How many quasiparticles can be in a superconductor?

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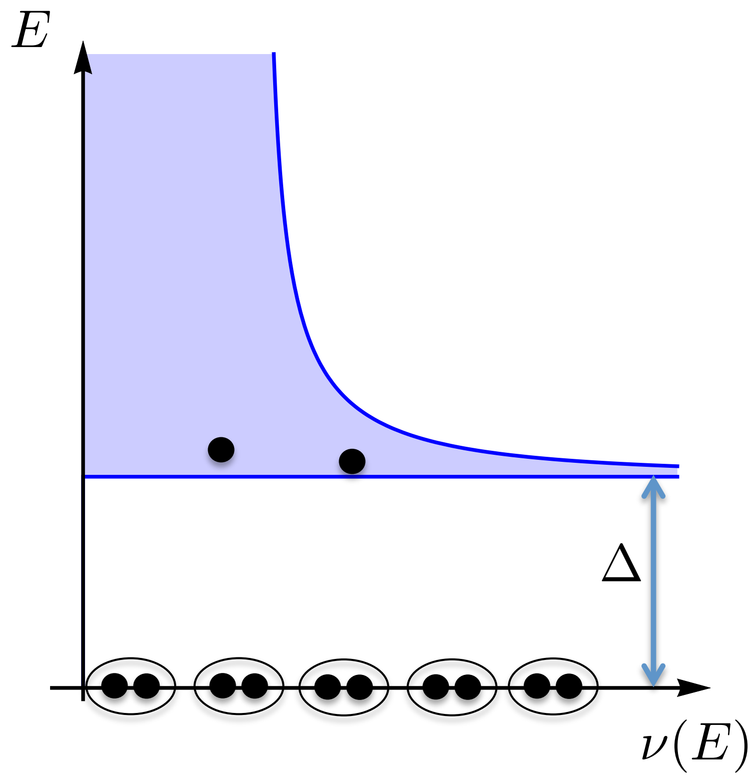
quasiparticles in superconductors



conventional superconductors (e.g. aluminum)

- superfluid condensate of spin-singlet, s-wave Cooper pairs
- fully gapped excitation spectrum

$\rightarrow$ exponential protection from dissipation at $T << \Delta$

$$c(T) \simeq \nu_0 \sqrt{8\pi k_B T \Delta} e^{-\Delta/k_B T}$$

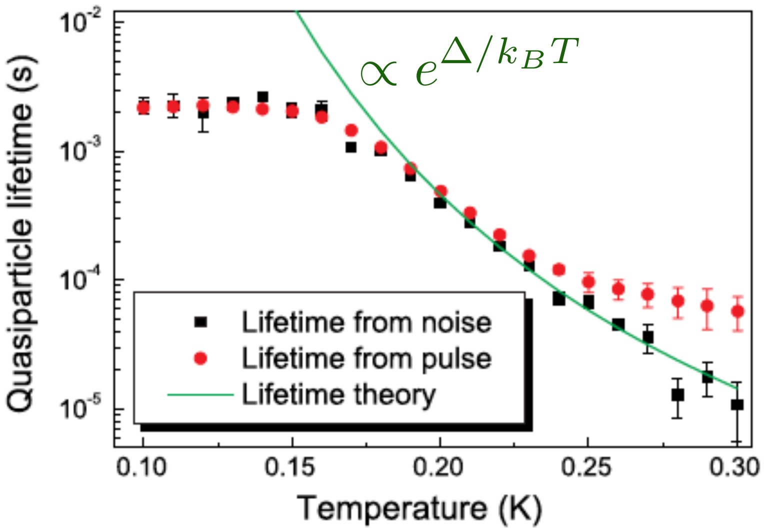
typical values in aluminum: $\Delta = 200\,\mu\text{eV}$

$$c(100\,\text{mK}) \approx 1\,\mu\text{m}^{-3}$$

$$c(50\,\text{mK}) \approx 10^{-6}\,\mu\text{m}^{-3}$$

$$c(10\,\text{mK}) \approx 10^{-51}\,\mu\text{m}^{-3}$$

experiment: excess quasiparticles



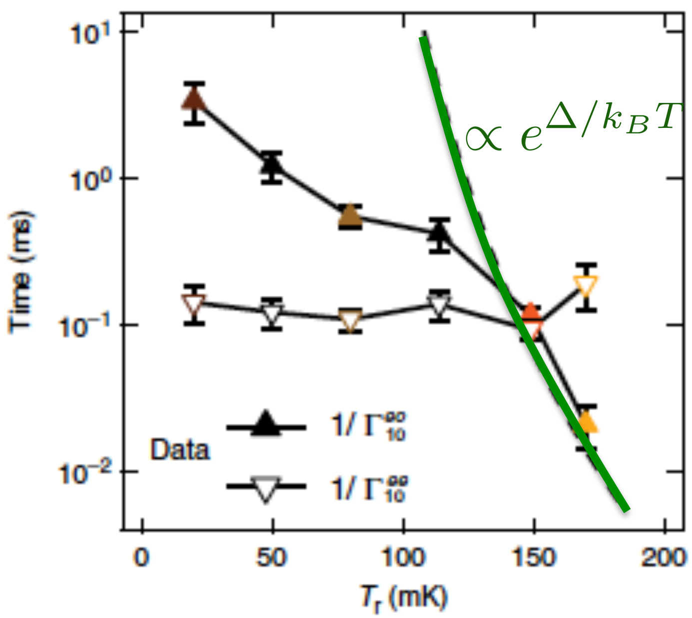
saturation of the lifetime in superconducting resonators at low T

$$\tau_r \simeq \frac{\tau_0}{2c} \frac{\nu_0 (k_B T_c)^3}{\Delta^2}$$

← de Visser *et al.*, PRL 2011

$$c \sim 25 - 55\,\mu\text{m}^{-3}$$

τ_0 *normal-metal electron-phonon relaxation rate at energy* Δ



saturation of the coherence time of superconducting qubits at low T

$$\Gamma^{eo} \simeq \frac{c}{\pi \nu_0} \sqrt{\frac{2\omega_{01}}{\Delta}}$$

← Ristè *et al.*, Nat. commun. 2013

$$c \sim 0.04\,\mu\text{m}^{-3}$$

ω_{01} *qubit frequency*

Main results

Observation:

excess quasiparticles in virtually all superconducting devices
which limit their performances

$$c \gtrsim 0.04\,\mu\mathrm{m}^{-3} \qquad\qquad c \gg c(T) \propto e^{-\Delta/k_B T}$$

