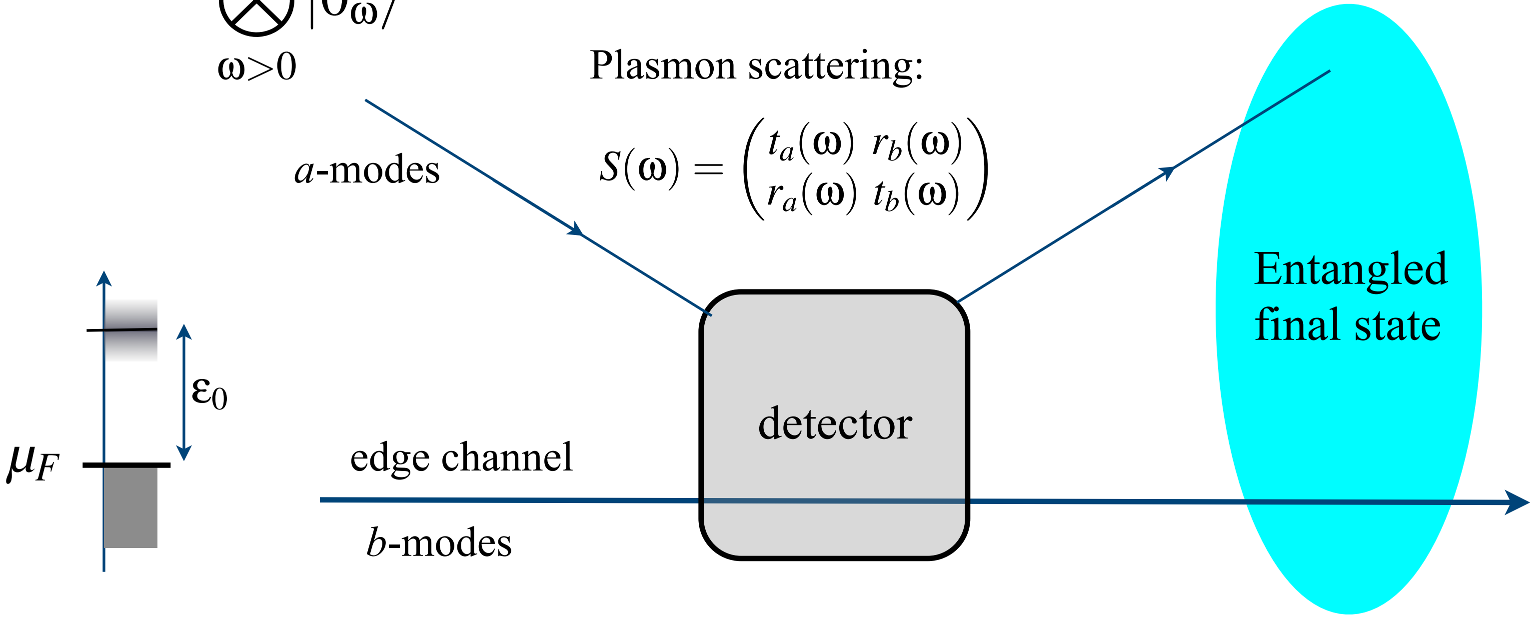
Scattering of a single electron wave packet

$$T = 0\ \mathrm{K}$$

$$\bigotimes_{\omega>0} |0_\omega\rangle$$

Plasmon scattering:

$$S(\omega) = \begin{pmatrix} t_a(\omega) & r_b(\omega) \\ r_a(\omega) & t_b(\omega) \end{pmatrix}$$



$$\int \varphi_0(y) \left(\bigotimes_{\omega>0} |-\lambda_\omega(y)\rangle \right) dy$$

$$\int \varphi_t(y) \left(\bigotimes_{\omega>0} |-t_b(\omega)\lambda_\omega(y)\rangle \otimes |-r_b(\omega)\lambda_\omega(y)\rangle \right) dy$$

edge excitations

detector's excitations

Where is the electron ?

