Dynamics of Majorana fermions: topological protection and teleportation

A. S. Mel'nikov

Institute for Physics of Microstructures RAS

Nizhny Novgorod, Russia

I. M. Khaymovich **J. P. Pekola**

Aalto University, Low Temperature Laboratory, Finland

Outline

- **Introduction.**

Majorana states. Bogolubov transformation etc.
Examples: p-wave superconductors, semiconducting nanowires and topological insulators with induced superconductivity.

- **Teleportation as a test for Majorana particles.**

Nonlocality vs absence of noise correlations.
Coulomb blockade as a recipe to restore nonlocality

- **NISIN. DC transport**

- **NISIN. AC response.**

- **Teleportation paradox and topological stability of Majorana states.**

Ettore Majorana
1906-?

Bosons – $\psi(r_1,r_2) = \psi(r_2,r_1)$ $[c_i, c_j^{+}] = \delta_{ij}$ $[c_i, c_j] = 0$

Fermions – $\psi(r_1,r_2) = -\psi(r_2,r_1)$ $\{c_i, c_j^{+}\} = \delta_{ij}$ $\{c_i, c_j\} = 0$ $c_i^2 = 0$

Majorana fermions –

$\{\gamma_i, \gamma_j\} = 2\delta_{ij}$

$\gamma_i = \gamma_i^{+}$ *particle = antiparticle*

$\gamma_i^2 = 1$ **no** Pauli principle but distinct from bosons

$c = {}^1/_2(\gamma_1 + i\gamma_2)$ with $\gamma_i = \gamma_i^{+}$

$\gamma_1 = c + c^{+}$ $\gamma_2 = (1/i)(c - c^{+})$

BCS mean field theory.
Bogolubov canonical transformation.
No changes in the operator commutation rules

Annihilation and creation electron operators →

$$\hat{\Psi}_\alpha(\vec{r}) = \sum_n \left(u_{\alpha n}(\vec{r})\hat{c}_n + v^*_{\alpha n}(\vec{r})\hat{c}^+_n \right)$$

$$\hat{\Psi}^+_\alpha(\vec{r}) = \sum_n \left(u^*_{\alpha n}(\vec{r})\hat{c}^+_n + v_{\alpha n}(\vec{r})\hat{c}_n \right)$$

← Annihilation and creation quasiparticle operators

Inverse transformation

$$\hat{c}_n = \sum_\alpha \int d^3r \left(u^*_{\alpha n}(\vec{r})\hat{\Psi}_\alpha + v^*_{\alpha n}(\vec{r})\hat{\Psi}^+_\alpha \right)$$

$$\hat{c}^+_n = \sum_\alpha \int d^3r \left(u_{\alpha n}(\vec{r})\hat{\Psi}^+_\alpha + v_{\alpha n}(\vec{r})\hat{\Psi}_\alpha \right)$$

Fermi commutation rules:

$$\left\{c_n^+ c_m\right\} = \delta_{nm} \qquad \left\{c_n c_m\right\} = 0$$

Orthogonality condition:

$$\sum_\alpha \int \left(u_{\alpha\lambda}(\vec{r})u^*_{\alpha\nu}(\vec{r}) + v_{\alpha\lambda}(\vec{r})v^*_{\alpha\nu}(\vec{r})\right) d^3r = \delta_{\lambda\nu}$$

Complete set of functions:

$$\sum_\lambda u_{\alpha\lambda}(\vec{r})u^*_{\beta\lambda}(\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')\delta_{\alpha\beta}$$

$$\sum_\lambda v_{\alpha\lambda}(\vec{r})u^*_{\beta\lambda}(\vec{r}\,') = 0$$

Bogolubov – de Gennes equations and their symmetry

$$\left(\hat{H}-\mu\right)u_\alpha + \int \Delta_{\alpha\beta}(r,r')v_\beta(r')d^3r' = \varepsilon u_\alpha$$

$$\int \Delta^{+}_{\alpha\beta}(r',r)u_\beta(r')d^3r' + \left(\mu-\hat{H}^*\right)v_\alpha = \varepsilon v_\alpha$$

$$\Delta_{\alpha\beta}(\mathbf{r},\mathbf{r}') = -\Delta_{\beta\alpha}(\mathbf{r}',\mathbf{r})$$

$$\varepsilon \to -\varepsilon$$

$$\begin{pmatrix} u_\alpha \\ v_\alpha \end{pmatrix} \to \begin{pmatrix} v^*_\alpha \\ u^*_\alpha \end{pmatrix}$$

All states come in pairs???

Self-consistency condition.

$$\Delta_{\alpha\beta} = \frac{1}{4} U_{\alpha\beta,\delta\gamma}(\mathbf{r},\mathbf{r}') \sum_n (1 - 2f(\epsilon_n)) \left(v^*_{\gamma,n}(\mathbf{r}) u_{\delta,n}(\mathbf{r}') - v^*_{\delta,n}(\mathbf{r}') u_{\gamma,n}(\mathbf{r}) \right)$$

Singlet pairing

$$\Delta_{\alpha\beta}(r,r') = i\sigma_y D(r,r')$$

$$D(r,r') = D(r',r)$$

$$\varepsilon \to -\varepsilon$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \to \begin{pmatrix} -v^* \\ u^* \end{pmatrix}$$

Triplet pairing

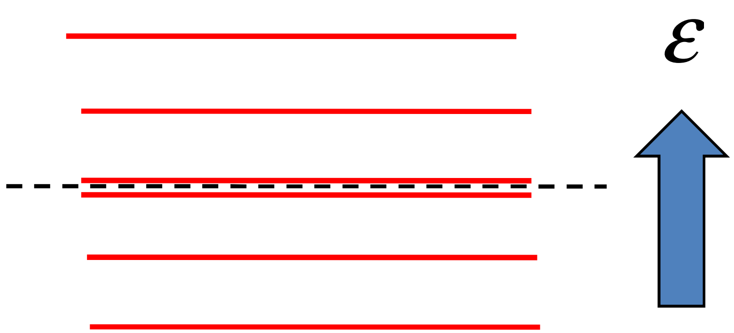
$$\Delta_{\alpha\beta}(r,r') = i\sigma_y \vec{\sigma}\vec{D}(r,r')$$

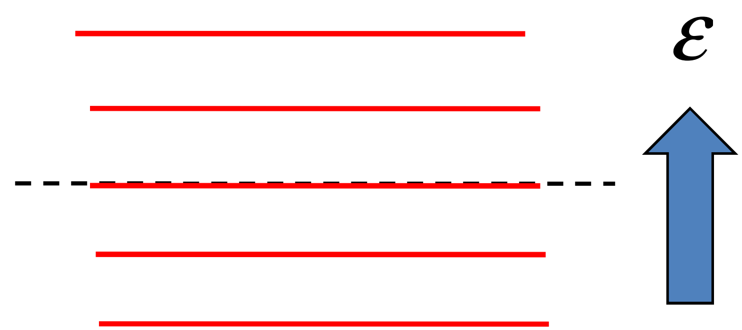
$$\vec{D}(r,r') = -\vec{D}(r',r)$$

$$\varepsilon \to -\varepsilon$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \to \begin{pmatrix} v^* \\ u^* \end{pmatrix}$$

Is it possible to get a state without a partner?
Majorana state





$$c^{+} \neq c$$

Standard fermions (with usual commutation rules)

$$u_{\alpha} = v_{\alpha}^{*} \qquad c^{+} = c$$

???? Majorana fermions (not fermions at all)

$$c^{+}c + cc^{+} = 1 \qquad cc + cc = 0$$

Obvious contradiction:
We can not change statistics using canonical Bogolubov tranformation

Partly defined quasiparticle

$$\hat{\Psi}_\alpha(\vec{r}) = u_{\alpha 0}(\vec{r})(\hat{c}_0 + \hat{c}_0^+) + \sum_{n \neq 0} \left(u_{\alpha n}(\vec{r})\hat{c}_n + v_{\alpha n}^*(\vec{r})\hat{c}_n^+ \right)$$

$$\hat{\Psi}_\alpha^+(\vec{r}) = u_{\alpha 0}^*(\vec{r})(\hat{c}_0 + \hat{c}_0^+) + \sum_{n \neq 0} \left(u_{\alpha n}^*(\vec{r})\hat{c}_n^+ + v_{\alpha n}(\vec{r})\hat{c}_n \right)$$

$$\hat{a} = (\hat{c}_0 + \hat{c}_0^+)/2 = \sum_\alpha \int d^3 r \left(u_{\alpha 0}^*(\vec{r})\hat{\Psi}_\alpha + u_{\alpha 0}(\vec{r})\hat{\Psi}_\alpha^+ \right)$$

$$\hat{c}_0 = \hat{a} + i\hat{b}$$

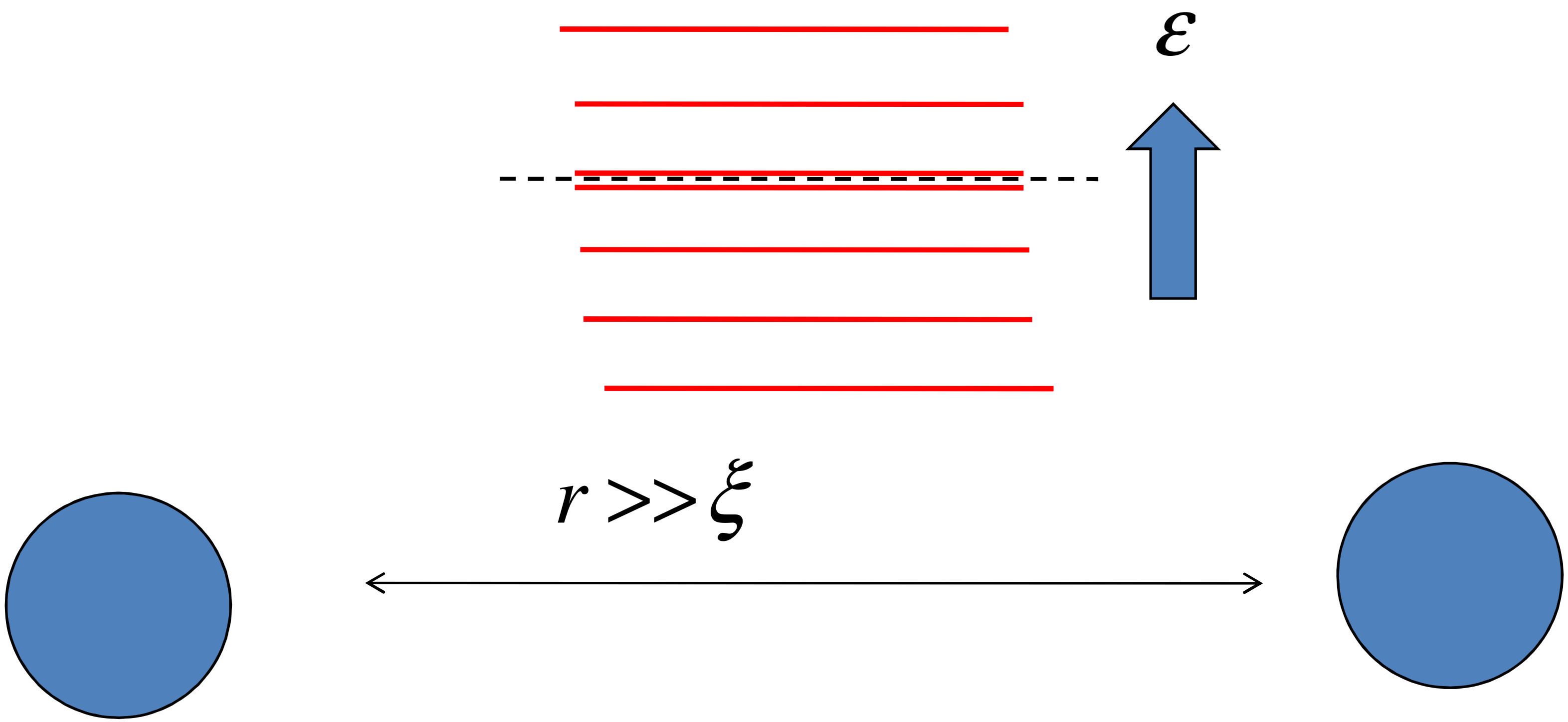
How to define this b-part???