Our work:

generation-recombination model
→ residual quasiparticle concentration

- for delocalized quasiparticles above the superconducting gap

  $$c \propto \sqrt{A}$$ A: rate due a non-equilibrium agent

- for localized quasiparticles at mesoscopic fluctuations of the gap edge

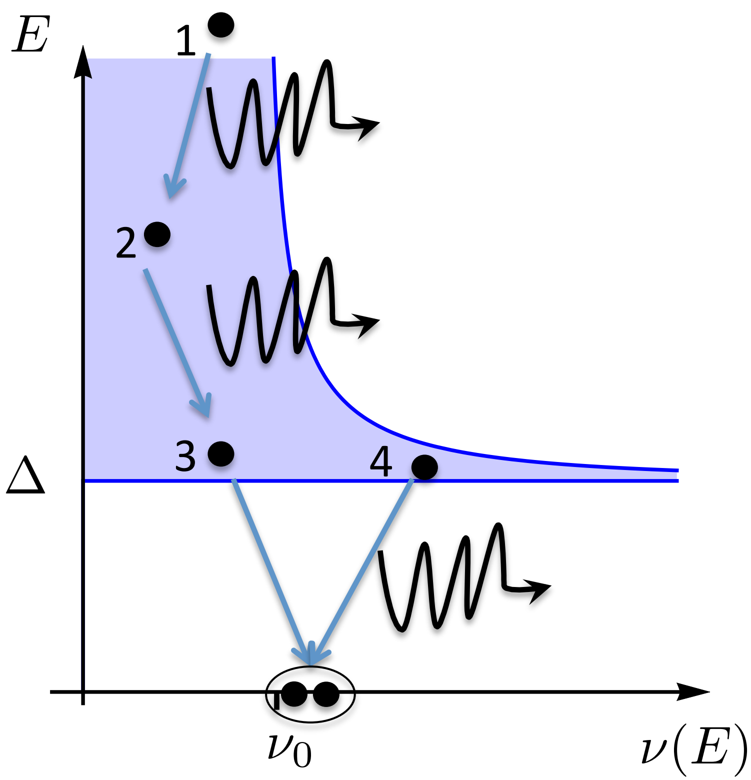
  $$c \propto \frac{1}{\ln^3(1/A)}$$ **poor efficiency of shielding**

- **full spin-polarization** of quasiparticles in small superconducting islands

Outline

- Motivation
- Generation/recombination model for delocalized quasiparticles
- localized states in disordered superconductors
- Our work: extremely slow relaxation of localized quasiparticles
    - Packing coefficient from a bursting bubbles model
    - Polarized and unpolarized states at weak spin-flip rate
- Conclusions & Perspectives

generation/recombination model



- generation due to a non-equilibrium agent:
  - EM and blackbody radiation,
  - cosmic rays
  - natural radioactivity
  - … Martinis *et al.*, PRL 2009
- fast energy relaxation by emitting phonons
- slow annihilation of two quasiparticles near the gap edge with rate

$$\Gamma_{34} = \bar{\Gamma} \int d\mathbf{r}\, p_3(\mathbf{r}) p_4(\mathbf{r})$$

Balance between generation (rate per volume A) and annihilation for delocalized quasiparticles near gap edge:

$$A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$$

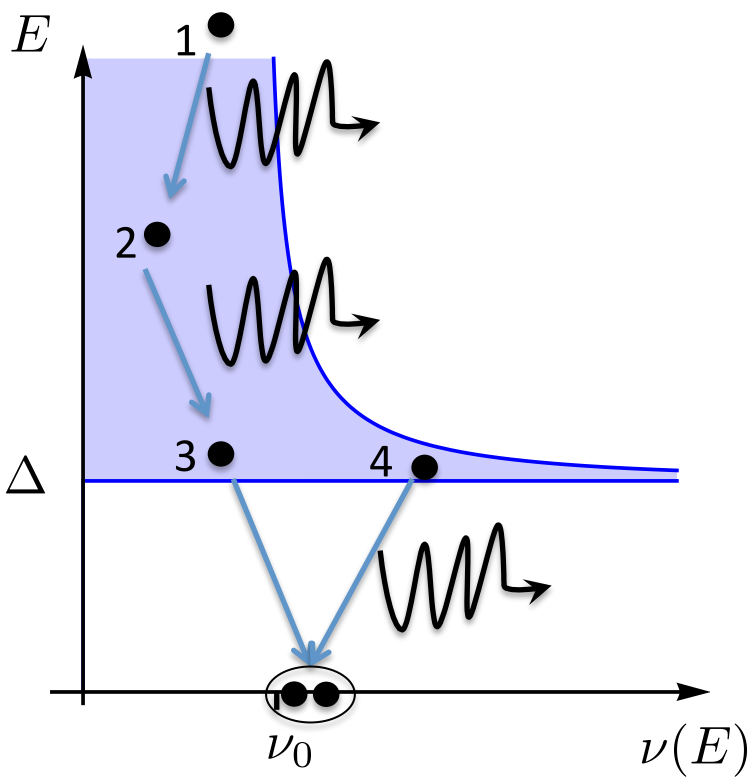
material constant in Aluminum

$$\bar{\Gamma} = 8 \frac{(\Delta/k_B T_c)^3}{\tau_0 \nu_0 \Delta} \approx 40\,\mu\mathrm{m}^{-3}\mathrm{s}^{-1}$$

→ scale for shielding the device

$$c < 25\,\mu\mathrm{m}^{-3} \quad \rightarrow \quad P_{\mathrm{inj}} = A \Delta \mathcal{V} < 1\,\mathrm{fW}$$

generation/recombination model



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Martinis *et al.*, PRL 2009

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Balance between generation (rate per volume A) and annihilation for delocalized quasiparticles near gap edge:

$$A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$$

… but, if quasiparticles are localized,
strong correlations in their positions modify c

disordered superconductors

ℓ elastic mean free path

ξ superconducting coherence length

- clean metal $\ell > \xi$:

  pairing of electrons with opposite spins and momenta

- dirty metal $\ell < \xi$:

  pairing of electrons in time-reversed states

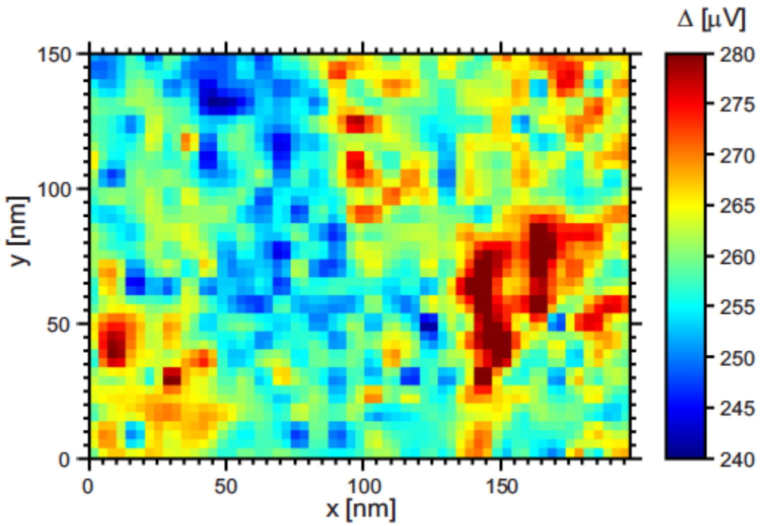
"Anderson theorem" (mean-field):

