Ultrasonic attenuation in a pseudogapped superconductor

A. Shtyk and M. Feigelman

I. Background and motivation

II. Collective modes in a pseudogapped state

III. Electron-phonon interaction

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   • Amplitude

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I. Background and motivation
Two scenarios of SIT*

- Spectral gap drops together with $T_c$:
  “fermionic” mechanism
- Spectral gap is nearly constant while $T_c$ drops to zero:
  “bosonic” mechanism ("preformed pairs")

*Superconductor to insulator transition
SIT: preformed pairs

- The spectral gap appears much before (with T down) than superconductive coherence does.
- Coherence peaks in the DoS appear together with resistance vanishing.

Baturina et al. 2008

Sacepe et al., 2011
Insulating state induced by magnetic field

G.Sambandamurthy, L. Engel, A. Johansson, E. Peled and D. Shahar

Inelastic electron-phonon scattering time in the insulating state

- Electron-phonon decoupling in disordered insulators
- Hysteresis in I-V curves. Overheating of electronic subsystem
- Fits the theory for e-ph interaction in disordered conductors

\[
\frac{V^2}{R(T_{el})} = \Gamma \Omega (T_{el}^\beta - T_{ph}^\beta),
\]

However the coupling is stronger by orders of magnitude

Altshuler et al. 2009

Ovadia et al. 2009; 2016
Electron-phonon cooling and ultrasound attenuation rate

\[ J = \frac{dE_{ph}}{dt} = J(T_{el}) - J(T_{ph}) \]

\[ J(T) = \int_0^\infty d\omega \omega \nu_{ph}(\omega) \frac{B_{ph}(\omega/T)}{\tau_{ph}(\omega)} \]

The phonon decay rate \( \tau_{ph}^{-1} \)

\[ \tau_{ph}^{-1}(\omega, T_{el}) = \frac{1}{\rho_m \omega} \text{Im} \Sigma^R(\omega, q, T_{el})|_{\omega = v_s q} \]

A. Shtyk, M. Feigelman & V. Kravtsov  PRL 111, 166603 (2013)
Previous search for enhanced inelastic e-ph rate

• **Before** (weak disorder) the core idea behind was:
  • Phonon couples to a slow diffusive mode of a conserved physical quantity (charge/spin/energy)
  • Competition with a “normal” attenuation due to standard local mechanism

  A.Shtyk et al,  *PRL* 111, 166603 (2013);  *PRB* 92, 195101 (2015)

• **Now**, within bosonic scenario of SIT:
  • Kill the single electron excitations completely and directly probe collective modes!
Collective SC gap versus single-particle gap


Motivation summary

• Preformed Cooper pairs: single electron gap $\Delta_1 > \Delta$ and “normal” attenuation is strongly suppressed

• Ultrasonic attenuation as a probe of collective excitations in a pseudogapped state

• Ultrasonic attenuation should be dominated by collective modes in a superconductor and exhibit activation behavior $\propto e^{-2\Delta/T}$
II. Collective modes in a pseudogapped state
Contribution of single-electron states is suppressed by pseudogap $\Delta_1 \gg T_c$.

**Pseudo spin** representation:

\[
\begin{align*}
S_+^\mu &= a_{\mu}^+ a_{\mu}, \\
S_-^\mu &= a_{\mu} a_{\mu}^+, \\
2S_z^\mu &= a_{\mu}^+ a_{\mu} + a_{\mu} a_{\mu}^+.
\end{align*}
\]

$\hat{H}_{\text{Bcs}}$ acts on **Even Sector**: all states which are 2-filled or empty.

\[
\overline{M}_{\mu\nu} = \frac{1}{V} M(3\mu - 3\nu)
\]

\[Z \sim \nu_0 T_c L_{\text{loc}}^d\]

**Pseudospin** approximation

Effective number of interacting **“neighbours”**
Represent spins-1/2 as semions with imaginary chemical potential. No additional constraints.
Fedotov-Popov trick