Possible answer: **Let us find another ill-defined quasiparticle!**

A standard way to overcome the problem:

We construct the operator b from another zero energy state

The states which define a and b are far away from each other



Examples:

vortices in p-wave superconductors (G.E.Volovik, 1997)

Edge states (Kitaev 1D p-wave superconductor)

Systems with induced superconductivity

P-wave superconductors. Sr_2RuO_4 as a possible candidate? He-3

Free vortex

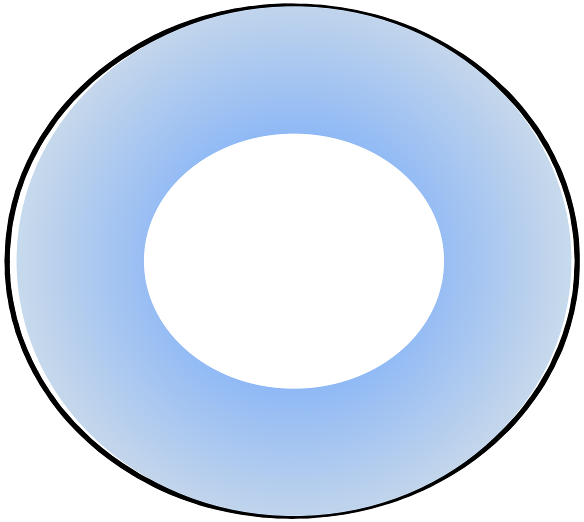
$$\Delta = |\Delta(r)|\, e^{i\theta_p \pm i\theta}$$

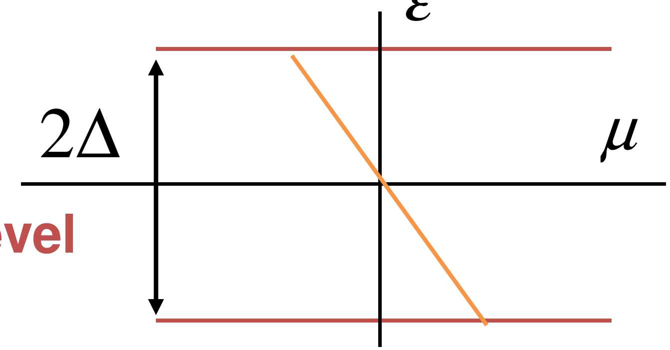
$$\varepsilon_\mu(k_\perp) = -\omega\mu \approx -\frac{\mu\Delta_0}{k_\perp \xi}$$

$$\int_0^{2\pi} \mu(\theta_p)\, d\theta_p = 2\pi(n+\beta)$$

$$\beta = 0$$

Edge states



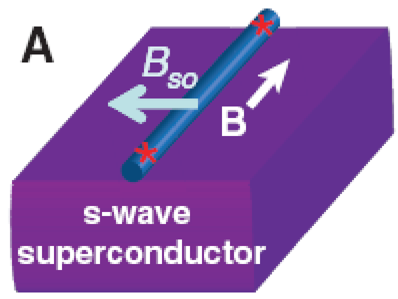


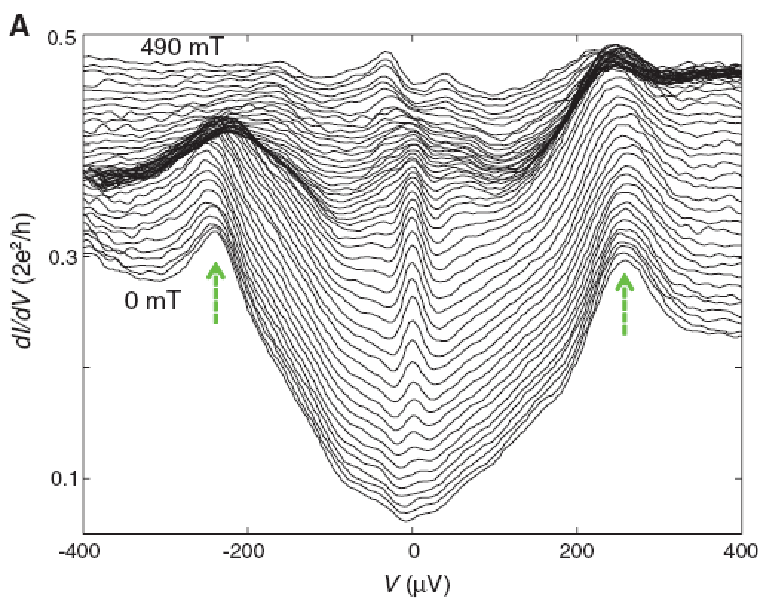
Systems with induced superconducting order

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1]* K. Zuo,[1]* S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

We use InSb nanowires (*15*), which are known to have strong spin-orbit interaction and a large g factor (*16*). From our earlier quantum-dot experiments, we extract a spin-orbit length $l_{so} \approx 200$ nm corresponding to a Rashba parameter $\alpha \approx 0.2$ eV·Å (*17*). This translates to a spin-orbit energy scale $\alpha^2 m^*/(2\hbar^2) \approx 50$ µeV (m^* =





nature physics

ARTICLES

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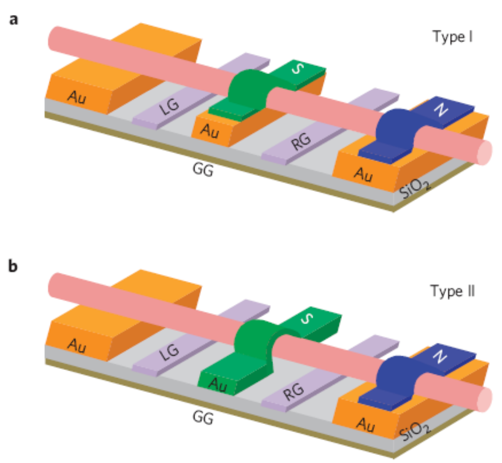
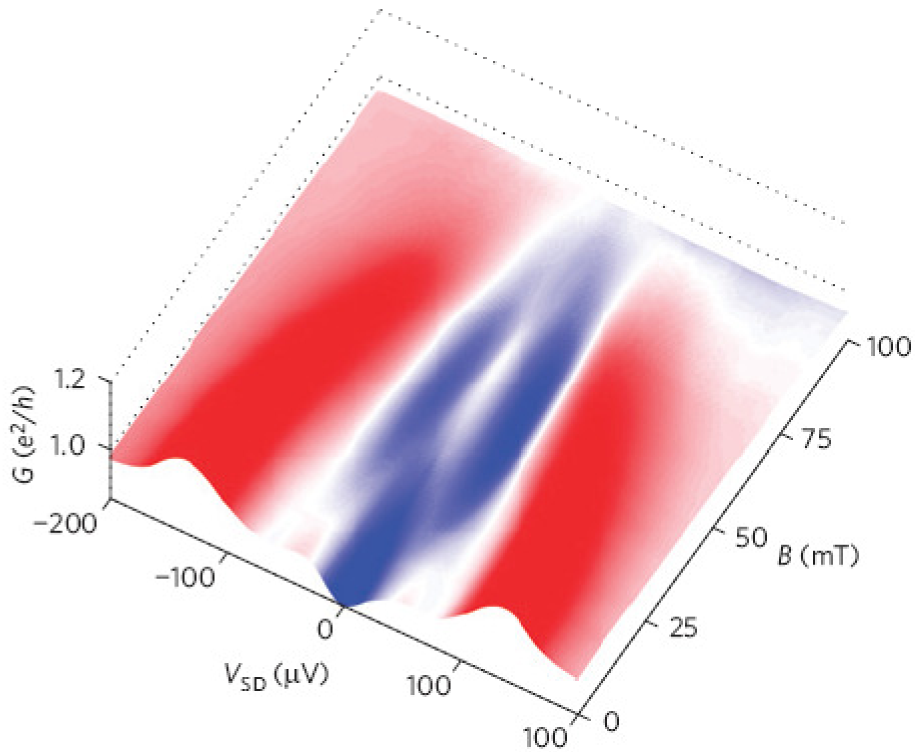
Zero-bias peaks and splitting in an Al–InAs nanowire topological superconductor as a signature of Majorana fermions

Anindya Das†, Yuval Ronen†, Yonatan Most, Yuval Oreg, Moty Heiblum* and Hadas Shtrikman





nature physics

LETTERS

PUBLISHED ONLINE: 14 SEPTEMBER 2015 | DOI: 10.1038/NPHYS3461

Parity lifetime of bound states in a proximitized semiconductor nanowire

A. P. Higginbotham[1,2†], S. M. Albrecht[1†], G. Kiršanskas[1], W. Chang[1,2], F. Kuemmeth[1], P. Krogstrup[1], T. S. Jespersen[1], J. Nygård[1,3], K. Flensberg[1] and C. M. Marcus[1*]