Δ is unaffected by non-magnetic disorder and remains spatially uniform

Abrikosov-Gorkov 1958

Anderson 1959

At large disorder:



← STM study of TiN films

Sacépé et al., PRL 2008

disordered superconductors

mesoscopic fluctuations of the gap

Larkin and Ovchinnikov, JETP 1972

$$\delta\Delta(\mathbf{r}) = \Delta(\mathbf{r}) - \Delta$$

$$\langle \delta\Delta(\mathbf{r})\delta\Delta(\mathbf{r}')\rangle = (\delta\Delta)^2\delta(\mathbf{r}-\mathbf{r}')$$

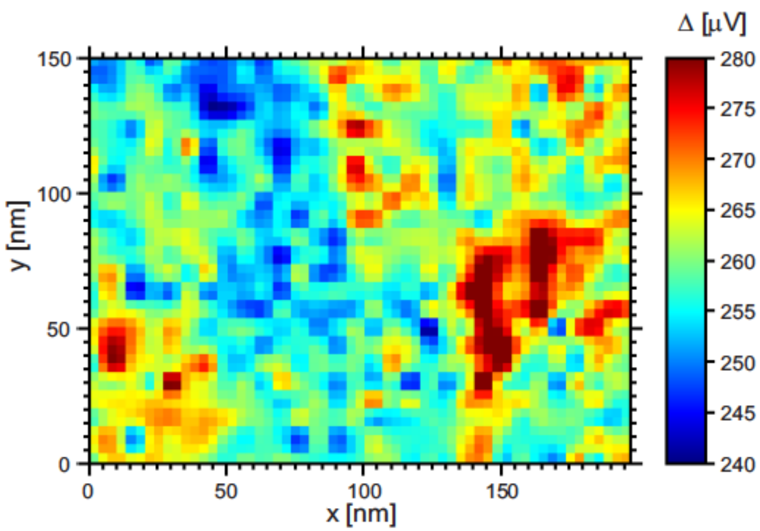
correlation radius $\lesssim \xi$

magnitude $\dfrac{(\delta\Delta)^2}{\Delta^2\xi^3} \sim \dfrac{1}{g^2} \ll 1$ g dimensionless conductance on the scale ξ

in bulk Al: $g \sim 10^4$

but $\delta\Delta$ can be larger in films with Coulomb repulsion

Skvortsov and Feigelman, PRL 2012



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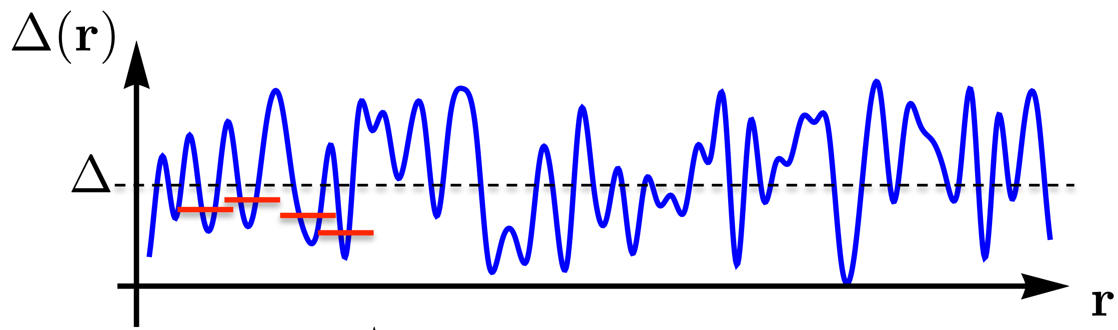

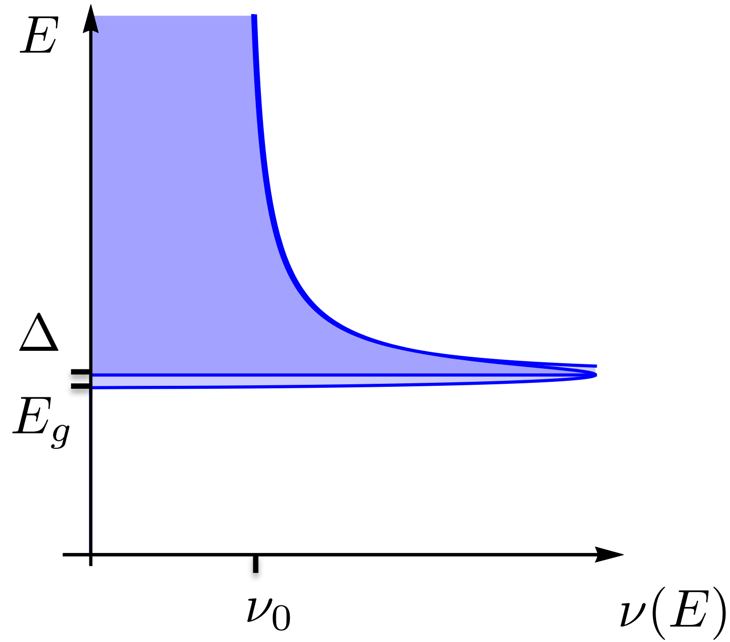

overlapping bound states:

- reduced gap
  $$E_g = \Delta - \varepsilon_g$$
  $$\varepsilon_g \sim \frac{\Delta}{g^{4/3}}$$
- rounding of the BCS singularity

tail states in disordered superconductor

Larkin and Ovchinnikov,
JETP 1972

mesoscopic fluctuations of the gap

$$\delta\Delta(\mathbf{r}) = \Delta(\mathbf{r}) - \Delta$$

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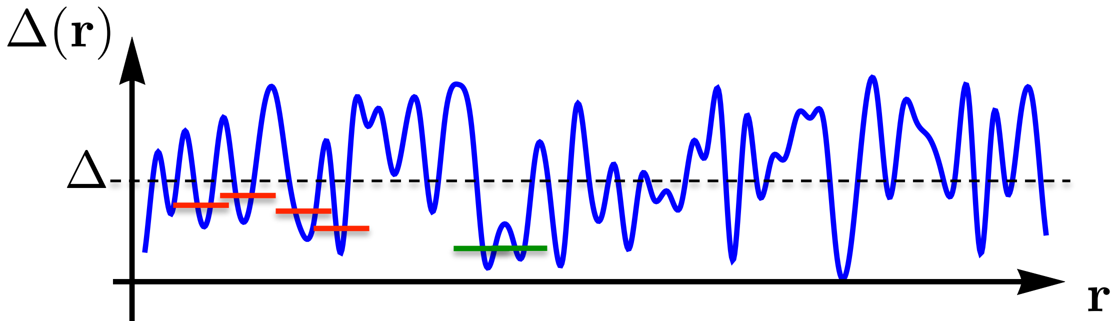
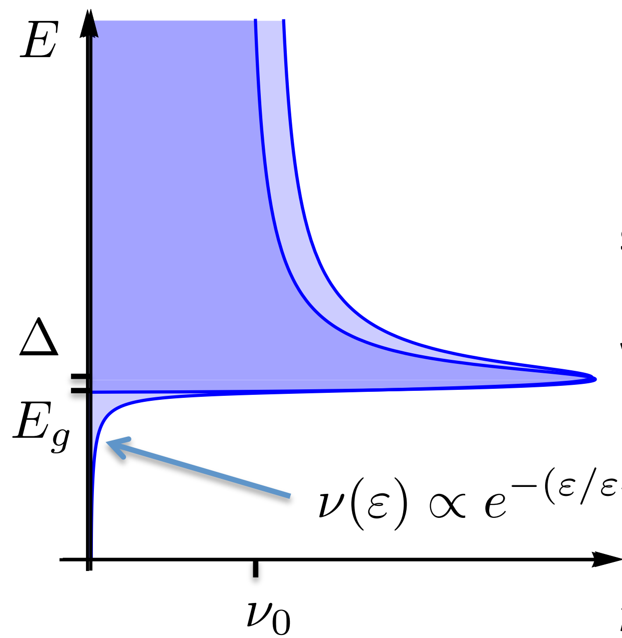




$$\nu(\varepsilon) \propto e^{-(\varepsilon/\varepsilon_T)^{5/4}}$$

rare, optimal fluctuations generate localized tail states with energy

$$E = E_g - \varepsilon$$

such that

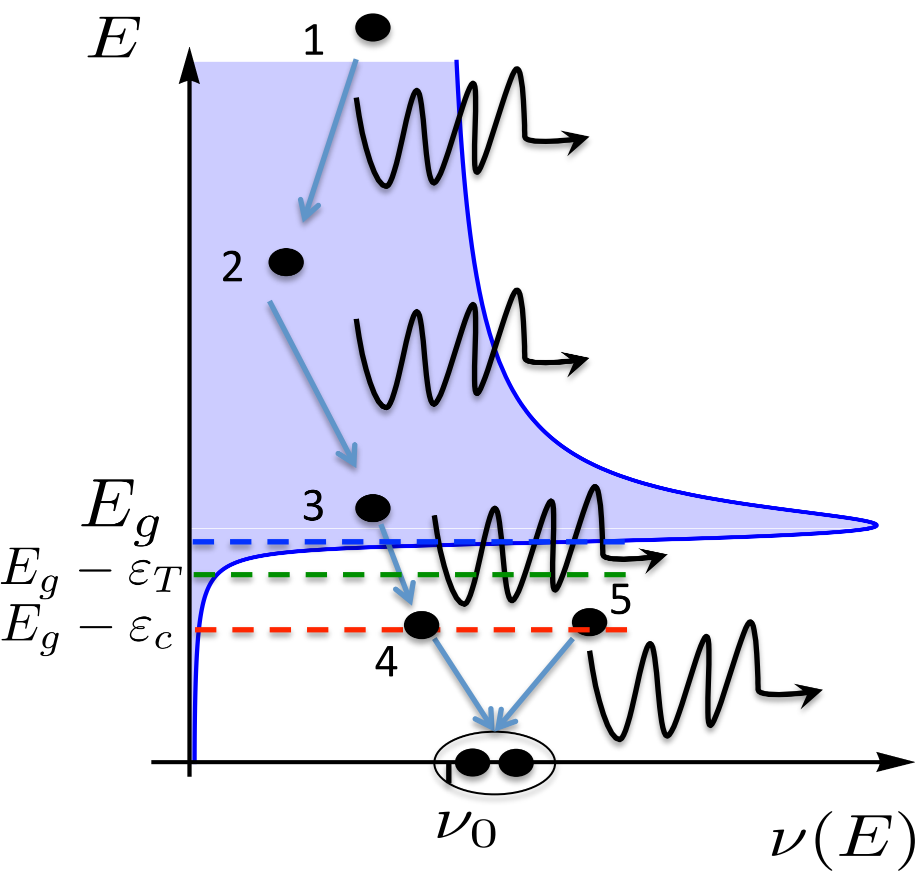
$$\varepsilon^2 \sim (\delta\Delta)^2/L^3(\varepsilon)$$

with localization radius

$$L(\varepsilon) \sim \xi(\Delta/\varepsilon)^{1/4}$$

energy scale: $\varepsilon_T \sim \dfrac{\Delta}{g^{8/5}}$

Bottleneck for energy relaxation



fast energy relaxation does not stop at Δ: quasiparticles relax into the tail states

number of overlapping states with energy $< E_g - \varepsilon$:

$$N(\varepsilon) = L^3(\varepsilon) \int_{\varepsilon}^{\infty} d\varepsilon' \nu(\varepsilon')$$

typically $N(\varepsilon_T) \sim g^{17/15} \gg 1$

relaxation stops when no more overlapping states with lower energy are available:

$$N(\varepsilon_c) \sim 1 \quad \text{at} \quad \varepsilon_c \sim \varepsilon_T (\ln g)^{4/5}$$

localization radius at energy ε_c

$$r_c \simeq 0.5\xi(\varepsilon_T/\Delta)^{-1/4}[\ln N(\varepsilon_T)]^{-1/5}$$

$$10^{-4} < \varepsilon_T/\Delta < 10^{-2}$$

$$\xi < r_c < 3\xi$$

generation/recombination model for localized states

- quasiparticles are generated at random points with rate A per unit volume
- they keep their positions
- they annihilate pairwise with the rate

$$\Gamma(\mathbf{R}) = \bar{\Gamma} \int d\mathbf{r}\, p_c(\mathbf{r}) p_c(\mathbf{r}+\mathbf{R}) \propto \frac{\bar{\Gamma}}{r_c^3} e^{-R/r_c}$$

$$p_c(\mathbf{r}) \propto e^{-r/(2r_c)}$$

most probable shape of the state at ε_c

balance between generation and annihilation:

typical distance between particles $r \sim c^{-1/3}$

$r \ll r_c$ (dense) $\quad A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$