Bare electrons

$$\psi(x)=\frac{1}{\sqrt{2\pi a}}U^{\dagger}e^{i\sqrt{4\pi}\phi_R(x,t)}$$

$$\psi^{\dagger}(x)\,|F\rangle=\frac{1}{\sqrt{2\pi a}}\bigotimes_{\omega>0}|-\lambda_{\omega}(x)\rangle \quad \text{where} \quad \lambda_{\omega}(x)=-\frac{1}{\sqrt{\omega}}e^{-i\omega x/v_F}$$

Dressed electrons

$$\frac{1}{\sqrt{2\pi a}}\bigotimes_{\omega>0}|-t_b(\omega)\lambda_{\omega}(x)\rangle=e^{i\int_0^{\infty}\frac{d\omega}{\omega}\Im(t_b(\omega))}\psi^{\dagger}(x)\,|g(x)\rangle$$

bare electron

e/h pairs

$$|g(y)\rangle=\bigotimes_{\omega>0}|(1-t_b(\omega))\lambda_{\omega}(y)\rangle$$

Neutral charge density wave

From single electron coherence

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The chiral Fermi gas

Coherent wave packet above the Fermi surface: $\int_{-\infty}^{+\infty} \varphi_0(x)\,\psi^\dagger(x)\,|F\rangle$

Wick's theorem: $\mathcal{G}^{(e)}(x,y) = \mathcal{G}_F^{(e)}(x,y) + \mathcal{G}_{\mathrm{WP}}^{(e)}(x,y)$

$$\mathcal{G}_F^{(e)}(x,y) = \frac{i}{2\pi}\frac{1}{y-x+i0^+} \qquad \mathcal{G}_{\mathrm{WP}}^{(e)}(x,y) = \varphi_0(x)\,\varphi_0^*(y)$$

The chiral edge coupled to the detector

Generalization of Wick's theorem: $\mathcal{G}^{(e)}(x,y) = \mathcal{G}_{\mathrm{mv}}^{(e)}(x,y) + \mathcal{G}_{\mathrm{wp}}^{(e)}(x,y)$

where $\mathcal{G}_{\mathrm{mv}}^{(e)}(x,y) \mapsto \mathcal{G}_{\mathrm{F}}^{(e)}(x,y)$ in the limit of vanishing coupling

$\mathcal{G}_{\mathrm{wp}}^{(e)}(x,y) \mapsto \mathcal{G}_{\mathrm{WP}}^{(e)}(x,y)$

to energy relaxation

Energy resolved single electron excitation:

$$\int e^{ik_0x}\psi^\dagger(x)\,|F\rangle\,dx \xrightarrow{\text{propagation}} \mathcal{G}^{(e)}_{k_0}(x,y)$$

Electron distribution function

$$\int \mathcal{G}^{(e)}_{k_0}(x,y)e^{-ik(x-y)}d(x-y) = \boxed{L\,n_F(k)} + \boxed{\delta n(k)}$$

Fermi sea

Single electron

$$\delta n(k) = \boxed{Z(k_0)\delta(k-k_0)} + \boxed{\delta n_r(k,k_0)}$$

$$\int \delta n(k)\,dk = 1$$

Quasi particle peak

Regular part

Single particle limit at low energy

Simple relaxation model

$p(q)$ probability for loosing momentum q

Outcoming electron distribution:

$$\delta n(k) = p(k_0 - k) + Z(k_0)\delta(k - k_0) \qquad k > 0$$

$$\delta n(k) = 0 \qquad k < 0$$

Particle conservation: $p(k) = -Z'(k)$

Validity range

At low coupling: the Fermi sea remains spectator.

For low energy excitations: probes frequencies where $|t_b(\omega) - 1|$ is small enough.