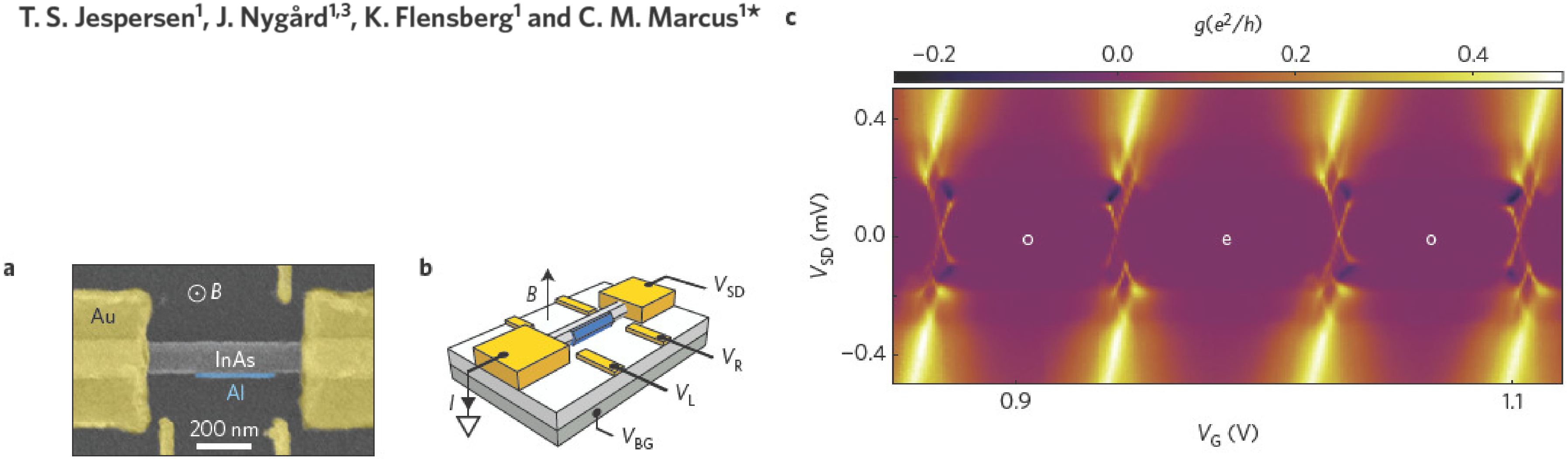
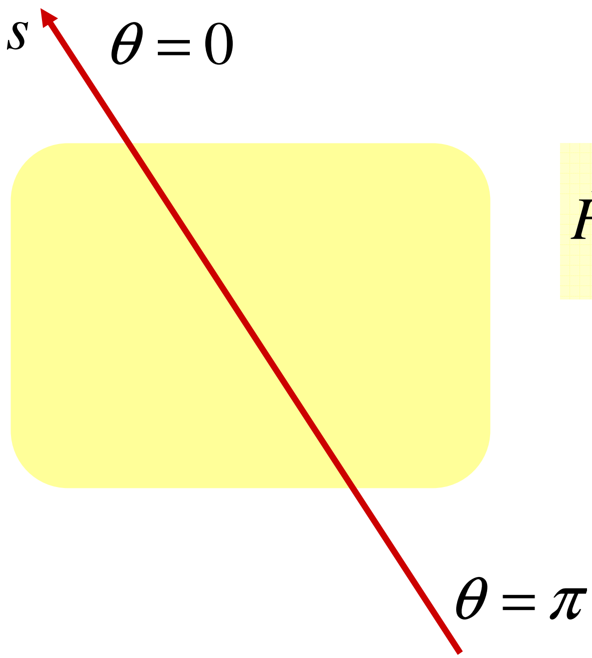


Figure 1 | Nanowire-based hybrid quantum dot. a, Scanning electron micrograph of the reported device, consisting of an InAs nanowire (grey) with a segment of epitaxial Al on two facets (blue) and Ti/Au contacts and side gates (yellow) on a doped silicon substrate with 100 nm oxide. **b**, Device schematic and measurement set-up, showing the orientation of the magnetic field, *B*. **c**, Differential conductance, *g*, as a function of effective gate voltage, V_G, and source-drain voltage, V_{SD}, at $B=0$. Even (e) and odd (o) occupied Coulomb valleys are labelled.

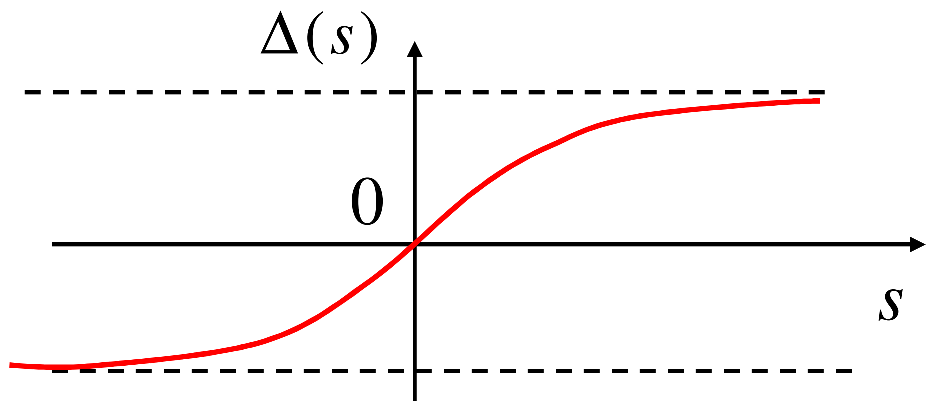
General recipe how to arrange zero energy states (at the Fermi level).

superconducting phase should change by pi



$$\hat{H} = -i\hbar V_{\perp}\hat{\sigma}_z \frac{\partial}{\partial s} + \hat{\sigma}_x \,\mathrm{Re}\,\Delta(s) - \hat{\sigma}_y \,\mathrm{Im}\,\Delta(s,b)$$

$$\hat{\Psi}_0 = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix} \exp\left(-\frac{1}{\hbar V_{\perp}} \int_0^s \mathrm{Re}\,\Delta(t)\,dt \right)$$

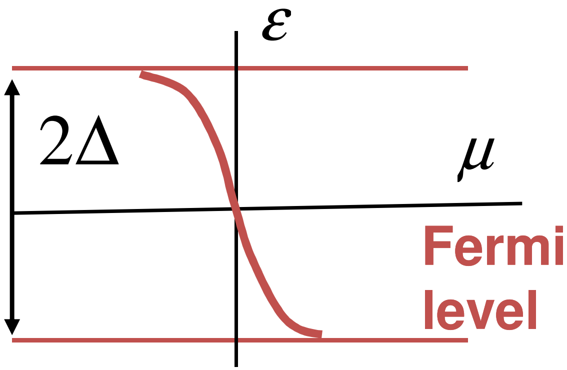


energy=0

Examples:

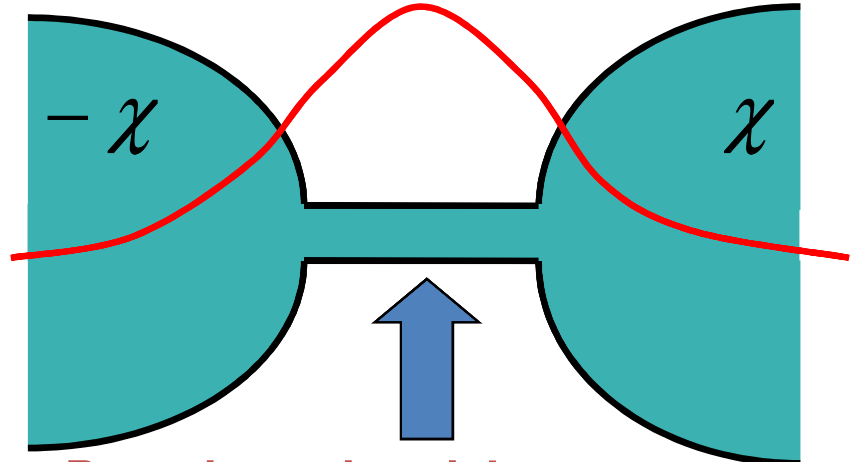
vortex

Anomalous spectral branch.



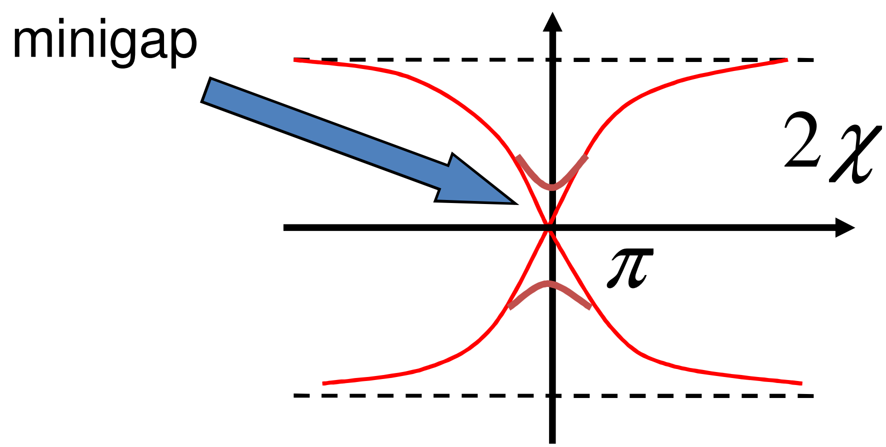
$$\varepsilon_\mu(k_\perp) = -\omega\mu \approx -\frac{\mu\Delta_0}{k_\perp \xi}$$

