Ultrasonic attenuation in a pseudogapped superconductor

A. Shtyk and M.Feigelman

I. Background and motivation
II. Collective modes in a pseudogapped state
III. Electron-phonon interaction
IV. Ultrasonic attenuation due to collective modes
   - Phase
   - Amplitude
V. Effects of Coulomb interaction

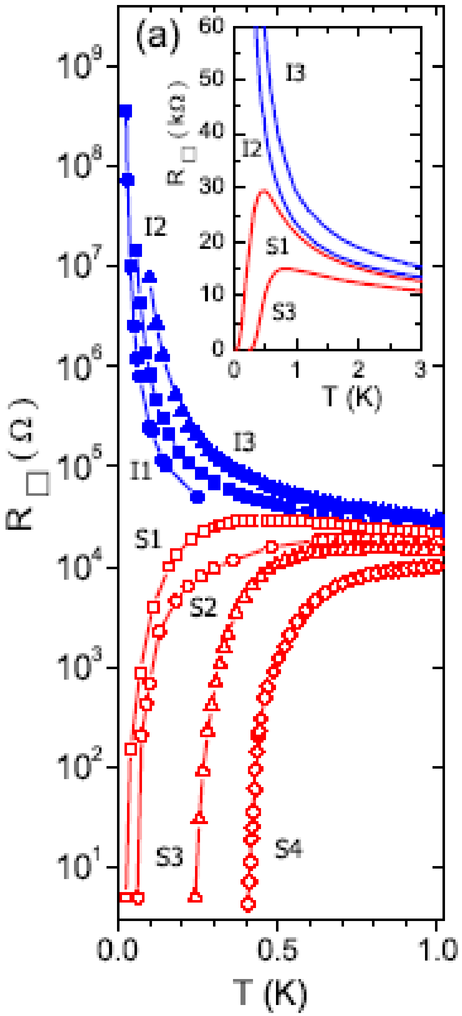
I. Background and motivation

Two scenarios of SIT*

- Spectral gap drops together with T_c : "fermionic" mechanism
- Spectral gap is nearly constant while T_c drops to zero: "bosonic" mechanism ("preformed pairs")

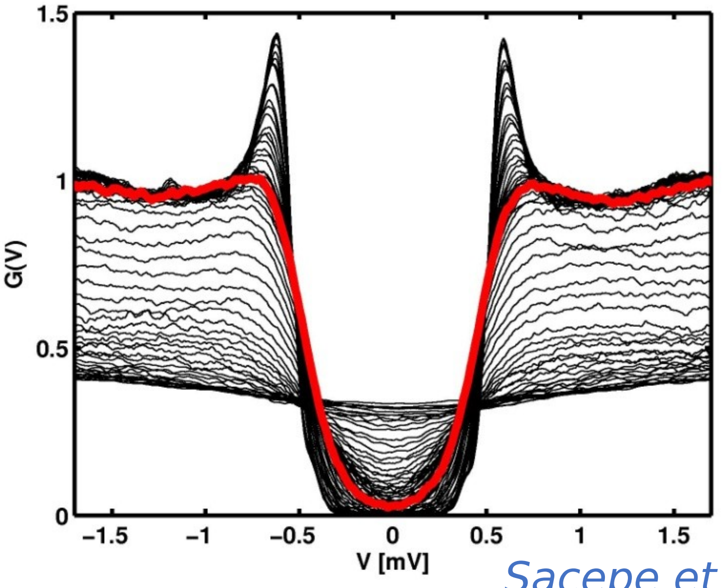
*Superconductor to insulator transition

SIT: preformed pairs



Baturina et al 2008

- The spectral gap appears much before (with T down) than superconductive coherence does
- Coherence peaks in the DoS appear together with resistance vanishing

