Quantum dynamics in Josephson junction circuits

Wiebke Guichard
Université Joseph Fourier/ Néel Institute
Nano Department
Equipe Cohérence quantique

Josephson junction team
Olivier Buisson, Bernard Pannetier, Laurent Lévy

Current PhD students:
Ioan Pop (Josephson junction arrays, Topologically protected qubit)
Florent Lecocq (Qubit de phase)
Zhihui Peng (Epitaxial Josephson junction)

Former PhD students and postdocs:
Aurélien Fay (Coupled qubit circuit)
Emile Hoskinson
Julien Claudon
Franck Balestro

Collaborations:
Frank Hekking, Lev Ioffe, Bénoit Doucot, Léonid Glazman, Ivan Protopopov,
Michael Gershenson

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Outline

• Quantum dynamics in a coupled qubit circuit

• Josephson junction arrays: Towards the realization of a topologically protected qubit?
Ultrasmall Josephson junction

$$E_J = \frac{R_Q \Delta}{R_T} \frac{1}{2}$$

Josephson effect:

$$I(\phi) = \frac{2e}{h} E_J \sin(\phi)$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos \hat{\phi}$$

Coulomb blockade:

$$I=0 \text{ for } V<e/C$$

$$\Delta \phi \Delta N \geq 1/2$$
Bloch Bands

$$\frac{d^2 \psi}{d \varphi^2} + \left( \frac{E}{E_c} + \frac{E_J}{E_C} \cos \varphi \right) \psi = 0$$

$E_J \gg E_C$ → Tight Binding Model

$H = -E_c \frac{d^2}{d \varphi^2} - E_J \cos \varphi$

Movement of a particle in a periodic potential

Tunnelling amplitude for phase slip:

$$\nu \approx \left( E_J^3 E_C \right)^{1/4} \exp \left( -\sqrt{8 E_J / E_C} \right)$$
$E_C \gg E_J$ Weak Binding Model

$|0\rangle \quad |1\rangle$

$E(q)$

$E_{\text{Pot}}$
The coupled circuit

Asymmetric Cooper pair transistor

\[ \hat{H}_T = \frac{(2e)^2(n - n_g)^2}{2C_\Sigma} - \sum e_j \cos(\delta/2)\cos(\Theta_d) \]
\[ - \Delta e_j \sin(\delta/2)\sin(\Theta_d) \]

\[ \hat{H}_S = \frac{1}{2} \hbar \omega_p (\hat{P}^2 + \hat{X}^2) - \sigma \hbar \omega_p \hat{X}^3 \]

Charge qubit

\[ \hbar \omega_T (I_b, \Phi, V_g) \]

E\_J \sim E\_C

Phase qubit

\[ |0\rangle \rightarrow |3\rangle \]

\[ \Delta U (I_b, \Phi_b) \]

E\_J \gg E\_C
Asymmetric Cooper pair transistor: charge qubit

\[ \hat{H}_T = \frac{(2e)^2(\hat{n} - n_g)^2}{2C_{\Sigma}} - 2e_j \cos(\delta/2) \cos(\hat{\Theta}_d) - 2\Delta e_j \sin(\delta/2) \sin(\hat{\Theta}_d) \]

with \[ [\hat{n}, \hat{\Theta}_d] = i \]

A charge qubit:

\[ |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]
\[ |-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \]

\( \hbar \omega_T (I_b, \Phi, V_g) \)

Josephson energy
\[ 2e_j \sim 21.8 \text{ GHz} \]
\[ 2\Delta e_j \sim 8.8 \text{ GHz} \]

Transistor asymmetry \( \sim 40\% \)

Charge energy
\[ \frac{(2e)^2}{2C_{\Sigma}} \sim 19.3 \text{ GHz} \]
Manipulation of the qubit at the optimal point $\delta=\pi$

- $\nu_{MW} = \nu |\rightarrow \rightarrow |\rightarrow$ Measurement on the SQUID
- $T_{MW}$: tunable
- $T_{\text{delay}}$

Rabi oscillations:

- $T_{\text{decay,Rabi}} = 110$ ns
- $\Delta \nu = 24.2$ MHz
- $T_2 = 15$ ns
- $T_1 = 811.0$ ns

Energy relaxation
Spectroscopy measurement of the two quantum systems

\[ n_g = \frac{1}{2} \]
\[ I_b = 1910 \text{nA} \]
Spectroscopy versus $V_g$

**Resonant coupling**

- $|0, +\rangle - |1, -\rangle = \sqrt{2}$
- $|0, +\rangle + |1, -\rangle = \sqrt{2}$

- $g \sim 110 \text{MHz}$
- $\delta v_T \sim 40 \text{MHz}$ (charge noise limitation)
- $\delta v_S \sim 20 \text{MHz}$ (fluctuator and flux noise limitation)

**Dispersive coupling**

- $\Delta \sim 200 \text{MHz}$
- $\chi \sim g^2/4\Delta \sim 10 \text{MHz}$
Spectroscopy versus flux