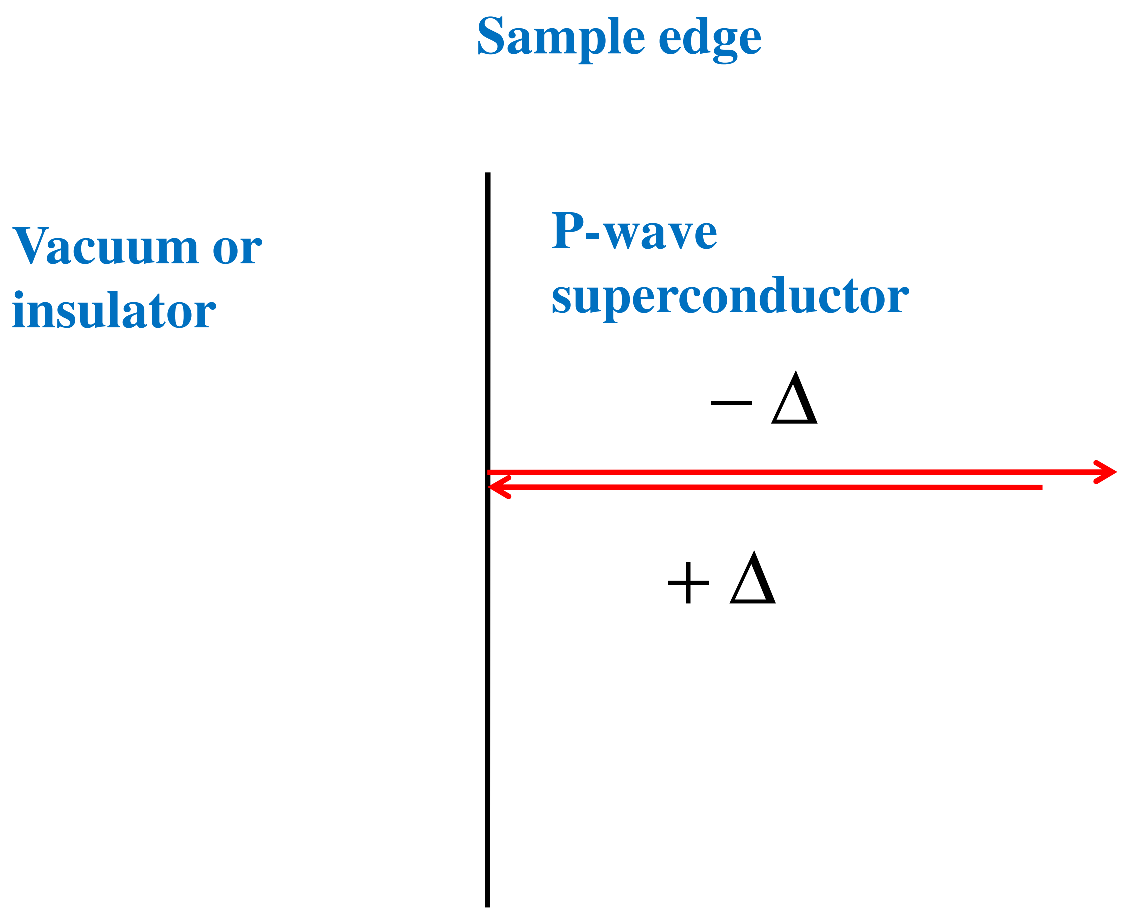
Josephson junction



Bound quasiparticle states

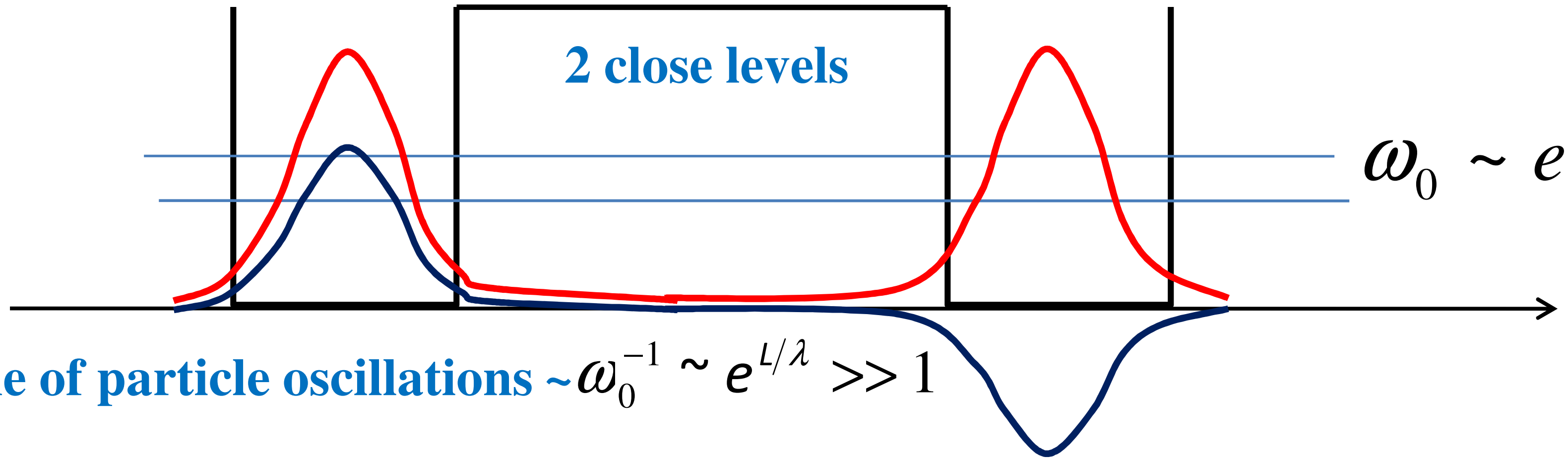
$$E/\Delta = \pm\cos\chi$$




Sample edge
Vacuum or insulator
P-wave superconductor
$-\Delta$
$+\Delta$

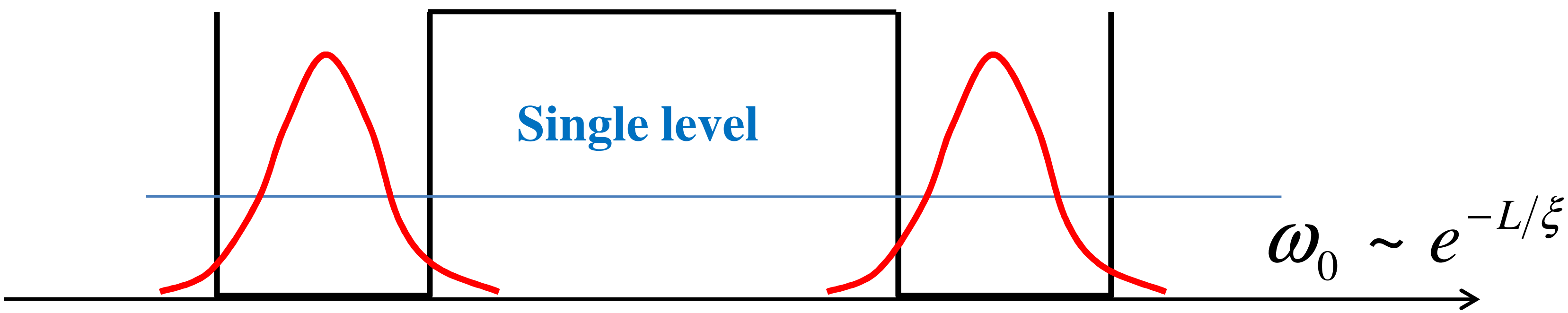
Nonlocality as an inherent property of Majorana particles. Teleportation. Topological protection against perturbations.

Oscillations of the wavefunction between two quantum wells



Time of particle oscillations $\sim \omega_0^{-1} \sim e^{L/\lambda} >> 1$

Teleportation of the wavefunction between two Andreev wells



Time of particle transfer =0

IOP PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **40** (2007) 1479–1488 doi:10.1088/0953-4075/40/8/002

Stretched quantum states emerging from a Majorana medium

Gordon W Semenoff[1] and Pasquale Sodano[2,3]

PRL **100,** 027001 (2008) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2008

Testable Signatures of Quantum Nonlocality in a Two-Dimensional Chiral *p*-Wave Superconductor

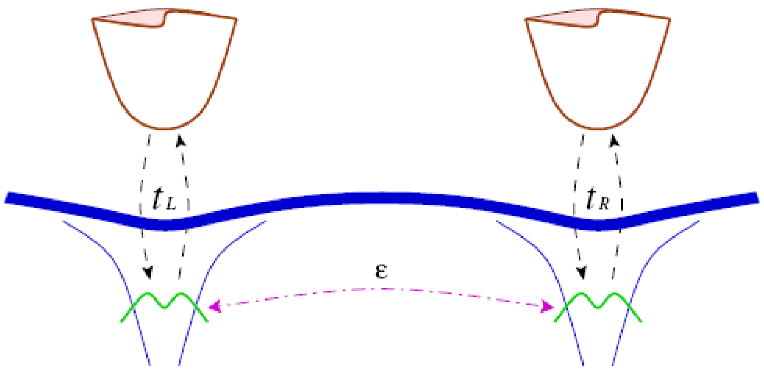
Sumanta Tewari,[1] Chuanwei Zhang,[1] S. Das Sarma,[1] Chetan Nayak,[2,3] and Dung-Hai Lee[4,5]

Nonlocality vs absence of noise correlations.

PRL **98,** 237002 (2007) PHYSICAL REVIEW LETTERS week ending 8 JUNE 2007

Observing Majorana bound States in *p*-Wave Superconductors Using Noise Measurements in Tunneling Experiments

C. J. Bolech[1,2] and Eugene Demler[1]
[1]*Physics Department, Harvard University, Cambridge Massachusetts 02138, USA*
[2]*Physics&Astronomy Department, Rice University, Houston Texas 77005, USA*
(Received 27 July 2006; published 5 June 2007)



$$\frac{S_{\alpha\beta}(\omega=0)}{e^2}=\coth\left(\frac{eV}{2T}\right)\left\{2\delta_{\alpha\beta}\frac{I}{e}-\frac{\Gamma^2}{2\pi}\left[\frac{(\omega'-\varepsilon)}{(\omega'-\varepsilon)^2+\Gamma^2}\right]_{-\mu}^{+\mu}-\alpha\beta\frac{\Gamma^2}{4\pi\varepsilon}\ln\frac{(\varepsilon+\mu)^2+\Gamma^2}{(\varepsilon-\mu)^2+\Gamma^2}\right\}.$$

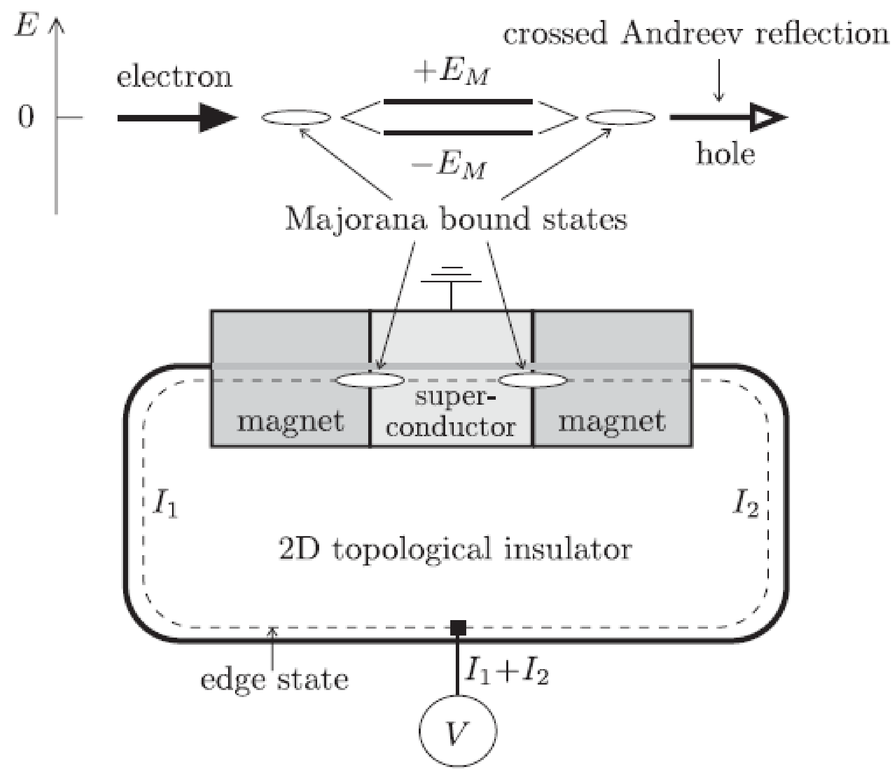
Notice that the diagonal and off-diagonal matrix components of $S_{\alpha\beta}$ are different now. In particular, we remark that $\lim_{\varepsilon\to 0}S_{\alpha\bar{\alpha}}=0$. Taken together with the result given above for the current, this indicates that in the $\varepsilon\to 0$ limit, the right and left tunneling processes are completely independent even at the level of current fluctuations. It is