$r \gg r_c$ (dilute) $\quad A r^3 \sim \frac{\bar{\Gamma}}{r_c^3} e^{-r/r_c} \implies c \propto \frac{1}{r_c^3 \ln^3\left(\frac{\bar{\Gamma}}{A r_c^6}\right)}$

simplified model: bursting bubbles

characteristic lengthscale: $\frac{r}{r_c} \approx \ln\left(\frac{\bar{\Gamma}}{Ar_c^6}\right)$ $r \gg r_c$

annihilation varies quickly with the distance d between two quasiparticles:

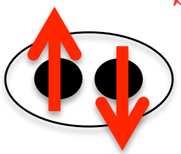
- fast annihilation if $d < r$
- slow annihilation if $d > r$

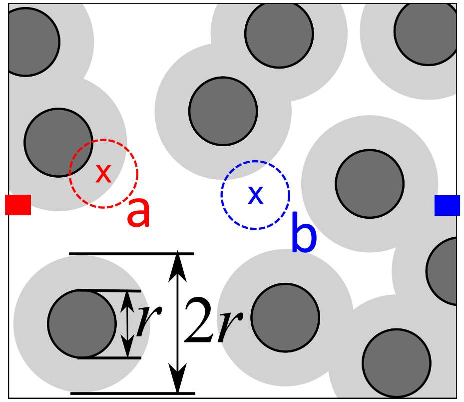
$\rightarrow$ describe quasiparticles as bubbles with radius $r/2$ than cannot overlap

spin-selection rule: annihilation in singlet state

$\rightarrow$ let's assume a fast spin-flip rate

$S = 0$

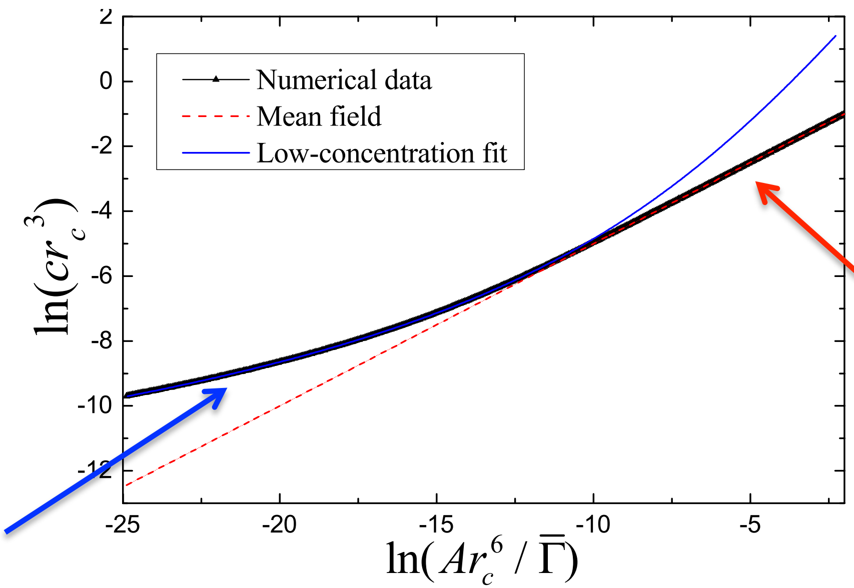




$$c = \frac{C_p}{(4\pi/3)r^3}$$ with packing coefficient: $$C_p \approx 0.605 \pm 0.008 \gtrsim \frac{1}{2}$$

Full dynamical simulation

use annihilation rate $$\Gamma(\mathbf{R}) = \bar{\Gamma} \int d\mathbf{r}\, p_c(\mathbf{r}) p_c(\mathbf{r}+\mathbf{R})$$



$$cr_c^3 = \sqrt{Ar_c^6/\bar{\Gamma}}$$

$$c = \frac{C_p}{(4\pi/3)r^3}$$

with improved estimate for r:

$$Ar_c^6/\bar{\Gamma} = b(r/r_c)^{\beta-3} e^{-r/r_c}$$

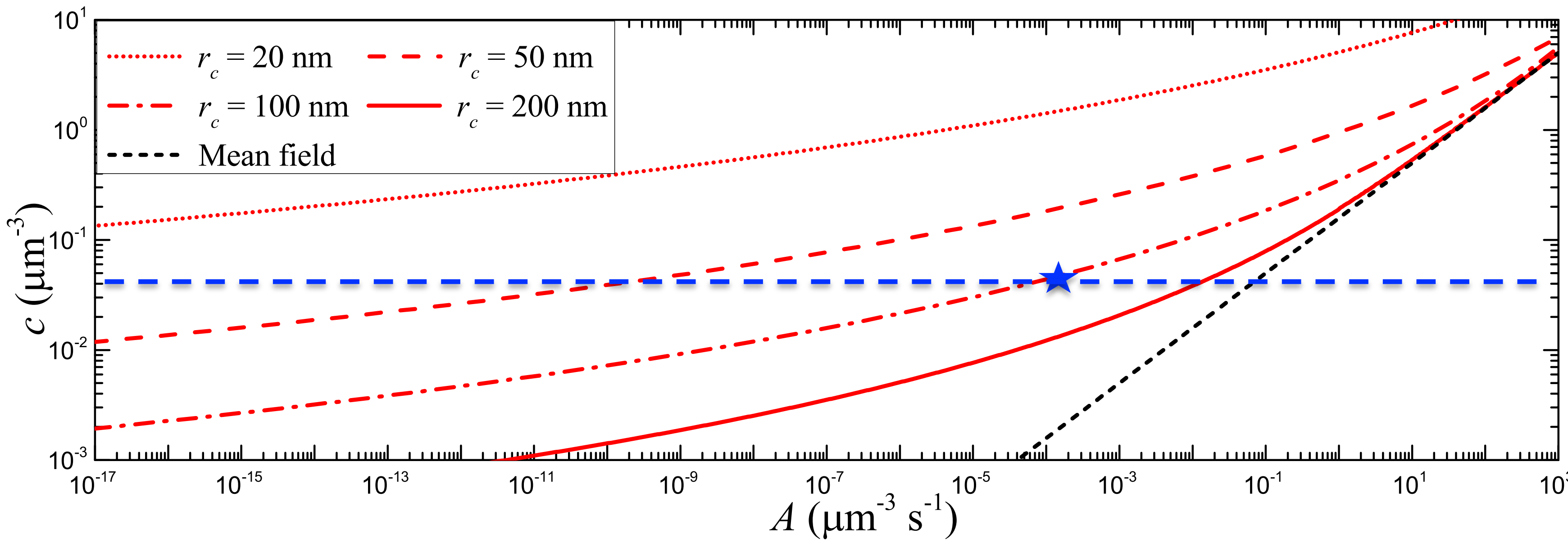
$$b = 0.008$$

$$\beta = 0.41$$

cosmic radiation?