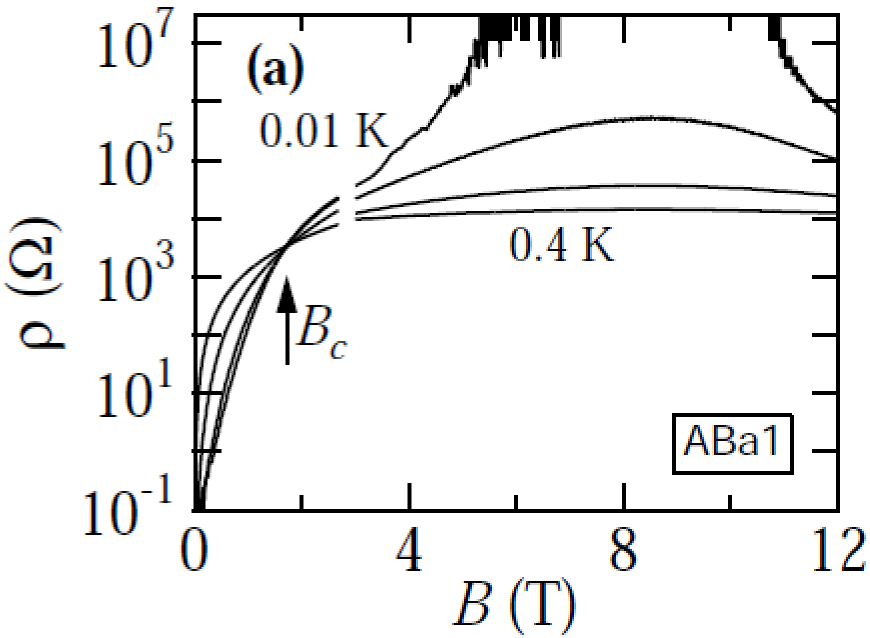


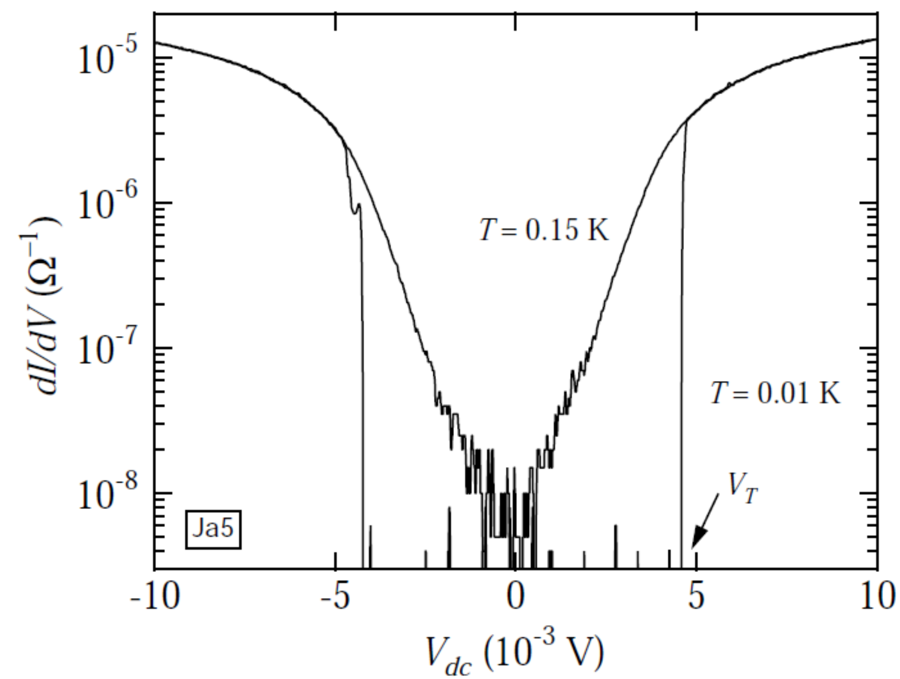
Sacepe et al, 2011

Insulating state induced by magnetic field

G.Sambandamurthy, L. Engel, A. Johansson, E.Peled and D.Shahar

Physical Review Letters, 94, 017003 (2005)