Single particle limit at high energies

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Large energy excitations $k_0 \longrightarrow +\infty$

Validity condition: The electron remains far from the Fermi surface

Wave packet contribution

$$\mathcal{G}^{(e)}_{\mathrm{wp}}(x,y) \mapsto \varphi_t(x)\varphi_t(y)^* \times \mathcal{D}_{\mathrm{tot}}(x,y)$$

$$\mathcal{D}_{\mathrm{tot}}(x,y) = \exp\left(\int_0^{+\infty} 2\,\Re(1-t_b(\omega))\,(e^{i\frac{\omega}{v_F}(y-x)}-1)\,\frac{d\omega}{\omega}\right)$$

$$2\,\Re(1-t_b(\omega)) = |r_b(\omega)|^2 + |1-t_b(\omega)|^2$$

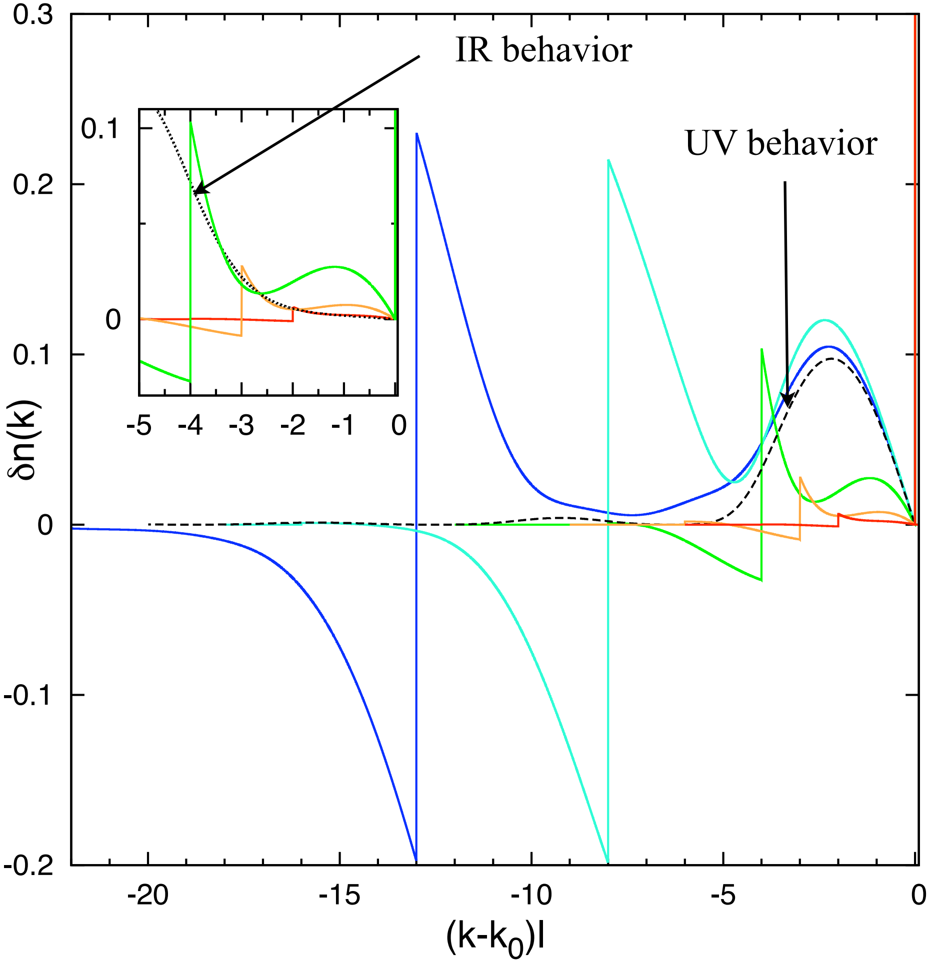
Extrinsic decoherence due to the detector   Intrinsic decoherence (e/h pairs)

Very similar to the **dynamical Coulomb Blockade theory**: see Ingold & Nazarov review for example.

But here it arises as a limiting regime of a more general approch!