Fedotov, Popov (1988)

- Representing spins-1/2 as semions with imaginary chemical potential. **No additional constraints**

\[
S_i^\alpha \rightarrow (1/2)\psi_i^\dagger \sigma^\alpha \psi_i
\]

\[
H[S_i] \rightarrow H \left[ \frac{1}{2} \psi_i^\dagger \sigma \psi_i \right] + (i\pi/2\beta)N_F
\]

- Similar to Abrikosov pseudofermions but unphysical states are automatically taken care of by complex chemical potential \( \mu \in \mathbb{C} \)

- Standard fermionic diagrammatic rules but with Matsubara energies

\[
\varepsilon_l = 2\pi T(l + 1/4)
\]
Superconducting action

1. Start with pseudospins. XY-Heisenberg in a random field

\[ H[S_i] = -2 \sum_{i=1}^{N} \xi_i S_i^z - \sum_{i,j=1}^{N} J_{ij} \left[ S_i^x S_j^x + S_i^y S_j^y \right] \]

2. Introduce semions

\[ S_i^\alpha = (1/2) \psi_i^\dagger \sigma^\alpha \psi_i \]

3. Introduce \( \Delta \) via Hubbard-Stratonovich transformation

\[ H = - \sum_i \psi_i^\dagger \xi_i \sigma^z \psi_i - \frac{1}{4} \sum_{i,j} J_{ij}(\psi_i^\dagger \sigma^\alpha \psi_i)(\psi_j^\dagger \sigma^\alpha \psi_j) \]

\[ \mathcal{A}[\Delta] = - \text{Tr} \left[ \Delta^* J^{-1} \Delta \right] + \text{Tr} \ln \left[ i \varepsilon_l + \xi \sigma^z + \frac{1}{2} (\Delta \sigma^- + \Delta^* \sigma^+) \right] \]

\[ \left[ G(i\varepsilon_l) \right]^{-1} \]
Superconducting action

\[ A[\Delta] = -\text{Tr} \left[ \Delta^* J^{-1} \Delta \right] + \text{Tr} \ln \left[ i\varepsilon_l + \xi \sigma^z + \frac{1}{2}(\Delta \sigma^- + \Delta^* \sigma^+) \right] \]

- Box-shaped distribution of disorder. Mean field.
- The saddle point gives the self-consistency equation

\[ \nu(\xi) = (2W)^{-1} \Theta(W - |\xi|) \]

\[ 1 = g \int_{\xi} \frac{1}{\sqrt{\xi^2 + \Delta^2}} \tanh \frac{\sqrt{\xi^2 + \Delta^2}}{T} \]

\[ J(k) = g \left[ 1 - R^2 k^2 + O(k^4) \right] \]

- Two collective modes. Massive amplitude (Higgs) and massless phase (Goldstone)

\[ \Delta = \Delta_0 (1 + \eta)e^{i\phi} \]
SC propagator

\[ L_{\alpha\beta}^{-1} = J^{-1}(q)\delta_{\alpha\beta} \]

- The phase mode: a pole

\[ L_{\phi}^{R-1}(\Omega, k) = (4W)^{-1} [(\Omega + i0)^2 - v^2 k^2] \]

- The amplitude mode: a branch cut

\[ L_{\eta}^{R-1}(\Omega, Q) = -\frac{\Delta^2}{W} \left[ i\pi \sqrt{\frac{\Omega - 2\Delta}{4\Delta}} + \frac{v^2 k^2}{4\Delta^2} \right] , \quad (\Omega > 2\Delta, |\Omega - 2\Delta| \ll \Delta) \]
III. Electron-phonon interaction:

How to include phonons?
How to include phonons?