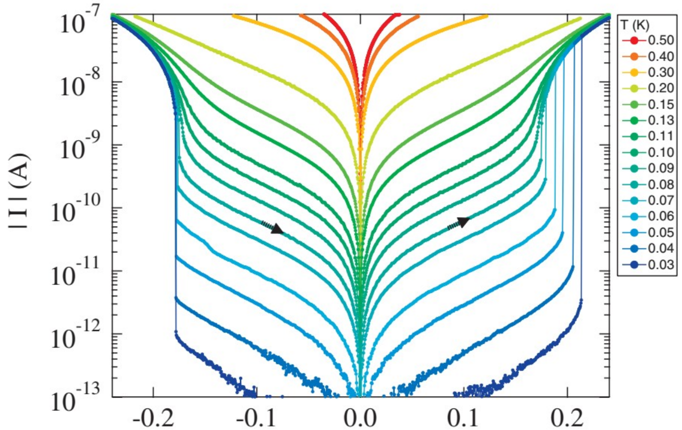


Inelastic electron-phonon scattering time in the insulating state

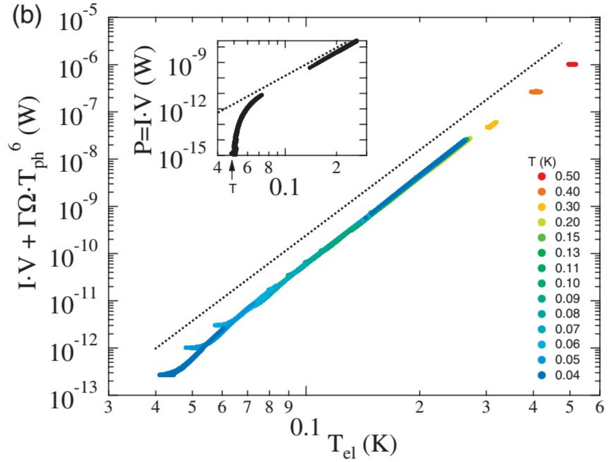
- Electron-phonon decoupling in disordered insulators
- Hysteresis in I-V curves. Overheating of electronic subsystem
- Fits the theory for e-ph interaction in disordered conductors

$$\frac{V^2}{R(T_{\rm el})} = \Gamma\Omega(T_{\rm el}^{\beta} - T_{\rm ph}^{\beta}),$$

However the coupling is stronger by orders of magnitude



Ovadia et al 2009; 2016



Altshuler et al 2009

Electron-phonon cooling and ultrasound attenuation rate

energy flow $\mathcal{J} = \frac{dE_{ph}}{dt} = J(T_{el}) - J(T_{ph})$

$$J(T) = \int_0^\infty d\omega\, \omega\, \nu_{ph}(\omega) \frac{B_{ph}(\omega/T)}{\tau_{ph}(\omega)}$$

The phonon decay rate τ_{ph}^{-1}

$$\tau_{ph}^{-1}(\omega, T_{el}) = \frac{1}{\rho_m \omega} \mathrm{Im}\Sigma^R(\omega, q, T_{el})|_{\omega = v_s q}.$$

A.Shtyk, M. Feigelman & V. Kravtsov PRL **111***, 166603 (2013)*

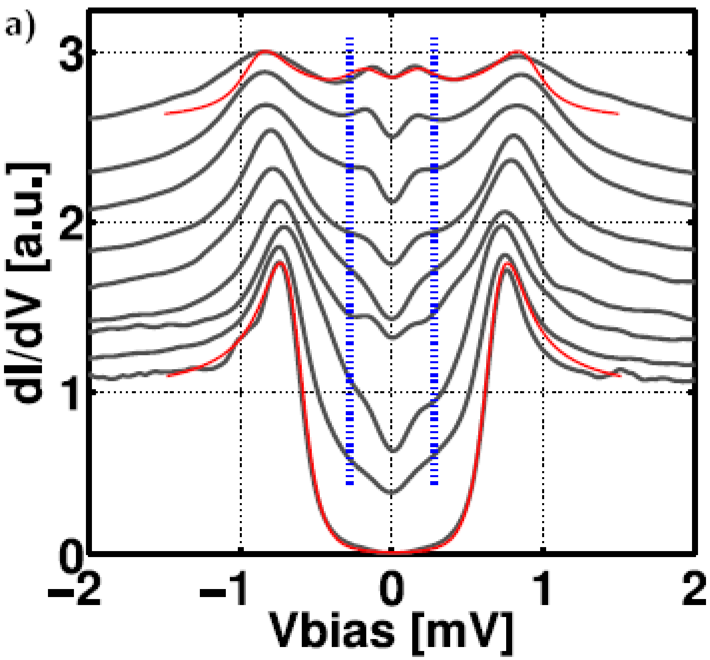
Previous search for enhanced inelastic e-ph rate