Interplay between the Andreev reflection at the end of the wire and crossed Andreev reflection with the transmission of a hole to the second lead

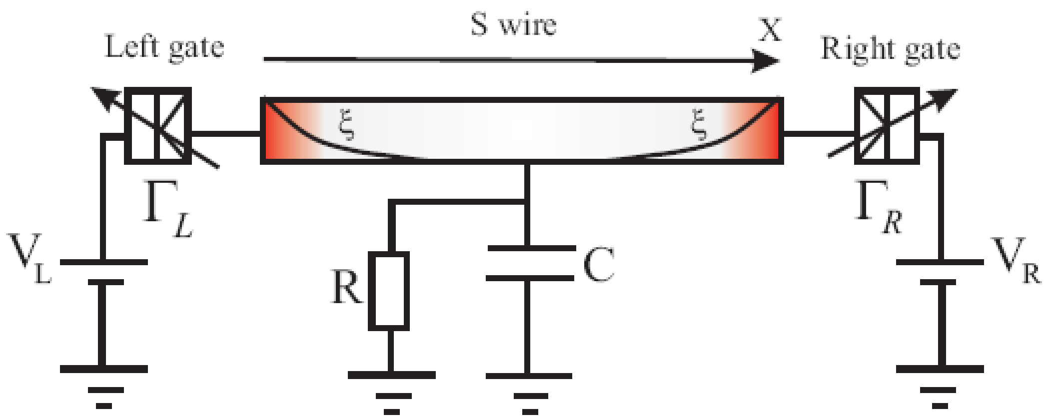
PRL **101,** 120403 (2008) PHYSICAL REVIEW LETTERS week ending 19 SEPTEMBER 2008

Splitting of a Cooper Pair by a Pair of Majorana Bound States

Johan Nilsson, A. R. Akhmerov, and C. W. J. Beenakker
Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
(Received 3 July 2008; revised manuscript received 15 August 2008; published 18 September 2008)



Coulomb blockade as a recipe to restore nonlocality?



PRL **104,** 056402 (2010) PHYSICAL REVIEW LETTERS week ending 5 FEBRUARY 2010

Electron Teleportation via Majorana Bound States in a Mesoscopic Superconductor

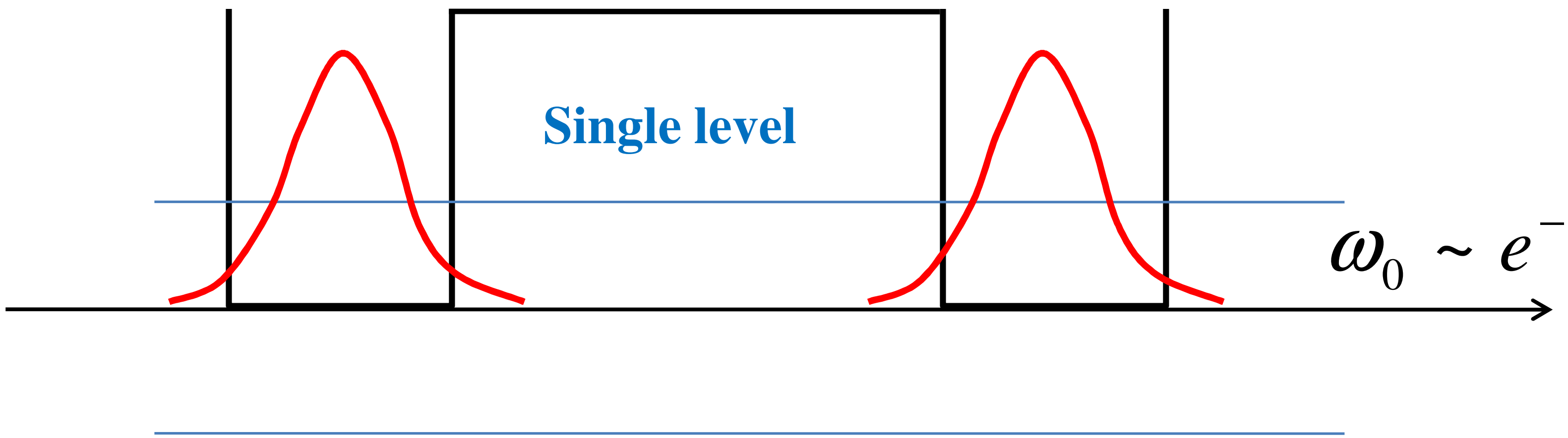
Liang Fu

$$\tilde{H} = H_L + \delta\left(f^\dagger f - \frac{1}{2}\right) + (\lambda_1 c_1^\dagger f + \text{H.c.}) + (-1)^{n_0}(-i\lambda_2 c_2^\dagger f + \text{H.c.}).$$

How to avoid teleportation?

We need to excite another level with negative energy!

Time of particle transfer~ $\omega_0 \sim e^{-L/\xi}$



$\omega_0 \sim e^{-L/\xi}$

Is it possible to excite both the positive and negative energy levels on equal footing and to get a two level problem?

It seems to be impossible
since there is only 1 fermion corresponding to these states!

$$\hat{\Psi}_\alpha(\mathbf{r},t) = \sum_n \left(u_{\alpha,n}(\mathbf{r},t)\hat{c}_n + v^*_{\alpha,n}(\mathbf{r},t)\hat{c}^\dagger_n \right)$$

$$\hat{\Psi}^\dagger_\alpha(\mathbf{r},t) = \sum_n \left(u^*_{\alpha,n}(\mathbf{r},t)\hat{c}^\dagger_n + v_{\alpha,n}(\mathbf{r},t)\hat{c}_n \right)$$

$$i\frac{\partial}{\partial t}\hat{g}_n = \begin{pmatrix} \hat{H}_0 - \mu & \hat{\Delta} \\ \hat{\Delta}^\dagger & \mu - \hat{H}^*_0 \end{pmatrix} \hat{g}_n$$

Despite of the obvious fact that both levels correspond to the only fermion the nonequilibrium time-dependent solutions $\hat{g}_n(\mathbf{r},t)$ of the BdG equations contain contributions corresponding to both levels.

In equilibrium:

One fermion → ***Wavefunction:***

One BdG level (with positive energy)

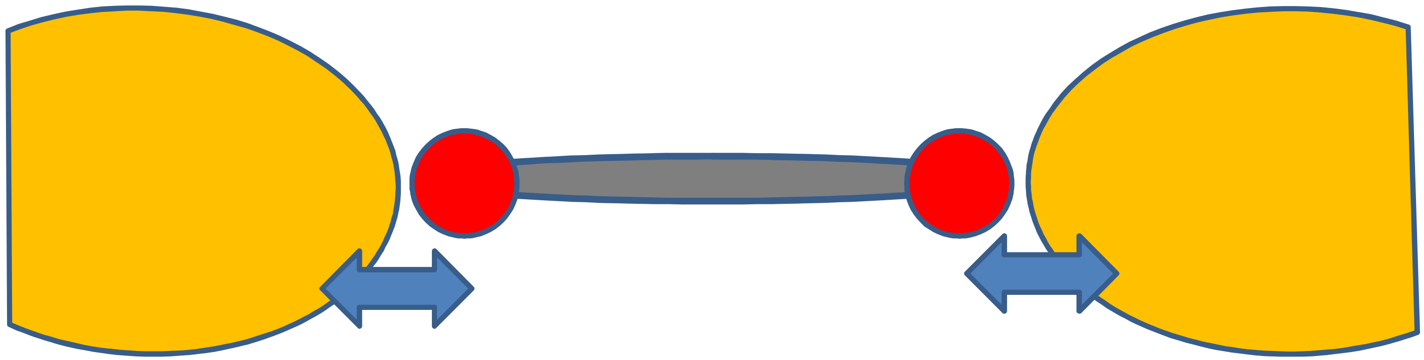
In non-equilibrium:

One fermion → ***Wavefunction:***

Superposition of many BdG levels (with positive and negative energies)