- Modification of the interaction/hopping
  \[ H_{\text{e-ph}}[S_i] = \kappa \sum_{i,j=1}^N (J_{ij} \text{ div } u) \left[ S_i^x S_j^x + S_i^y S_j^y \right] \]

- Within adiabatic limit \( \omega, q \to 0 \) \( (\lambda = J(0)/W) \)
  \[ \lambda \to \lambda(1 - \kappa \text{ div } u) \]
  \[ \Delta = 2We^{-1/\lambda} \to \Delta \left(1 + \frac{\kappa}{\lambda} \text{ div } u\right) \]

- Electron-phonon vertex reduces to modification of the (inverse) propagator. Similar to Ward identities
  \[ \Gamma_{\eta/\phi} = -\frac{\kappa}{\lambda} \cdot \frac{\partial L_{\eta/\phi}^{-1}}{\partial \ln \Delta} \]
IV. Ultrasonic attenuation due to collective modes
Ultrasound Attenuation: phase

- UA (after analytical continuation)
  \[
  \text{Im} \Sigma_{\text{ph-} \phi}^A (\omega, q) = q^2 \int_{Q,\Omega} |\Gamma_\phi|^2 (B_+ - B_-) (\text{Im} L_\phi^R) + (\text{Im} L_\phi^R)_-
  \]

- Propagator and vertex
  \[
  L_{\phi}^{-1}(\Omega,k) = (4W)^{-1} \left[(\Omega + i0)^2 - v^2 k^2 \right]
  \]
  \[
  \Gamma_\phi = -\frac{\kappa}{\lambda} \frac{\partial L_{\phi}^{-1}}{\partial \ln \Delta} = \frac{\kappa_\star v^2}{4W} [k \cdot (k + q)]
  \]

- Ultrasonic attenuation
  \[
  \alpha = \frac{1}{\rho_m \omega} \text{Im} \Sigma_{\text{ph-} \phi}^A (\omega, q)
  \]
  \[
  Q^{-1} = \frac{\alpha}{\omega}
  \]
  \[
  Q_{\phi}^{-1} = \frac{2\pi^4 \kappa_\star^2}{15} \frac{T^4}{\rho_m s v^4}
  \]
  \[
  Q_{\phi}^{-1} = \frac{2\pi^4}{15} \kappa^2 \left(\frac{a_0}{R}\right)^4 \left(\frac{T}{\Delta}\right)^4
  \]
Ultrasound Attenuation: amplitude

- Phonon modifies $\Delta$. Formally, it moves a branching point, which determines the most singular part of the electron-phonon vertex.

- Propagator and vertex

$$L_{\eta}^{R^{-1}}(\Omega, Q) = -\frac{\Delta^2}{W} \left[ i\pi \sqrt{\frac{\Omega - 2\Delta}{4\Delta}} + \frac{v^2 k^2}{4\Delta^2} \right],$$

$$\Gamma_{\eta} = -\frac{\kappa}{\lambda} \cdot \frac{\partial L_{\eta}^{-1}}{\partial \ln \Delta} \approx -\frac{i\pi}{2} \frac{\kappa_\ast \Delta^2}{W} \sqrt{\frac{\Delta}{\Omega - 2\Delta}}.$$

- Due to the strong singularity, UA is dominated by energies near threshold.

$$Q_{\text{ph}^{-1} - \eta} = \frac{64 \sqrt{\pi}}{3} \frac{k^2}{\rho m s^2 v^3} \left(\frac{\Delta}{T}\right) \left(\frac{\omega}{\Delta}\right)^{3/4} e^{-2\Delta/T}.$$
Phase vs amplitude

• Different frequency and temperature behavior for phase and amplitude contributions:

• Both contributions can be identified. Crossovers in frequency and temperature dependence

\[ Q^{-1} \propto T^4 \]

\[ Q^{-1} \propto \omega^{3/4} T^{-1} e^{-2\Delta/T} \]
V. Coulomb interaction
Coulomb interaction

