Spin correlation functions and decay of quasiparticles in *XXZ* spin chain at $T > 0$

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  - One-dimensional anisotropic XXZ spin-$\frac{1}{2}$ model: introduction and phase diagram
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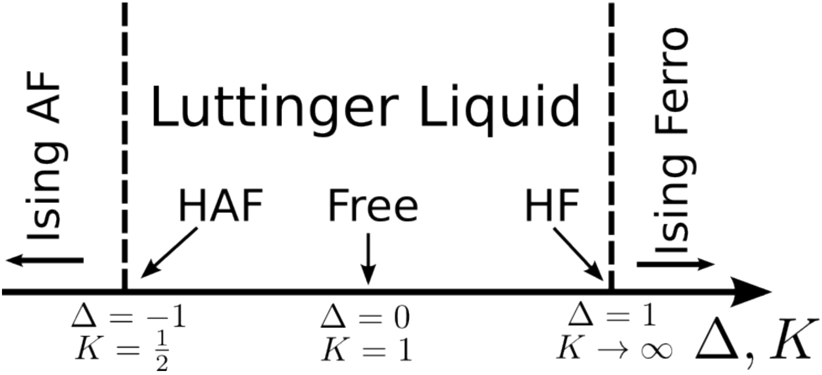
- Resonant processes near the light cone and self-consistent diagram approach
- Dynamic spin-spin correlation function
- Corrections beyond self-consistent approximation

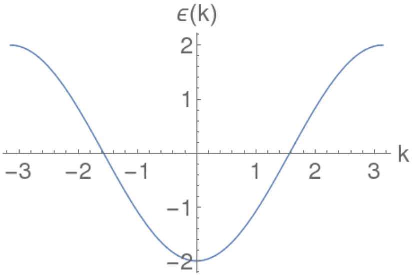
Model and phase diagram of the clean system

$$\hat{H} = -J \sum_n \left(\hat{S}_n^x \hat{S}_{n+1}^x + \hat{S}_n^y \hat{S}_{n+1}^y + \Delta \hat{S}_n^z \hat{S}_{n+1}^z \right)$$

At $-1 < \Delta < 1$ elementary exciations are bosons with linear spectrum $\omega(k) \propto |k|$ (Luttinger Liquid, LL)



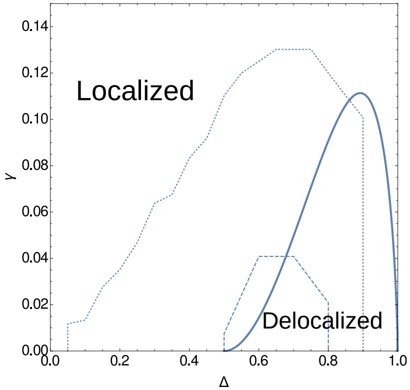
Luttinger Liquid description



Free scalar bosonic theory with linear spectrum $\omega(k) = uk$:

$$\hat{H}_{LL} = \frac{1}{2\pi} \int dx \left(\frac{u}{K} (\partial_x \phi)^2 + uK (\pi \Pi)^2 \right), \quad \begin{cases} K = \frac{\pi}{2 \arccos \Delta} \\ u = \frac{Ja}{2} \frac{\sin \pi/2K}{1 - 1/2K} \end{cases}$$

"Dirty" system (pt. 1)



$$\hat{V} = -\sum_n h_n \hat{S}_n^z, \quad \langle h_n h_m \rangle = \delta_{nm} \left\langle h^2 \right\rangle$$

Renormalization group treatment:

$$\begin{cases} \frac{dg}{d\xi} = (3-2K)g \\ \frac{du}{d\xi} = -\frac{uK}{2} g \\ \frac{dK}{d\xi} = -\frac{K^2}{2} g \end{cases}$$

Dimensionless disorder constant:

$$g = \frac{8(1-1/2K)^2}{\pi \sin^2(\pi/2K)} \frac{\left\langle h^2 \right\rangle}{J^2}$$

T. Giamarchi, H.J.Shulz, PRB **37**, 325 (1988)
P. Schmitteckert et al., PRL **80**, 560 (1998)
J. M. Carter and A. MacKinnon, PRB **72**, 024208 (2005) (2005)

"Dirty" system (pt. 2)

- Momentum relaxation rate:

$$1/\tau(T) \sim (ug/a)(T/J)^{2K-2}$$

- Thermal conductivity:

$$\kappa = \frac{\pi}{3} u T \tau \propto T^{3-2K}$$

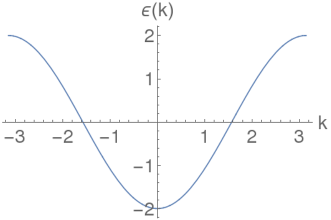
- "Interference" corrections are small by parameter $1/T\tau$.

IP and M.V.Feigel'man, PRB **92**, 235448 (2015)

Nonlinearity of Fermionic spectrum: where it is important

- At the point $\Delta = 1$ our model is isotropic Heisenberg ferromagnet with quadratic excitation spectrum $\omega \propto q^2$. Close to this point linear approximation easily breaks down.
- Nonlinearity of the spectrum leads to finite width of the spectral function $\delta\epsilon(k) \sim (u/a)(ka)^3$. Thermal excitations: $k \sim T/u$. Thus $\delta\epsilon(T) \sim T(Ta/u)^2$, to be compared with $1/\tau(T)$
- At $K < 5/2$ nonlinearity effect is irrelevant at low $T \ll T_* \sim J(h^2/J^2)^{1/(5K-2)}$ and relevant at $T > T_*$.
- At $K > 5/2$ in the whole low-T region nonlinearity is relevant for the correct description of the disorder.
- $K = 5/2$ corresponds to $\Delta = \cos\frac{\pi}{5} \approx 0.81$