Some details of charge transfer through the low energy states in NISIN system

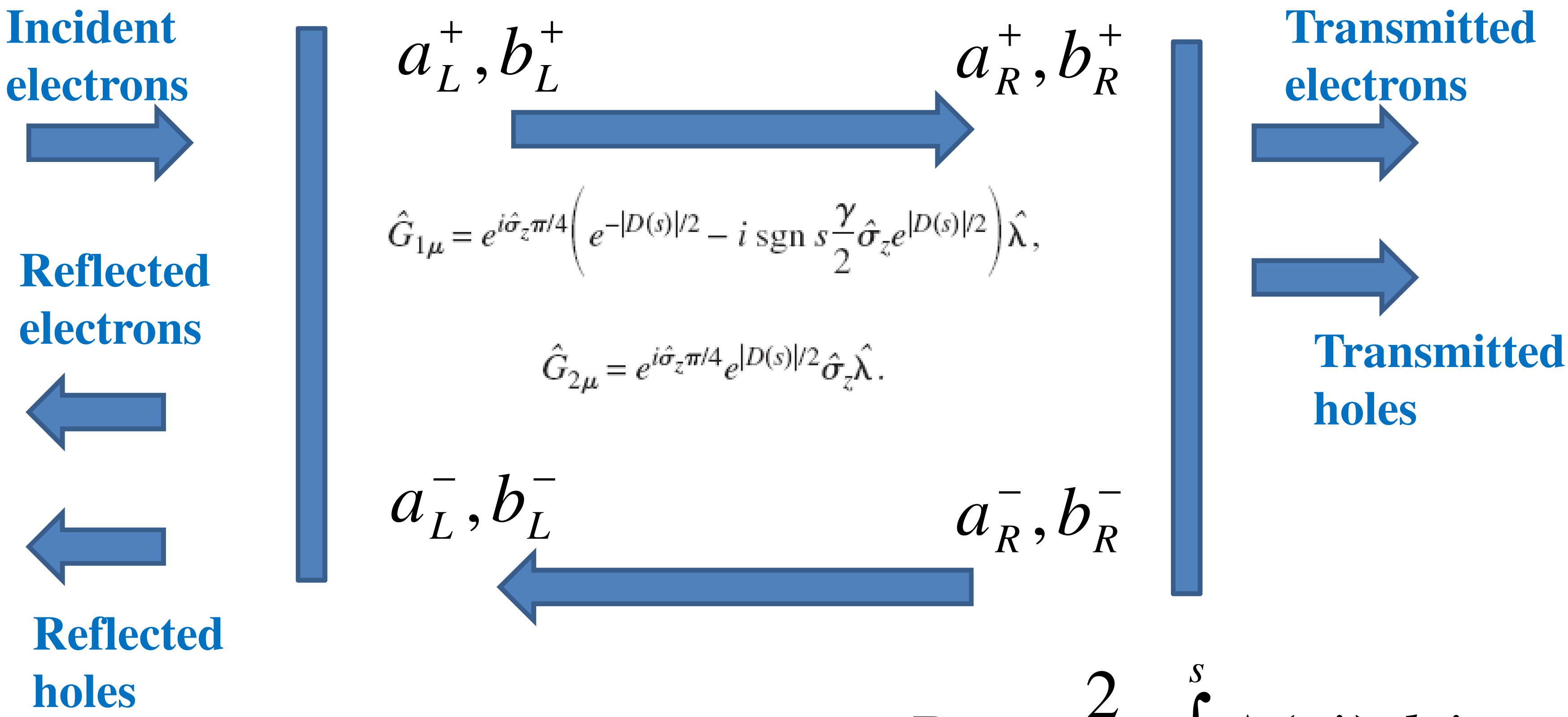
Left normal lead **Right normal lead**



We can construct the operator b from the states in the lead.

Then we can get a 2 level system

Solution of the scattering problem



$$\hat{G}_{1\mu} = e^{i\hat{\sigma}_z \pi/4}\left(e^{-|D(s)|/2} - i \,\mathrm{sgn}\, s \frac{\gamma}{2} \hat{\sigma}_z e^{|D(s)|/2} \right)\hat{\lambda},$$

$$\hat{G}_{2\mu} = e^{i\hat{\sigma}_z \pi/4} e^{|D(s)|/2} \hat{\sigma}_z \hat{\lambda}.$$

$$D = \frac{2}{\hbar V_F} \int_0^s \Delta(s')ds'$$

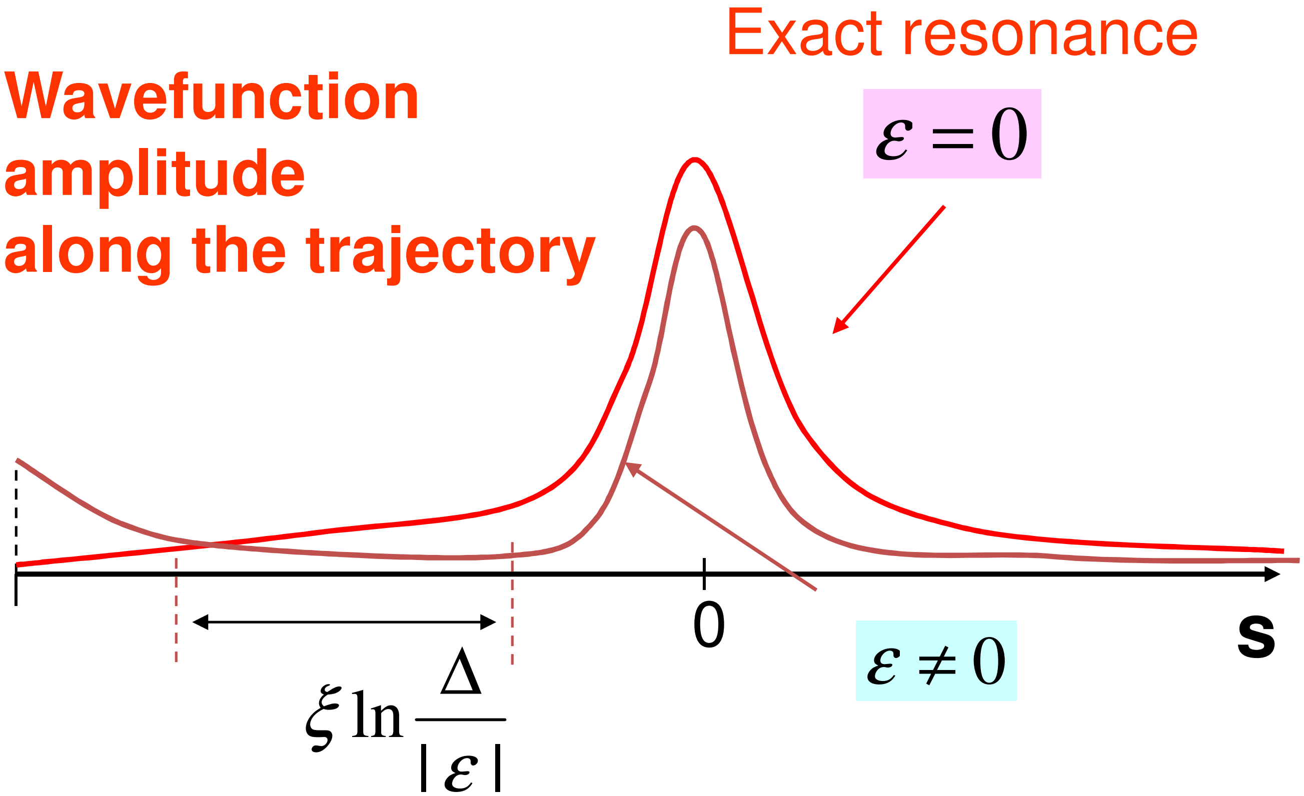
N. B. Kopnin et al
PRB **68,** 054528 (2003);
PRB **75**, 024514 (2007)

Wave functions near the resonance Andreev level

$$\hat{G}_{1\mu} = e^{i\hat{\sigma}_z \pi/4}\left(e^{-|D(s)|/2} - i \,\mathrm{sgn}\, s \frac{\gamma}{2} \hat{\sigma}_z e^{|D(s)|/2} \right) \hat{\lambda},$$

$$\hat{G}_{2\mu} = e^{i\hat{\sigma}_z \pi/4} e^{|D(s)|/2} \hat{\sigma}_z \hat{\lambda}.$$

Wavefunction amplitude along the trajectory



Analogy: tunneling between two vortices

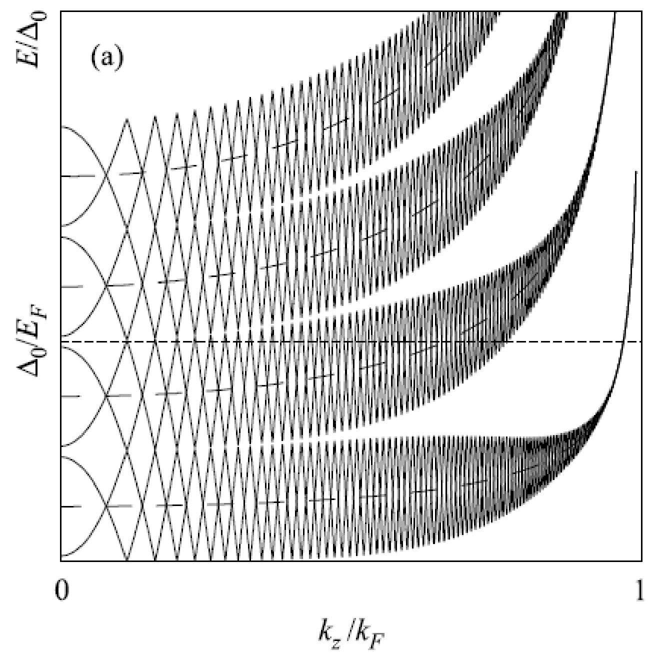
Pis'ma v ZhETF, vol. 83, iss. 12, pp. 675 – 680

Intervortex quasiparticle tunneling and electronic structure of multi-vortex configurations in type-II superconductors

A. S. Mel'nikov[1], *M. A. Silaev*

Splitting of vortex core levels:

$$\delta\varepsilon \sim \Delta e^{-L/\zeta} \cos(k_F L + \delta)$$



Equations for coupled Majorana states

$$a_L^+ + a_L^- = A_L \qquad a_R^+ + a_R^- = A_R$$

$$\left(\frac{1}{\Delta}\frac{\partial}{\partial t} + \gamma_L\right)A_L - i\frac{\omega_0}{\Delta}A_R = \sqrt{\gamma_L}\, e^{-i\varepsilon t}$$

$$\left(\frac{1}{\Delta}\frac{\partial}{\partial t} + \gamma_R\right)A_R - i\frac{\omega_0}{\Delta}A_L = 0$$

$$\gamma_L = \frac{1-|R_L|}{1+|R_L|} \qquad \gamma_R = \frac{1-|R_R|}{1+|R_R|}$$

$$\omega_0 = \Delta e^{-D_0}\sin(k_F L + \delta) \qquad D_0 = \frac{2}{\hbar V_F}\int_0^{L/2}\Delta(s')ds' \sim \frac{L}{\xi}$$

Equations for coupled Majorana states

$$A_L + A_R = A_+ \qquad\qquad A_L - A_R = A_-$$

Both states are involved (with positive and negative energies)

$$\left(\frac{1}{\Delta}\frac{\partial}{\partial t} + \frac{\gamma_L + \gamma_R}{2} - i\frac{\omega_0}{\Delta}\right)A_+ + \frac{\gamma_L - \gamma_R}{2}A_- = \sqrt{\gamma_L}\, e^{-i\varepsilon t}$$

$$\left(\frac{1}{\Delta}\frac{\partial}{\partial t} + \frac{\gamma_L + \gamma_R}{2} + i\frac{\omega_0}{\Delta}\right)A_- + \frac{\gamma_L - \gamma_R}{2}A_+ = \sqrt{\gamma_L}\, e^{-i\varepsilon t}$$