material parameter for Al $\bar{\Gamma} = 40\,\mu\mathrm{m}^{-3}\mathrm{s}^{-1}$

different values of r_c correspond to different disorder strengths

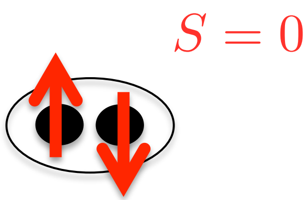


cosmic radiation at sea level dominated by muons with:

- mean energy in the GeV range
- flux of 1 muon/cm^2/min → $A \sim 10^{-4}\, s^{-1} \mu\mathrm{m}^{-3}$
- stopping power in Al of 1 MeV/cm

$$r_c \sim 0.1\,\mu\mathrm{m} \quad \rightarrow \quad c \sim 0.01\,\mu\mathrm{m}^{-3}$$

classical spin



spin selection rule:
annihilation of quasiparticles in singlet state only

bursting bubbles model with classical spin

$$c = \frac{C_p}{(4\pi/3)r^3}$$

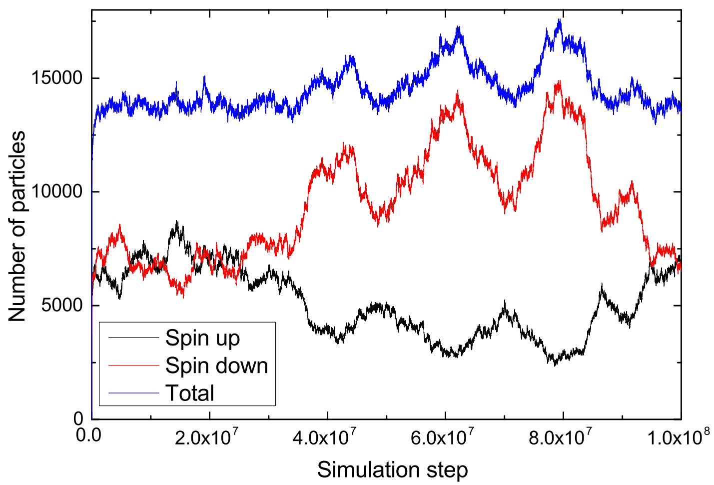
- with large spin-flip rate:

$$C_p \approx 0.605 \pm 0.008$$

- without spin-flip:

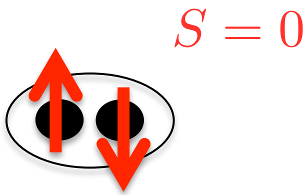
$$C_p \approx 2.19 \pm 0.05$$

$\leftarrow$ large fluctuations



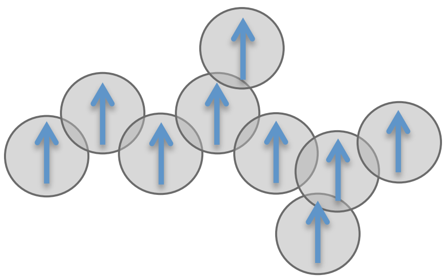
A. Bespalov *et al.* PRL **117**, 117002 (2016)

quantum spin



In progress

spin selection rule:
annihilation of quasiparticles in singlet state only



cluster: particles connected through a chain of overlapping particles

$$|r_i - r_j| < r$$

no decay if each pair of particles is in a spin-triplet state

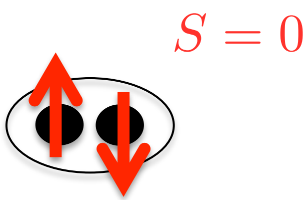
$\rightarrow$ cluster of N particles in maximal-spin state with $s = N/2$

$$|\psi\rangle = \sum_{m=-s}^{s} c_m |sm\rangle$$

spin-coherent basis

$$|s, \Omega\rangle = (\cos\frac{\theta}{2})^{2s} \exp[\tan\frac{\theta}{2} e^{i\varphi} \hat{S}_-] |s, s\rangle$$

survival vs decay



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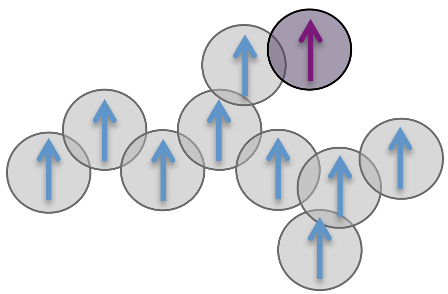
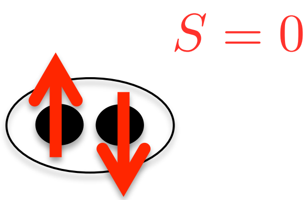


added particle overlapping with the cluster:

- if parallel spin
  $\longrightarrow$ cluster with $N+1$ particles

survival vs decay



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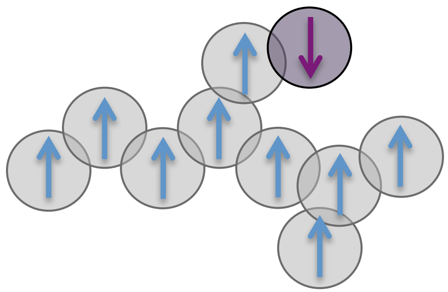


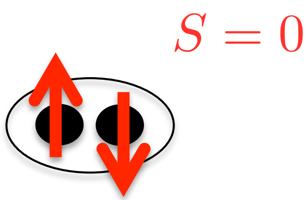
added particle overlapping with the cluster:

- if parallel spin
  $\longrightarrow$ cluster with $N+1$ particles

- if antiparallel spin
  $\longrightarrow$ either annhilates with a partner and leaves a cluster with N - 1 particles

  $\longrightarrow$ or new cluster with $N+1$ particles and tilted spin

survival wins over decay



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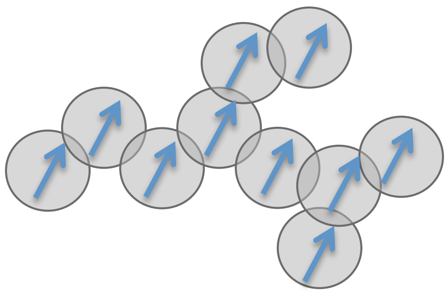