Spectroscopy at $I_{\text{bias}} = 2070 \text{ nA}$

- Two qubits can be in resonance from 9 GHz to 20 GHz.
- Strong variation of the coupling strength.
Resonant coupling

\[ \nu_r \text{ (GHz)} \]

\[ g \text{ (GHz)} \]

Coupling varies from 60 MHz to 1100 MHz, factor of 18
Electrical schematic of the circuit

\[ \hat{H} = \hat{H}_{ACPT} + \hat{H}_{SQUID} + \hat{H}_{COUPL} \]
Coupling in resonance

\[ \hat{H}_{COUPL} = \frac{1}{2} \hbar g \left( \hat{\sigma}_S^+ \hat{\sigma}_T^- + \hat{\sigma}_S^- \hat{\sigma}_T^+ \right) \]

\[ \hbar g = \frac{E_{c,c}}{2} - E_{c,j} \cos\left(\frac{\delta}{2} + \mu \tan(\delta)\right) \]

**Capacitive coupling**

\[ E_{c,c} = (1 - \lambda) \sqrt{\frac{E_C^S}{\hbar \nu_p}} \]

Capacitance asymmetry

\[ \lambda = \frac{C_1^T - C_2^T}{C_1^T + C_2^T} \]

**Josephson coupling**

\[ E_{c,j} = (1 - \mu) \sqrt{\frac{E_C^S}{\hbar \nu_p}} \frac{E_J^T}{2} \]

Josephson energy asymmetry

\[ \mu = \frac{E_{J,1}^T - E_{J,2}^T}{E_J^T} \]
Resonant coupling

We consider $\lambda = \mu = 41.6\%$

If transistor was symmetric ($\lambda = \mu = 0$) coupling would be zero
We consider $\lambda = 37.7\%$ and $\mu = 41.6\%$.

A. Fay et al., PRL 100, 187003 (2008)
Two qubits read-out

Quantronium read-out: classical Josephson junction $\omega_S \ll \omega_T$

In our case: $\omega_S \approx \omega_T$ !!!
Presence of an anti-level crossing

With anti-level crossing

SQUID alone

Very weak Landau-Zener transition

Adiabatic quantum transfer!

\[ \frac{\dot{\epsilon}}{\hbar} \approx 2.9 \, \text{GHz/ns} \]

\[ P_{LZ} = e^{-2\pi \frac{g^2}{\hbar \dot{\epsilon}}} \approx 0\% \]
Absence of an anti-level crossing

Without anti-level crossing

Squid alone

Very weak Landau-Zener transition
Quantum dynamics in Josephson junction arrays

Candidate for the realisation of a topologically protected qubit

Dual of Shapiro steps in a Josephson junction array

Phase bias and frustration:
\[ f = \frac{\Phi_R}{2\pi\Phi_0}, \]
\[ \delta = \frac{\Phi_c}{\Phi_0} \]

Chain of Josephson junctions with effective \( E_J \) and \( E_C \)

Chain of N spins \( 2^N \) possible states
Phase biased Josephson junction array

$$E_{\text{Pot}} = -E_J \cos(\varphi)$$

Tunneling-amplitude = Bloch band width

$$\nu \approx \left( \frac{E_J E_C}{8} \right)^{1/4} \exp \left( -\frac{\sqrt{8 E_J / E_C}}{4} \right)$$
Chain of single Josephson junctions: classical regime $E_J \gg E_C$

\[ \delta = \sum_{i=1}^{N} \varphi_i \]

\[ \varphi_i = \frac{\delta}{N} \]

\[ E_{pot} = \sum_i E_J [1 - \cos(\varphi_i)] \]

\[ = NE_J [1 - \cos(\delta/N)] \]

\[ = \frac{E_J}{2N} \delta^2 \]
Effect of phase slip in phase biased chain

Phase biased chain: \[ \delta = \sum_{i=1}^{N} \phi_i \quad \text{where} \quad \phi_i = \frac{\delta}{N} \]

Phase slip: energy unchanged but constraint violated!

Phase slip combined with small adjustments: constraint satisfied!
Chain of single Josephson junctions: classical regime $E_J \gg E_C$

$\delta = \sum_{i=1}^{N} \phi_i$

$\phi_i = \frac{\delta}{N}$

$\phi_j = \frac{\delta - 2\pi}{N} + 2\pi$

$E_{pot} = \sum_{i} E_J [1 - \cos(\phi_i)]$

$= NE_J [1 - \cos(\frac{\delta - 2\pi}{N})]$
Chain of single Josephson junctions: classical regime $E_J \gg E_C$

$\phi_i = \frac{\delta}{N}$

$E_m \approx \frac{E_J}{2N} (\delta + 2\pi n)^2$

Critical current of chain:

$I_c \approx \frac{\pi I_c}{N}$
Phase biased Josephson junction array

\[ \delta = \sum_{i=1}^{N} \phi_i \]

\[ H \psi_m = E_m \psi_m - N \nu (\psi_{m-1} + \psi_{m+1}) \]

\[ E_m = \frac{E_J}{2N} (2\pi \hat{n} - \delta)^2 \]

\[ E_L = \frac{\pi^2}{2L} \]

\[ L = \frac{N (\frac{\hbar}{2e})^2}{E_J} \]
Mathieu equation for Josephson array

\[ \frac{d^2 \psi(q)}{dq^2} + \left( \frac{E}{E_L} + \frac{2N \nu}{E_L} \cos 2q \right) \psi(q) = 0 \]

\[ E_{Pot} = -eV_c \cos(2q) \]

\[ L\ddot{q} + R\dot{q} + V_c \sin(2\pi q / (2e)) = V_{bias} \]
Energy spectrum and current-phase relation of chain

$E_J/E_c = 20$

$E_J/E_c = 3$

$E_J/E_c = 1.3$
Sample set-up

Sample characteristics:

Single junction in chain

$R_J = 6k\Omega$

$C_J = 1.2fF$

Read-out Josephson junction:

$R = 1k\Omega$

Area ratio between big loop and SQUID loop: 285
Current-phase relation yields information on the ground state

\[ I_{bias} = I_{chain} + I_J \]
\[ I_{bias} = I_{chain}(\gamma) + I_C \sin(\Gamma) \]

\[ I^\text{Chain} \ll I^J \]

\[ I_{bias} = I_{chain} \left( 2\pi \frac{\Phi}{\Phi_0} - \frac{\pi}{2} \right) + I_c \sin \left( \frac{\pi}{2} \right) \]

\[ I_S(\gamma) = \frac{\partial E_0}{\partial \gamma} \]
Measurement of the current-phase relation II

Large Josephson junction: $I_{sw} = 710 \text{nA}$

PhD: Ioan Pop, to be published
Measurement of the current-phase relation III

PhD: Ioan Pop, to be published
Josephson junction rhombi chain

Phase bias and frustration:

\[ f = \frac{\Phi_R}{2\pi\Phi_0}, \]
\[ \delta = \frac{\Phi_c}{\Phi_0} \]

Chain of Josephson junctions with effective \( E_J \) and \( E_C \)

Chain of \( N \) spins \( 2^N \) possible states
Measurement of the current-phase relation

Current-phase relation yields information on the ground state

\[ I_S = \frac{\partial E_0}{\partial \delta} \]

Current-phase relation at f=0: classical regime

\[ E_J/E_C \sim 20 \]

\[ I_s = \frac{\partial E_0}{\partial \delta} \]

Current-phase relation at $f=0$: quantum regime

$E_J/E_C \sim 2$

$\frac{I_S}{I_0} = \frac{\partial E_0}{\partial \delta}$

$H \psi_m = E_m \psi_m - N \nu(\psi_{m-1} + \psi_{m+1})$

$\nu_{\text{Rhombus}} = A \exp(-S_0)$

$S_0 = 2 \sqrt{\frac{8E_J}{E_C}}$

$A \approx 4.50(E_J^3E_C)^{1/4}$

Current phase relation at $f=0.5$

$\delta$

$I(\delta)$
Measurement of the ground state energy in the classical regime (1)

Phase slips across Rhombi

Measurement of the ground state energy in the classical regime (2)


$f=0.48$
Measurement of the ground state energy in the classical regime (3)

\( f = 0.5 \)

Even number of spin up

Odd number of spin up

\( f = 0.495 \)

\( f = 0.5 \)

Measurement of the ground state energy in the classical regime (4) 

Towards a topologically protected qubit?

Add quantum fluctuations at $f=0.5$ in order to lift the degeneracy of the states.
Idea of topologically protected qubit

\[ \delta E \approx \frac{[M]^N}{\Delta^{N-1}} \]

Energy spectrum of a rhombi chain at half flux frustration I

\[ \Phi_R = \Phi_0/2 \]

Even or odd number of Cooper pairs

\[ \gamma \]

\[ \gamma = 0 \]

\[ \gamma = \pi \]

\[ \gamma = -\pi \]

\[ |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \quad \rightarrow \quad |0; \gamma = 0\rangle \]

\[ |\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle \quad \rightarrow \quad |1; \gamma = 0\rangle \]

\[ |\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle \quad \rightarrow \quad |0; \gamma = \pi\rangle \]

\[ |\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle \quad \rightarrow \quad |1; \gamma = \pi\rangle \]
Energy spectrum of a rhombi chain at half flux frustration II

\[ \Psi_{\text{qubit}} = \Psi_0 e^{i(\varphi_0 + \delta \varphi)t} \]

\[ \delta \varphi \approx \left( \frac{\delta \Phi}{\Phi_0} \right) E_J \left[ \frac{N}{\Delta^{N-1}} \right] \]

Lev Ioffe and Beno\textit{i}t Douc\textit{o}t unpublished
Proposed sample

Lev Ioffe and Benoit Doucot unpublished
Phase-charge qubit:
Asymmetric Cooper pair transistor

$H_{ACPT} = \frac{(Q_\psi + C_g V_g)^2}{2C_\psi} - 2e_J \cos(\Psi - \delta/2) \cos(\delta/2) - 2\Delta e_J \sin(\Psi - \delta/2) \sin(\delta/2)$

Charging energy

Josephson energy

$E_J/E_C = 1$