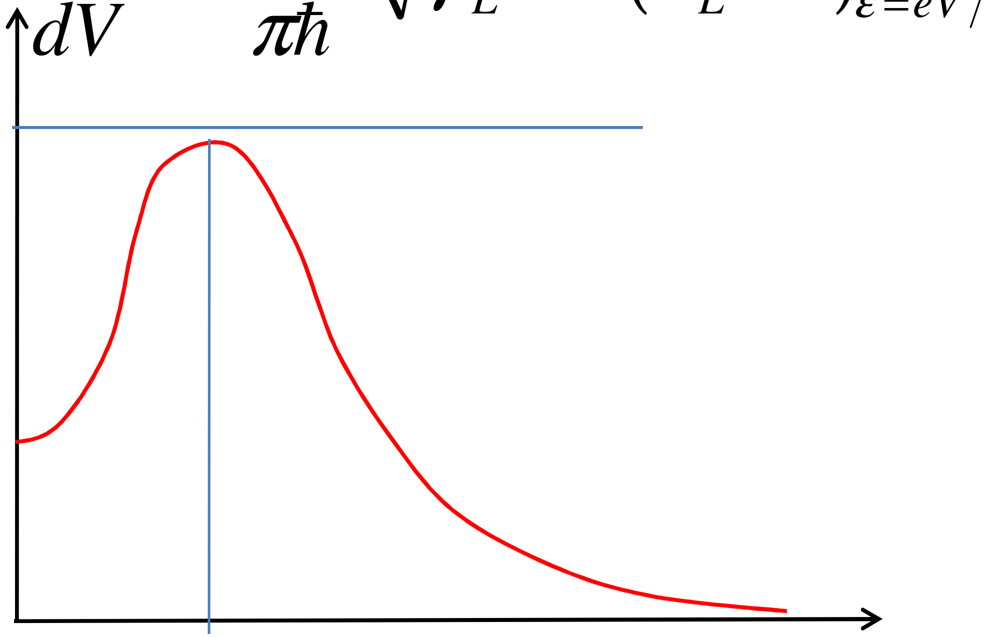
Single state dynamics:

$$\left(\frac{1}{\Delta}\frac{\partial}{\partial t} + \frac{\gamma_L + \gamma_R}{2} + i\frac{\omega_0}{\Delta}\right)A_- = \sqrt{\gamma_L}\, e^{-i\varepsilon t} \qquad\qquad A_+ = 0$$

DC transport. Zero bias peak and its splitting.

$$\frac{dI}{dV} = \frac{2e^2}{\pi\hbar}\sqrt{\gamma_L}\,\mathrm{Re}\left(A_L e^{i\varepsilon t}\right)_{\varepsilon = eV/\hbar}$$

$$\frac{2e^2}{\pi\hbar}$$

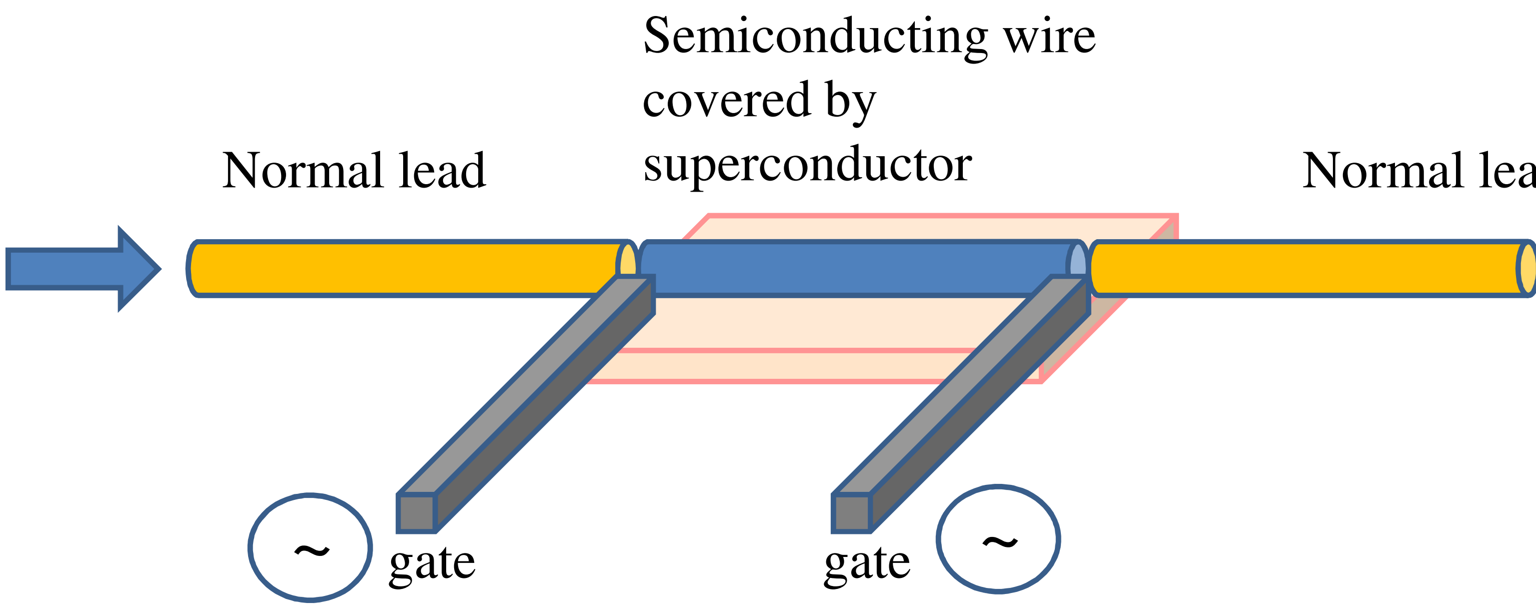


$$\sim \frac{\tilde{\Delta}\omega_0}{e}$$

$$\left.\frac{dI}{dV}\right|_{V=0} = \frac{2e^2}{\pi\hbar}\frac{\gamma_L\gamma_R}{\gamma_L\gamma_R + \omega_0^2}$$

Cf. P.Ioselevich and M.Feigelman (2013) for $\gamma_R = 0$

Possible setup for study of dynamics of Majorana states.



AC transport. Barriers with time-dependent transparency.

$$\gamma_L = \gamma_0 + \tilde{\gamma}\cos\omega t \qquad \gamma_R = \gamma_0 + \tilde{\gamma}\cos(\omega t + \varphi)$$

Resonances in the average current at: $$eV = \pm\omega_0 \pm n\hbar\omega$$

Zero bias conductance:

$$\frac{\pi}{e^2}\frac{dI_L}{dV_L}\bigg|_{V_L=0} \simeq \frac{2\Gamma_0^2}{\Gamma_0^2+\omega_0^2} + \frac{\tilde{\Gamma}\Gamma_0\cos\omega t}{\omega_0^2+\Gamma_0^2} + \frac{\pi\tilde{\Gamma}}{\omega_0^2+\Gamma_0^2}\left[\frac{\omega_0^2-\Gamma_0^2}{2}\sum_{\eta=\pm1}\mathcal{L}(\omega+\eta\omega_0)F_\eta(\omega t) - \omega_0\Gamma_0\sum_{\eta=\pm1}\eta\mathcal{L}(\omega+\eta\omega_0)F_\eta(\omega t+\varphi_0-\pi/2)\right],$$

$$\mathcal{L}(\varepsilon) = \Gamma_0/\pi(\varepsilon^2+\Gamma_0^2)$$

$$F_\pm(x) = \cos x + \sin x(\omega \pm \omega_0)/\Gamma_0.$$

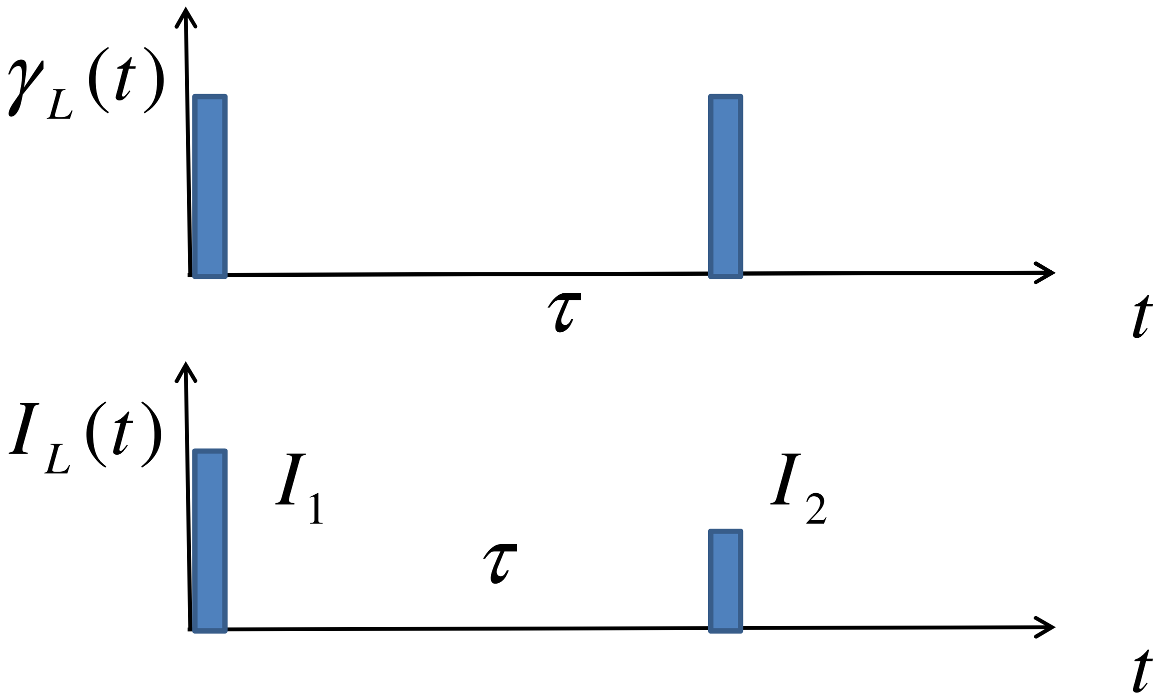
$$\omega \le \omega_0$$

Strong dependence on the phase shift φ

$$\omega >> \omega_0$$

No dependence on the phase shift φ

AC transport. Pump probe techniques.