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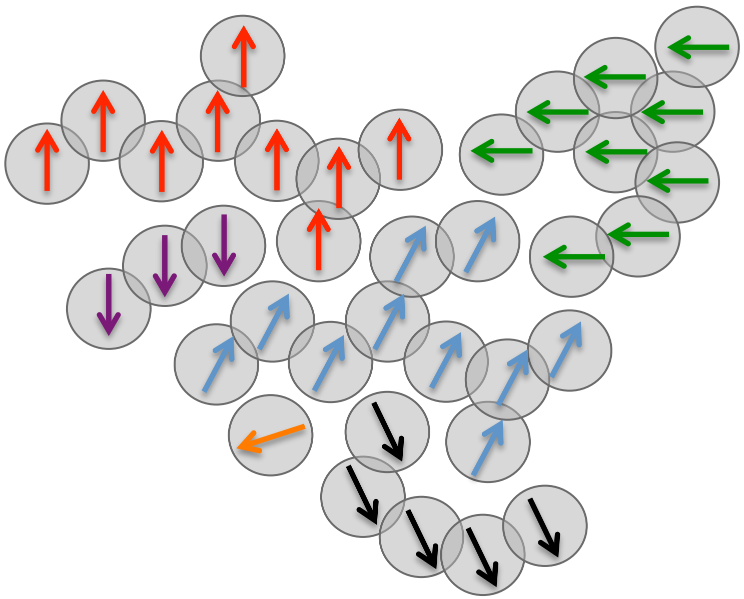
  $\rightarrow$ or new cluster with $N + 1$ particles and tilted spin

$$p_{\text{survival}} = \frac{1}{2} + \frac{1}{2(N+1)} > \frac{1}{2}$$

Polarized vs unpolarized state



Growth of particle number in a cluster
vs loss of particles to other cluster at its border:

$$\frac{dN}{dt} = \frac{AV}{N} - ASr$$

$$V \propto L^3$$

$$S \propto L^2$$

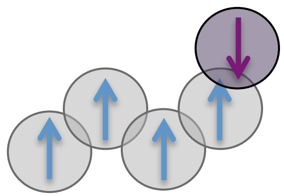
The cluster cannot be dense in a big island:

$$N \sim L/r \ll V/r^3 \quad \text{if} \quad L \gg r$$

In small islands, large fluctuations can result in a single cluster. Then:

$$N(t) \propto \sqrt{t}$$ (only limited by spin flips)

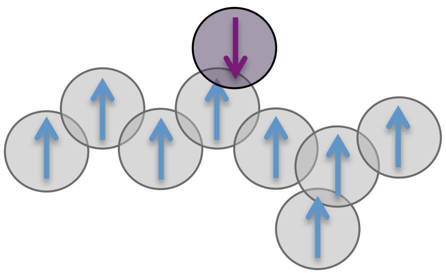
Model



Essential features of the model:

- spin direction vector per cluster
- clusters can loose particles

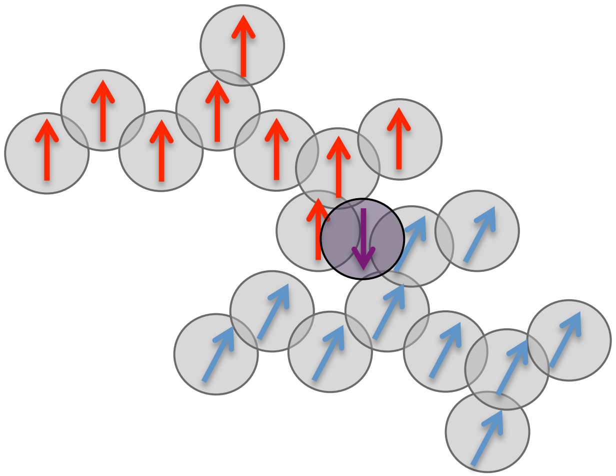
Model



Essential features of the model:

- spin direction vector per cluster
- clusters can loose particles, loose connections

Model



Essential features of the model:

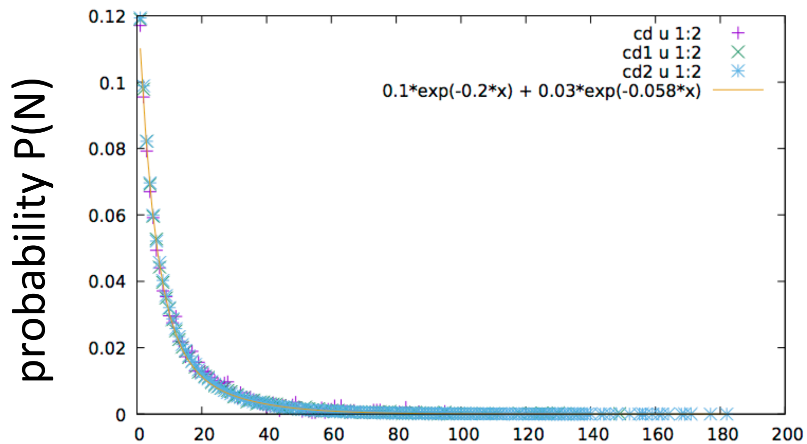
- spin direction vector per cluster
- clusters can loose particles, loose connections, and merge

some arbitrariness in the rules for merging, particle lost, and disconnection.

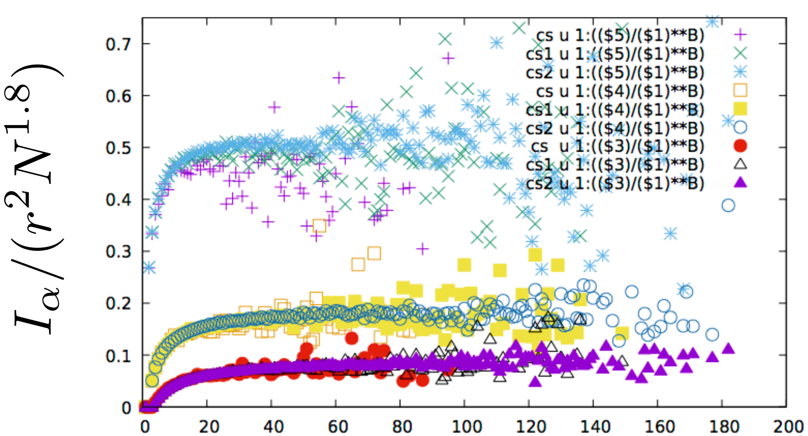
- reasonable classical limit: two clusters can merge if their spin vectors are close to parallel
- survival probality of two clusters with M and N particles:

$$p_{\mathrm{survival}} = \frac{M+N+1}{(M+1)(N+1)}$$

Unpolarized state



number N of particles in cluster



number N of particles in cluster

- C_p = 2.4 (close to classical spins)