- Before (weak disorder) the core idea behind was:
  - Phonon couples to a slow diffusive mode of a conserved physical quantity (charge/spin/energy)
  - Competition with a “normal” attenuation due to standard local mechanism

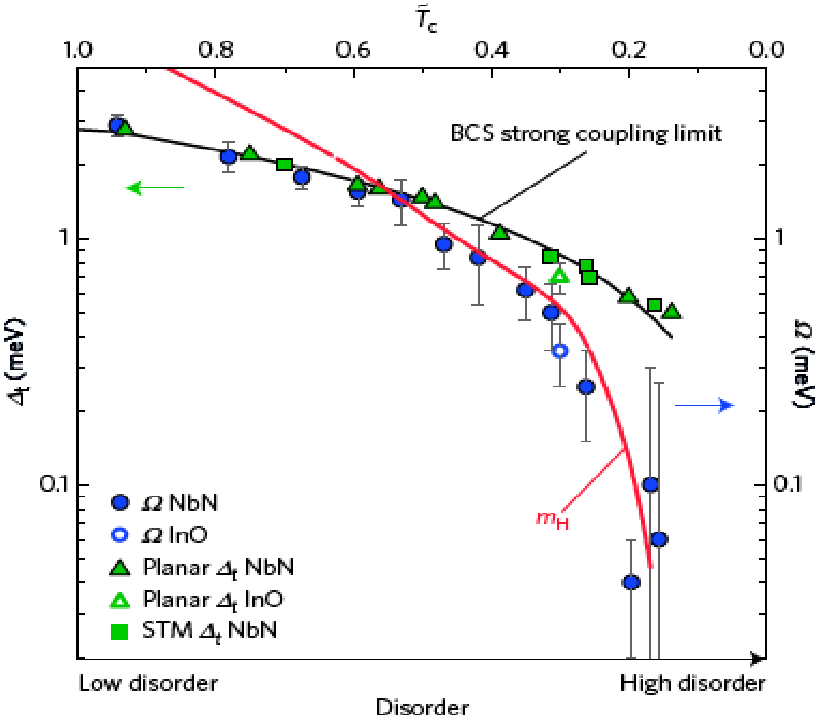
A.Shtyk et al, PRL **111**, *166603 (2013); PRB* **92**, *195101 (2015)*

- Now, within bosonic scenario of SIT:
  - Kill the single electron excitations completely and directly probe collective modes!

Collective SC gap versus single-particle gap



T.Dubouchet,
thesis, Grenoble
(11 Oct. 2010)



D. Sherman et al,
Nature Phys. (2015)

Motivation summary

- Preformed Cooper pairs: single electron gap $\Delta_1 > \Delta$ and “normal” attenuation is strongly suppressed
- Ultrasonic attenuation as a probe of collective excitations in a pseudogapped state
- Ultrasonic attenuation should be dominated by collective modes in a superconductor and exhibit activation behavior $\propto e^{-2\Delta/T}$

II. Collective modes in a pseudogapped state

Contribution of single-electron states Is suppressed by pseudogap $\Delta_1 \gg T_c$

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector: all states which are 2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g\sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu}\left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\overline{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

DoS, total volume

"Pseudospin" approximation

$$Z \sim \nu_0 T_c L^d_{loc}$$

Effective number of interacting "neighbours"