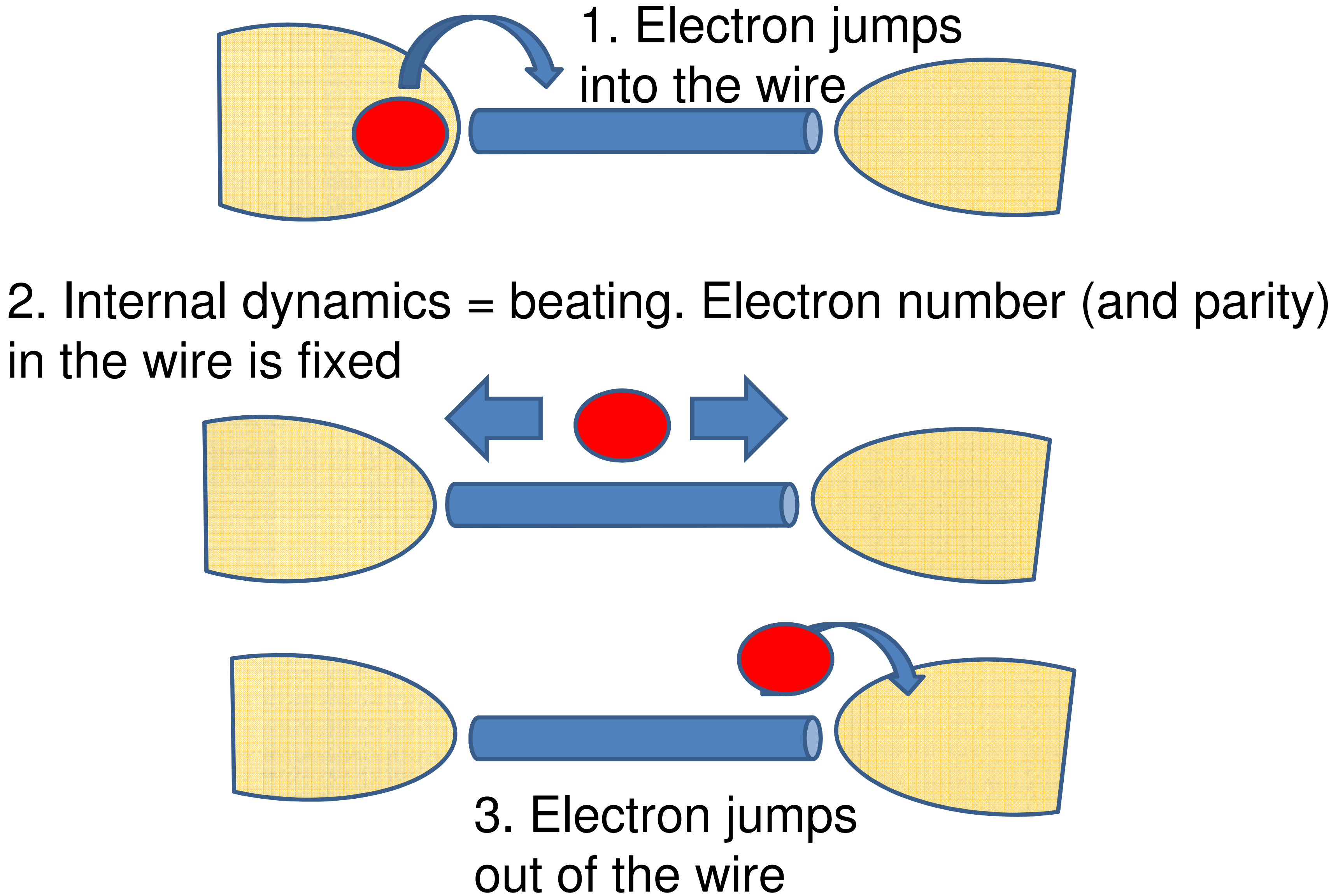


$$\delta I_2 \propto \cos(\omega_0 \tau)\cos(V\tau)$$

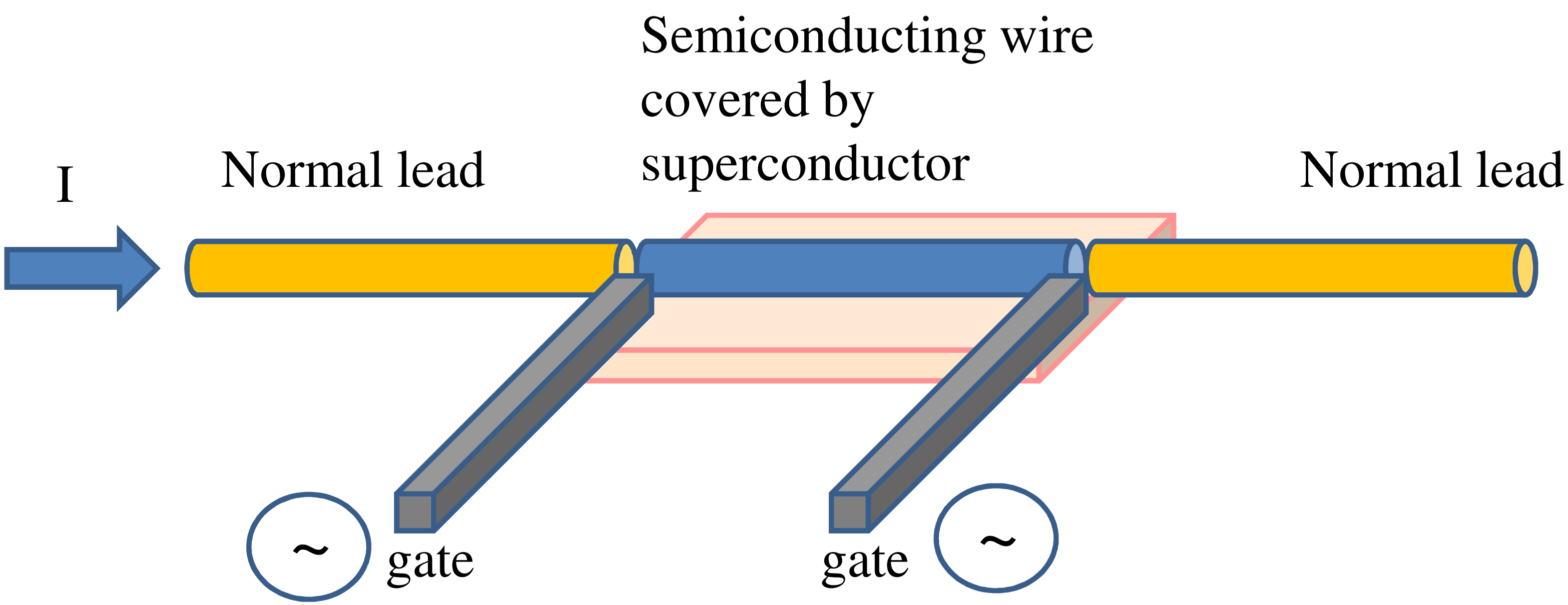
Two frequencies: $\frac{\omega_0 - V}{2}$ and $\frac{\omega_0 + V}{2}$

Single level: only one frequency $\frac{\omega_0 - V}{2}$

Can the Coulomb blockade destroy the beating phenomenon for a couple of the Majorana states?



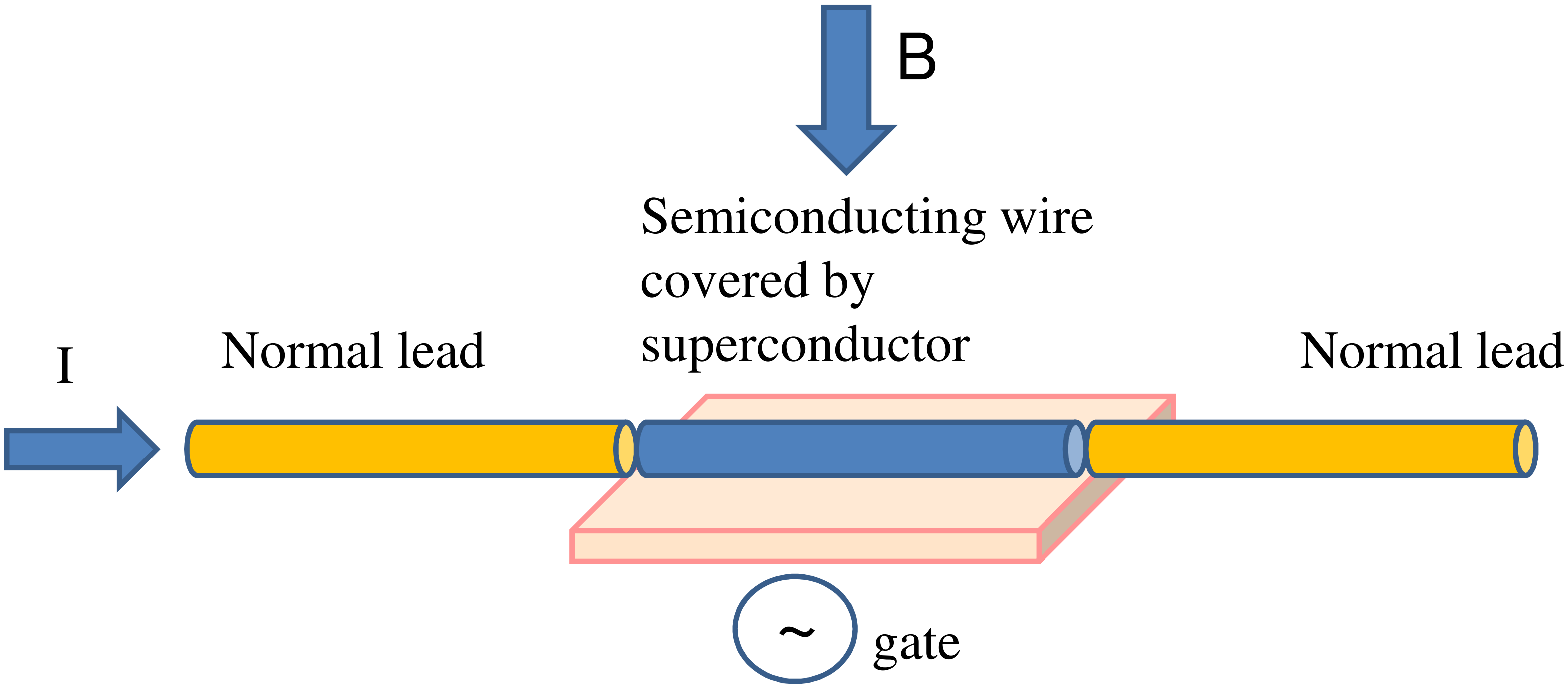
Teleportation paradox and topological stability of Majorana states.



Low frequency dispersion as an inherent property of Majorana states.

Stability range is restricted to low frequencies < ω_0

Outlook



Low frequency dispersion as an inherent property of Majorana states

Possible dynamic effects in oscillating magnetic field or applying an oscillating gate voltage

S leads

Coulomb effects

Some conclusions

- There is no teleportation

- Nonlocality of Majorana states in dynamical problems does exist only at very low frequencies $< \omega_0$

- Experimental study of dynamics of 2 Majorana states in semiconducting wires will give the characteristic time scales for their manipulation