- Introduce as

\[
H[S_i] = -2 \sum_{i=1}^{N} (\xi_i + \Phi_i) S_{i}^z - \sum_{i,j=1}^{N} J_{ij} \left[ S_{i}^x S_{j}^x + S_{i}^y S_{j}^y \right], \quad \langle \Phi \Phi \rangle_q = \frac{4\pi e^2}{\epsilon q^2}
\]

- Anderson-Higgs mechanism: massless Goldstone is eaten by the gauge field

\[
L_{\phi+\Phi}^{-1}(i\Omega_n, k) = -(4W)^{-1} \left[ A(i\Omega_n)(\Omega_n\phi + 2\Phi)^2 + v^2 k^2 \phi^2 + \frac{\epsilon k^2}{4\pi e^2} \Phi^2 \right]
\]

\[
\Delta_{\phi}^2 = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2}{\epsilon} \frac{\hbar^2 \rho_s}{e^2}
\]

- However there is also contribution from the branch cut

\[
Q_{\phi}^{-1} = f(\omega, T)e^{-2\Delta/T}
\]
Coulomb interaction

- Introduce as

\[ H[S_i] = -2 \sum_{i=1}^{N} (\xi_i + \Phi_i) S_i^z - \sum_{i,j=1}^{N} J_{ij} [S_i^x S_j^x + S_i^y S_j^y] , \quad \langle \Phi \Phi \rangle_q = \frac{4\pi e^2}{\epsilon q^2} \]

- Anderson-Higgs mechanism: massless Goldstone is eaten by gauge field

\[
L^{-1}_{\phi+\Phi} (i\Omega_n, k) = -(4W)^{-1} \left[ A(i\Omega_n)(\Omega_n\Phi + 2\Phi)^2 + v^2 k^2 \phi^2 + \frac{\epsilon k^2}{4\pi e^2} \Phi^2 \right]
\]

\[
\Delta^2_{\phi} = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2}{\epsilon} \frac{\hbar^2 \rho_s}{e^2}
\]

\[
(L^R_{\phi}(\Omega, k))^{-1} = \frac{\nu v^2 k^2}{4} \times \begin{cases} 
1 - \frac{\epsilon W \Omega^2}{4\pi e^2 v^2} \left( 1 + \left( \frac{\epsilon W k^2}{4\pi e^2} \right) \frac{\Omega}{2\Delta} \sqrt{1 - \left( \frac{\Omega}{2\Delta} \right)^2} \right)^{-1} & \Omega < 2\Delta \\
1 - \frac{\epsilon W \Omega^2}{4\pi e^2 v^2} \left( 1 + \left( \frac{\epsilon W k^2}{4\pi e^2} \right) \frac{\Omega}{2\Delta} \sqrt{1 - \left( \frac{\Omega}{2\Delta} \right)^2 - 1} \right)^{-1} & \Omega \geq 2\Delta
\end{cases}
\]
Coulomb interaction

- Plasmon gap

\[ \Delta_{\phi}^2 = \Delta^2 \cdot \frac{4\pi e^2}{\epsilon a J} = \frac{4\pi e^2 \hbar^2 \rho_s}{\epsilon e^2} \]

- InO\textsubscript{x}

\[ \frac{\Delta_{\phi}^2}{4\Delta^2} \approx \frac{500}{\epsilon} \quad \text{Most probably, phason gap}
\] \[ \text{Is larger than } 2\Delta \]

- Special case is given by SrTiO\textsubscript{3}, where \( \epsilon \approx 10^4 \).

Very light doping makes it superconducting.\textsuperscript{Lin et al (2015)}

Phase mode may remain relevant.
Conclusions

• Ultrasonic attenuation directly probes collective excitations in a pseudogapped state

• Two contributions to ultrasonic attenuation with contrasting temperature and frequency behaviors
  • Phase (Goldstone) \( Q^{-1} \propto T^4 \)
  • Amplitude (Higgs) \( Q^{-1} \propto \omega^{3/4} T^{-1} e^{-2\Delta/T} \)

• Coulomb interaction freezes the phase mode

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