Fedotov-Popov trick

Cooper pairs

1 *Anderson(1958)*

pseudospins

Fedotov, Popov (1988) 2

semions

3 *Stratonovich (1957), Hubbard (1959),*

SC order parameter field Δ

- Represent spins-1/2 as semions with imaginary chemical potential. No additional constraints

Fedotov-Popov trick

Fedotov, Popov (1988)

- Representing spins-1/2 as semions with imaginary chemical potential. No additional constraints

$$S_i^\alpha \to (1/2)\psi_i^\dagger \sigma^\alpha \psi_i$$

$$H[\boldsymbol{S}_i] \to H\left[\frac{1}{2}\psi_i^\dagger \boldsymbol{\sigma}\psi_i\right] + (i\pi/2\beta)N_F$$

- Similar to Abrikosov pseudofermions but unphysical states are automatically taken care of by complex chemical potential $\mu \in \mathbb{C}$
- Standard fermionic diagrammatic rules but with Matsubara energies

$$\varepsilon_l = 2\pi T(l + 1/4)$$

Superconducting action

1. Start with pseudospins. XY-Heisenberg in a random field

$$H[\boldsymbol{S}_i] = -2\sum_{i=1}^{N} \xi_i S_i^z - \sum_{i,j=1}^{N} J_{ij}\left[S_i^x S_j^x + S_i^y S_j^y\right]$$

2. Introduce semions $S_i^\alpha = (1/2)\psi_i^\dagger \sigma^\alpha \psi_i$

3. Introduce Δ via Hubbard-Stratonovich transformation

$$H = -\sum_i \psi_i^\dagger \xi_i \sigma^z \psi_i - \frac{1}{4}\sum_{i,j} J_{ij}(\psi_i^\dagger \sigma^\alpha \psi_i)(\psi_j^\dagger \sigma^\alpha \psi_j)$$

$$\mathcal{A}[\Delta] = -\mathrm{Tr}\left[\Delta^* J^{-1}\Delta\right] + \mathrm{Tr}\ln\Big[\underbrace{i\varepsilon_l + \xi\sigma^z + \frac{1}{2}(\Delta\sigma^- + \Delta^*\sigma^+)}_{[G(i\varepsilon_l)]^{-1}}\Big]$$

Superconducting action

$$\mathcal{A}[\Delta] = -\mathrm{Tr}\left[\Delta^* J^{-1} \Delta\right] + \mathrm{Tr}\ln\left[i\varepsilon_l + \xi\sigma^z + \frac{1}{2}(\Delta\sigma^- + \Delta^*\sigma^+)\right]$$

- Box-shaped distribution of disorder $\nu(\xi) = (2W)^{-1}\Theta(W - |\xi|)$. Mean field.
- The saddle point gives the self-consistency equation

$$1 = g\int_\xi \frac{1}{\sqrt{\xi^2+\Delta^2}} \tanh\frac{\sqrt{\xi^2+\Delta^2}}{T}$$

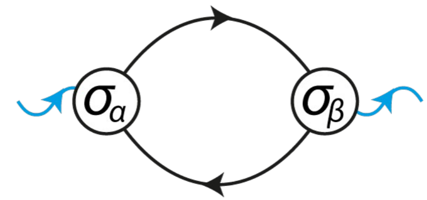
$$J(\boldsymbol{k}) = g\left[1 - R^2k^2 + O(k^4)\right],$$

- Two collective modes. Massive amplitude (Higgs) and massless phase (Goldstone)

$$\Delta = \Delta_0(1+\eta)e^{i\phi}$$

SC propagator

$$L_{\alpha\beta}^{-1} = J^{-1}(\boldsymbol{q})\delta_{\alpha\beta} \quad -$$



- The phase mode: a pole

$$L_{\phi}^{R-1}(\Omega, \boldsymbol{k}) = (4W)^{-1}\left[(\Omega + i0)^2 - v^2 k^2\right]$$

- The amplitude mode: a branch cut

$$L_{\eta}^{R-1}(\Omega, \boldsymbol{Q}) = -\frac{\Delta^2}{W}\left[i\pi\sqrt{\frac{\Omega - 2\Delta}{4\Delta}} + \frac{v^2 k^2}{4\Delta^2}\right], \quad (\Omega > 2\Delta,\ |\Omega - 2\Delta| \ll \Delta)$$

III. Electron-phonon interaction:

How to include phonons?