- 12% of single particles
- <N>=12
- rare big clusters

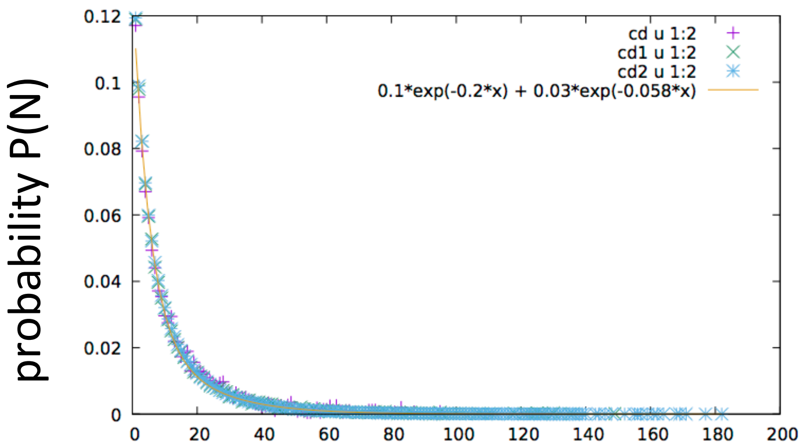
the inertia tensor characterizes the clusters' shape

- not much elongated
- eigenvalues $\sim NL^2_{\text{cluster}}$

$$L_{\text{cluster}}/r \sim N^{0.4} > N^{1/3}$$

$\longrightarrow$ clusters interpenetrate

Unpolarized state



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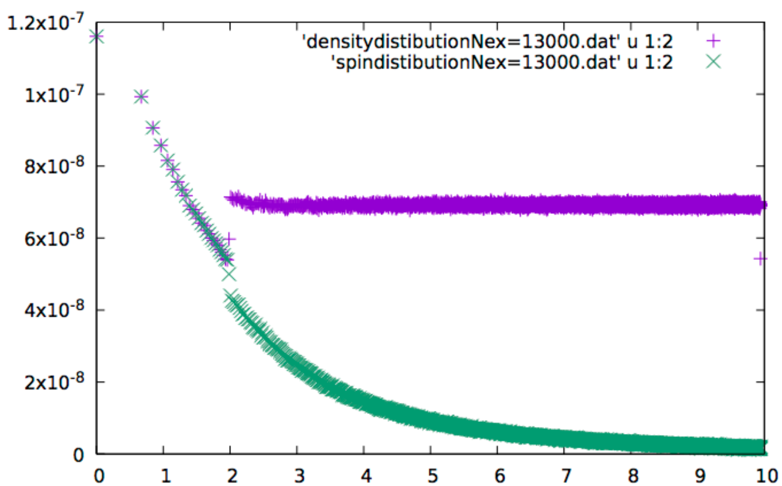


distance (in units of r)

- C_p = 2.4 (close to classical spins)

- 12% of single particles
- <N>=12
- big clusters

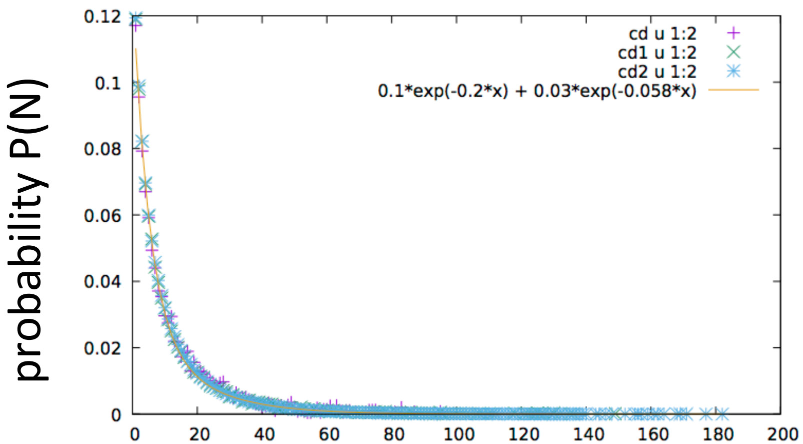
d<2r

- particle in the same cluster

d>2r:

- homogeneous state
- exponentially suppressed spin correlation

Unpolarized state



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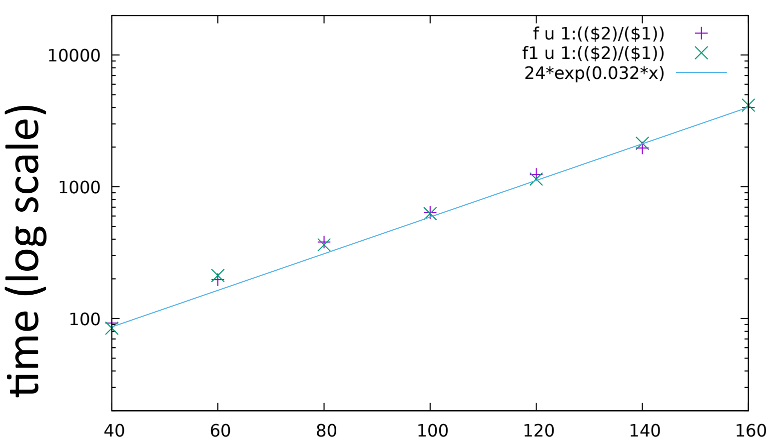

island's volume (in units of $4\pi r^3/(3C_p)$)

- C_p = 2.4 (close to classical spins)

- 12% of single particles
- <N>=12
- big clusters

- large fluctuations
- exponential time scale in island's volume for having all particles in a single cluster (which then eventually grows like $t^{0.5}$)

Conclusion and perspectives

- quasiparticles in localized states don't recombine easily

  $\rightarrow$ large excess quasiparticles in moderately disordered superconductors

- strategies to reduce their concentration
  - cleaner superconductors
  - shielding of the relevant non-equilibrium source
  - finite T (recombination of mobile quasiparticles is more efficient?)
- Physical observables?
  - EM absorption…
  - role of large space and time fluctuations?
- Polarized state:
  - more quasiparticles in isolated islands than in the bulk
  - mechanism for low-frequency flux noise due to unpaired spins in qubits (Faoro and Ioffe…)?

Refs: A. Bespalov *et al.* PRL **117**, 117002 (2016)

and *in progress...*