Relaxation of single electron excitations

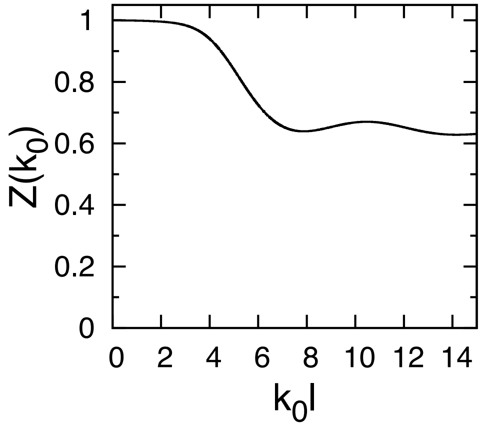
Energy relaxation



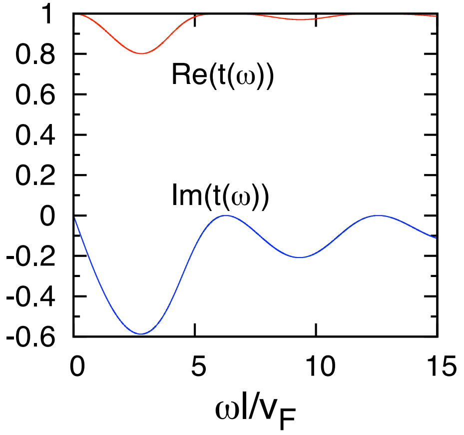
Quasi particle peak

$$R/R_K = 0.002$$

$$\frac{l}{v_F R_K C} = 1/2$$



Plasmon transmission

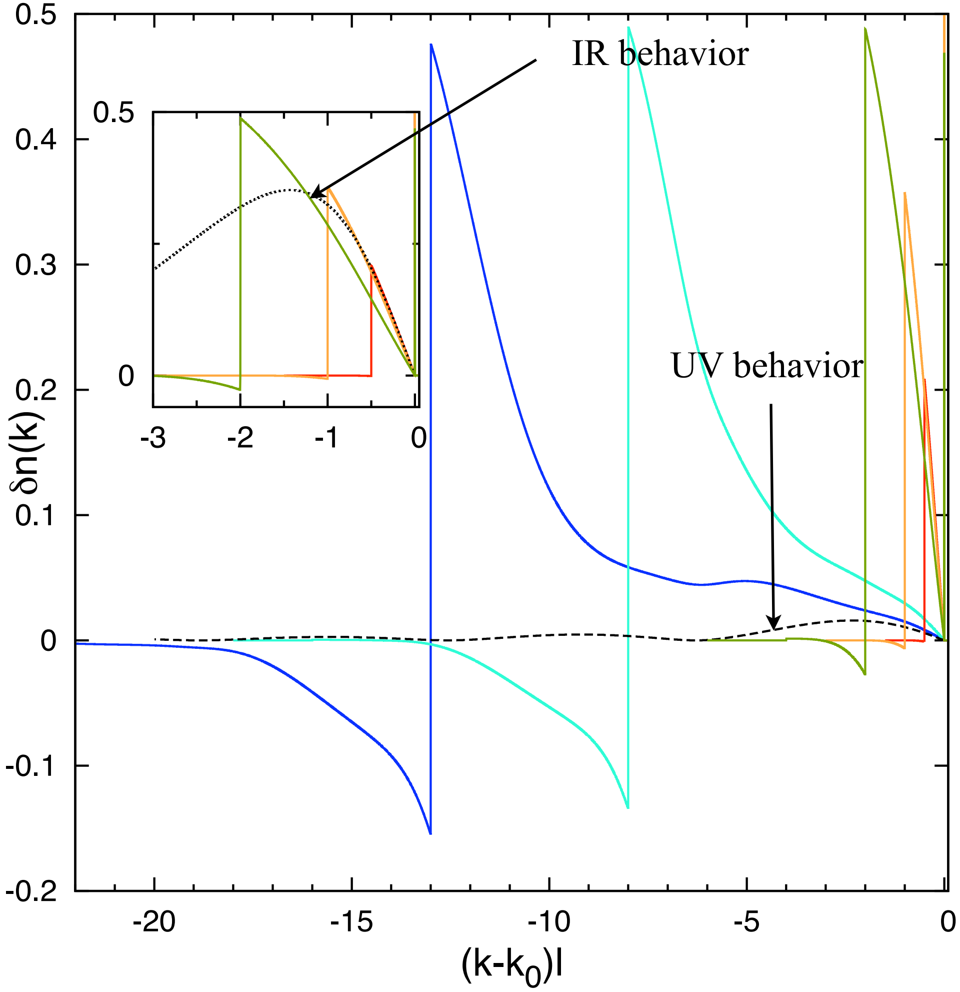


Relaxation of a single electron excitation

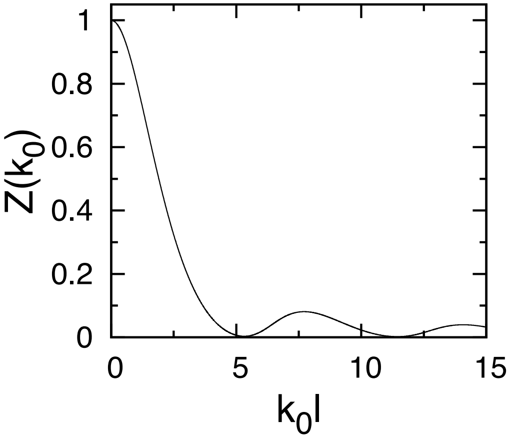
$$R = R_K/2$$

$$\frac{l}{v_F R_K C} = 1/2$$

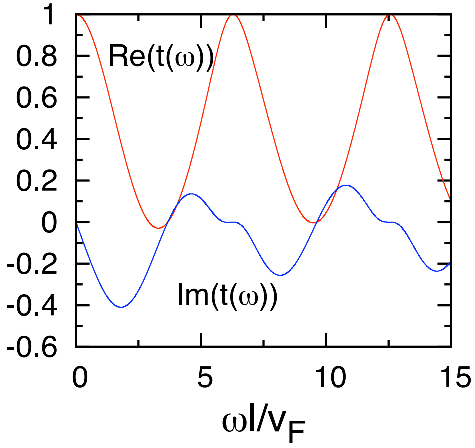
Energy relaxation



Quasi particle peak



Plasmon transmission



To summarize

Non perturbative approach to single electron relaxation

- Only depends on the finite frequency admittance;
- Has simple limiting regimes at high and low energies

IR: simple relaxation model

UV: analogous to the dynamical Coulomb blockade

- Low energy behavior of inelastic scattering probability: $\sigma_{\mathrm{in}}(\omega) = 1 - Z(\omega/v_F)$

$$g(\omega) = -iC_\mu\omega + R_q(C_\mu\omega)^2 + \dots \qquad R_q = R + R_K/2$$

At $R \neq 0$
$$\sigma_{\mathrm{in}}(\omega) \simeq \frac{R}{R_K}(\omega R_q C_\mu)^2$$

Quasi particle not destroyed by the detector !

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Conclusion and perspectives

Summary

Quantum optics formalism for electrons *à la Glauber*

Exact solution for the coupling of a chiral edge channel to a linear detector.

Equivalence of the finite frequency admittance and plasmon scattering

Exact results for detector induced decoherence and relaxation of a single electron excitation above the Fermi level in a chiral edge channel.

Conclusion and perspectives

Perspectives (*work in progress*)

Extension to finite temperatures (*easy*)

Discussion of *e/e* interactions

Problem of dephasing at low temperatures

Discussion of interchannel interactions

Experiments on $\nu = 2$ *edge states*

***Neel project:** spin propagation along edge channels*

Improvement of detector modeling

Description of the state emitted by the on-demand single electron source (e/h pairs ?)

Single electron quantum tomography

***LPA project:** quantum optics with coherent energy resolved single electron excitations*