How to include phonons?

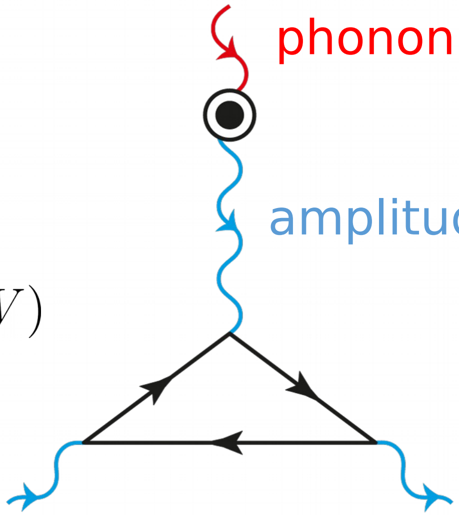
- Modification of the interaction/hopping

$$H_{\text{e-ph}}[\boldsymbol{S}_i] = \kappa \sum_{i,j=1}^{N} (J_{ij}\ \mathrm{div}\boldsymbol{u}) \left[S_i^x S_j^x + S_i^y S_j^y\right]$$

- Within adiabatic limit $\omega, q \to 0 \ \ (\lambda = J(\mathbf{0})/W)$

$$\lambda \to \lambda(1 - \kappa\, \mathrm{div}\boldsymbol{u})$$

$$\Delta = 2We^{-1/\lambda} \to \Delta\left(1 + \frac{\kappa}{\lambda}\mathrm{div}\boldsymbol{u}\right)$$

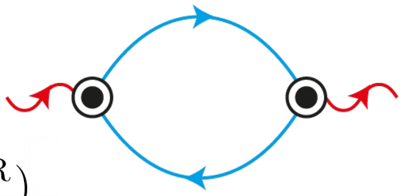


- Electron-phonon vertex reduces to modification of the (inverse) propagator. Similar to Ward identities

$$\Gamma_{\eta/\phi} = -\frac{\kappa}{\lambda} \cdot \frac{\partial L^{-1}_{\eta/\phi}}{\partial \ln \Delta}$$

IV. Ultrasonic attenuation due to collective modes

Ultrasound Attenuation: phase



- UA (after analytical continuation)

$$\mathrm{Im}\Sigma^{A}_{\text{ph-}\phi}(\omega,\boldsymbol{q}) = q^2 \int_{\Omega,\boldsymbol{Q}} |\Gamma_\phi|^2 \left(B_+ - B_-\right) (\mathrm{Im}L^{\mathrm{R}}_\phi)_+ (\mathrm{Im}L^{\mathrm{R}}_\phi)_-$$

- Propagator and vertex

$$L^{R-1}_\phi(\Omega,\boldsymbol{k}) = (4W)^{-1}\left[(\Omega+i0)^2 - v^2k^2\right]$$

$$\Gamma_\phi = -\frac{\kappa}{\lambda}\cdot\frac{\partial L^{-1}_\phi}{\partial \ln\Delta} = \frac{\kappa_* v^2}{4W}\left[\boldsymbol{k}\cdot(\boldsymbol{k}+\boldsymbol{q})\right]$$

- Ultrasonic attenuation

$$\alpha = \frac{1}{\rho_m\omega}\mathrm{Im}\Sigma^{A}_{\text{ph-}\phi}(\omega,\boldsymbol{q}) \qquad Q^{-1} = \frac{\alpha}{\omega}$$

$$Q^{-1}_{\mathrm{ph}-\phi} = \frac{2\pi^4\kappa_*^2}{15}\frac{T^4}{\rho_m s v^4} \qquad Q^{-1}_{\mathrm{ph}-\phi} = \frac{2\pi^4}{15}\kappa^2\left(\frac{a_0}{R}\right)^4\left(\frac{T}{\Delta}\right)^4$$

Ultrasound Attenuation: amplitude

- Phonon modifies Δ Formally, it moves a branching point, which determines the most singular part of the electron-phonon vertex
- Propagator and vertex

$$L_\eta^{R-1}(\Omega, \boldsymbol{Q}) = -\frac{\Delta^2}{W}\left[i\pi\sqrt{\frac{\Omega-2\Delta}{4\Delta}} + \frac{v^2k^2}{4\Delta^2}\right],$$

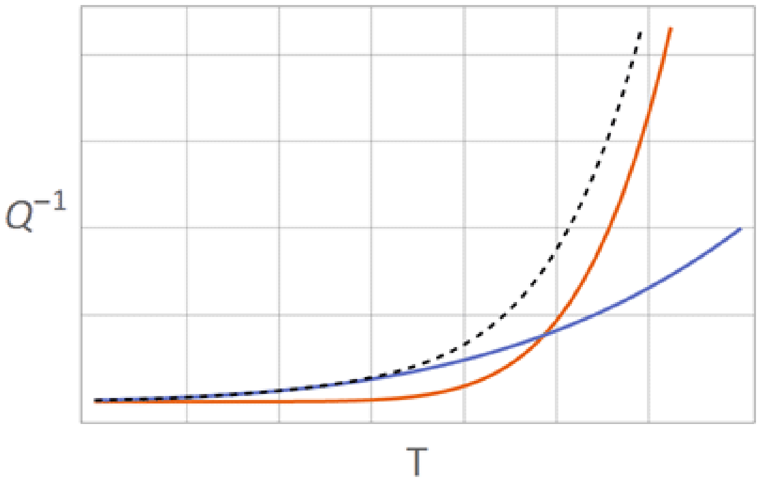
$$\Gamma_\eta = -\frac{\kappa}{\lambda}\cdot\frac{\partial L_\eta^{-1}}{\partial \ln\Delta} \simeq -\frac{i\pi}{2}\frac{\kappa_*\Delta^2}{W}\sqrt{\frac{\Delta}{\Omega-2\Delta}}$$

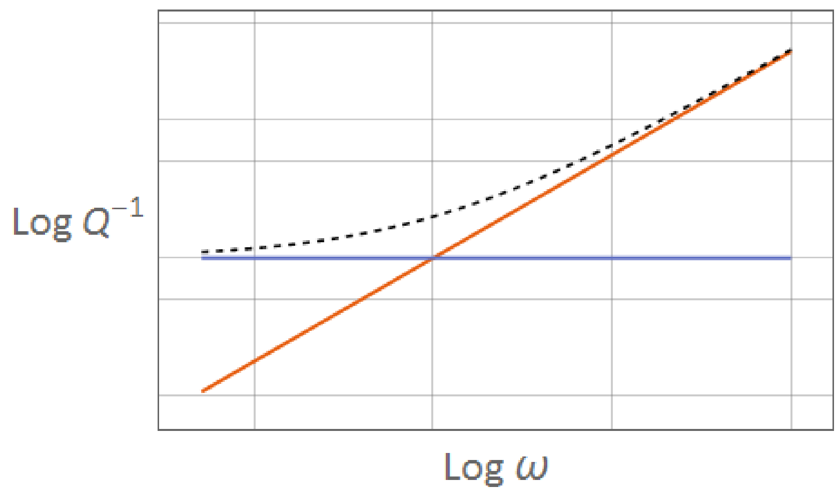
- Due to the strong singularity UA is dominated by energies near threshold

$$Q_{\mathrm{ph}-\eta}^{-1} = \frac{64\sqrt{\pi}}{3}\kappa_*^2\frac{\Delta^4}{\rho_m s^2 v^3}\left(\frac{\Delta}{T}\right)\left(\frac{\omega}{\Delta}\right)^{3/4} e^{-2\Delta/T}$$

Phase vs amplitude

- Different frequency and temperature behavior for phase and amplitude contributions:





- Both contributions can be identified. Crossovers in frequency and temperature dependence

$$Q^{-1} \propto T^4 \qquad Q^{-1} \propto \omega^{3/4} T^{-1} e^{-2\Delta/T}$$

V. Coulomb interaction

Coulomb interaction

- Introduce as

$$H[\boldsymbol{S}_i] = -2\sum_{i=1}^{N}(\xi_i + \Phi_i)S_i^z - \sum_{i,j=1}^{N} J_{ij}\left[S_i^x S_j^x + S_i^y S_j^y\right], \quad \langle\Phi\Phi\rangle_q = \frac{4\pi e^2}{\epsilon q^2}$$

- Anderson-Higgs mechanism: massless Goldstone is eaten by the gauge field

$$L^{-1}_{\phi+\Phi}(i\Omega_n, \boldsymbol{k}) = -(4W)^{-1}\left[A(i\Omega_n)(\Omega_n\phi + 2\Phi)^2 + v^2k^2\phi^2 + \frac{\epsilon k^2}{4\pi e^2}\Phi^2\right]$$

$$\Delta_\phi^2 = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2}{\epsilon}\frac{\hbar^2\rho_s}{e^2}$$

- However there is also contribution from the branch cut

$$Q^{-1}_{\mathrm{ph}-\phi} = f(\omega, T)e^{-2\Delta/T}$$

Coulomb interaction

- Introduce as

$$H[\boldsymbol{S}_i] = -2\sum_{i=1}^{N}(\xi_i + \Phi_i)S_i^z - \sum_{i,j=1}^{N} J_{ij}\left[S_i^x S_j^x + S_i^y S_j^y\right], \quad \langle\Phi\Phi\rangle_q = \frac{4\pi e^2}{\epsilon q^2}$$

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$$\Delta_\phi^2 = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2}{\epsilon}\frac{\hbar^2\rho_s}{e^2}$$

$$\left(L_\phi^R(\Omega, k)\right)^{-1} = \frac{\nu v^2 k^2}{4} \times \begin{cases} 1 - \frac{\epsilon W \Omega^2}{4\pi e^2 v^2}\left(1 + \left(\frac{\epsilon W k^2}{4\pi e^2}\right)\frac{\frac{\Omega}{2\Delta}\sqrt{1-\left(\frac{\Omega}{2\Delta}\right)^2}}{\arcsin\frac{\Omega}{2\Delta}}\right)^{-1} & \Omega < 2\Delta \\ 1 - \frac{\epsilon W \Omega^2}{4\pi e^2 v^2}\left(1 + \left(\frac{\epsilon W k^2}{4\pi e^2}\right)\frac{\frac{\Omega}{2\Delta}\sqrt{\left(\frac{\Omega}{2\Delta}\right)^2-1}}{\ln\left[\frac{\Omega}{2\Delta}+\sqrt{\left(\frac{\Omega}{2\Delta}\right)^2-1}\right]-i\frac{\pi}{2}}\right)^{-1} & \Omega \geq 2\Delta \end{cases}$$

Coulomb interaction

- Plasmon gap

$$\Delta_\phi^2 = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2}{\epsilon}\frac{\hbar^2 \rho_s}{e^2}$$

  - InO_x

$$\frac{\Delta_\phi^2}{4\Delta^2} \approx 500/\epsilon$$

Most probably, phason gap
Is larger than 2Δ

  - Special case is given by $SrTiO_3$, where $\epsilon \sim 10^4$.
    Very light doping makes it superconducting.
    Phase mode may remain relevant.

Lin et al (2015)

Conclusions

- Ultrasonic attenuation directly probes collective excitations in a pseudogapped state
- Two contributions to ultrasonic attenuation with contrasting temperature and frequency behaviors
  - Phase (Goldstone) $Q^{-1} \propto T^4$
  - Amplitude (Higgs) $Q^{-1} \propto \omega^{3/4} T^{-1} e^{-2\Delta/T}$
- Coulmb interaction freezes the phase mode

arXiv:1609.01683