Nonlinearity of Fermionic spectrum: Bosonic description



- Densities of the left-movers and right-movers:

$$\rho(x) \simeq -\frac{1}{\pi}\partial_x\phi = \underbrace{R(x)}_{k>0} + \underbrace{L(x)}_{k<0}$$

- Effective interaction:

$$\hat{H}^{(4)}_{b.c.} = -\frac{\alpha}{2}\int dx(\lambda_+ R^2 L^2 + \lambda_-(R^4 + L^4)) \tag{1}$$

- In presence of magnetic field:

$$\hat{H}^{(3)}_{b.c.} = \int dx\left(\frac{\alpha_1}{3}(R^3 + L^3) + \frac{\alpha_2}{2}(R^2 L + L^2 R)\right), \quad \begin{cases} \alpha_1 = \frac{3\alpha\sqrt{K}\lambda_- h}{\pi u} \\ \alpha_2 = \frac{\alpha\sqrt{K}\lambda_+ h}{\pi u} \end{cases} \tag{2}$$

Parameters of the Bosonic theory

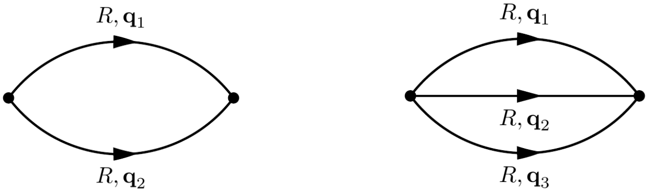
$$\alpha = 4\pi^3 u a^2$$

$$\lambda_+ = \frac{1}{2\pi} \tan \frac{\pi K}{2K-1},$$

$$\lambda_- = \frac{1}{24\pi K} \frac{\Gamma\left(\frac{3K}{2K-1}\right)}{\Gamma\left(\frac{3}{4K-2}\right)} \frac{\Gamma^3\left(\frac{1}{4K-2}\right)}{\Gamma^3\left(\frac{K}{2K-1}\right)}.$$

S. Lukyanov, Nuclear Physics B **522**, 533-549 (1998)

Lowest order of the perturbation theory



(a) Nonzero field, $h \neq 0$ (b) Symmetric case, $h = 0$

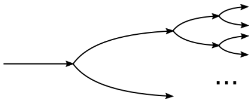
Figure: Singular diagrams for $\mathrm{Im}\Sigma^{(R)}_{ret}(\omega, q)$

Singularity comes from the form of the spectral weight $S(\omega, q) = \mathrm{Im}\,\langle RR\rangle_{\omega,q} \propto \delta(\omega - uq)$ with linear dispersion $\omega = uq$, and conservation of energy $\omega = \sum_i \omega_i$ and momentum, $q = \sum_i q_i$:

$$\Gamma^{(3)}(\omega, q) = \frac{\alpha_1^2}{48\pi^3}\delta(\omega - uq)q^2\left(q^2 + \left(\frac{2\pi T}{u}\right)^2\right)$$

$$\Gamma^{(4)}(\omega, q) = \frac{\alpha^2\lambda_-^2}{320\pi^5}\delta(\omega - uq)q^2\left[q^2 + \left(\frac{2\pi T}{u}\right)^2\right]\left[q^2 + 4\left(\frac{2\pi T}{u}\right)^2\right].$$

Self-consistent procedure (nonzero field)



- Decay into "dressed" quasiparticles ($\epsilon_i = \omega_i - uq_i$, $f(\epsilon) = \coth \frac{\beta\epsilon}{2}$):

$$\Gamma(\omega, \epsilon) = \frac{\alpha_1^2 q}{16\pi^3 u^3} \int d\omega_1 d\omega_2 \delta(\omega_1 + \omega_2 - \omega)\omega_1\omega_2(f(\omega_1) + f(\omega_2))J_1(\epsilon, \omega_1, \omega_2).$$

$$J_1(\epsilon, \omega_1, \omega_2) = \int d\epsilon_1 d\epsilon_2 \delta(\epsilon_1 + \epsilon_2 - \epsilon)\frac{1}{\pi^2}\frac{\Gamma(\omega_1, \epsilon_1)\Gamma(\omega_2, \epsilon_2)}{\left[\epsilon_1^2 + \Gamma^2(\omega_1, \epsilon_1)\right]\left[\epsilon_2^2 + \Gamma^2(\omega_2, \epsilon_2)\right]}$$

- Classical hydrodynamics: major contribution to the decay rate comes from $\omega_i \sim \omega$ instead of T.
- Decay rate varies considerably with ϵ at the scale $\epsilon \sim \Gamma(\omega)$.

$$\Gamma(\omega) = C_1 \frac{|\alpha_1|}{u^2} T^{1/2}|\omega|^{3/2} \sim \frac{|h| T^{1/2}|\omega|^{3/2}}{J^2}$$

Classical hydrodynamics: A.F. Andreev, Sov.Phys.JETP **51**, 1038 (1980)
Luttinger Liquid: K. Samokhin, J. Phys. Condens. Matter **10**, L533 (1998)

Self-consistent procedure (zero magnetic field)

$$\Gamma(\omega,\epsilon)=\frac{3\alpha^2\lambda_-^2}{32\pi^5u^5}q\int d\omega_1 d\omega_2 d\omega_3\delta(\omega_1+\omega_2+\omega_3-\omega)\omega_1\omega_2\omega_3\times$$
$$\times\left(1+f(\omega_2)f(\omega_3)+f(\omega_1)f(\omega_3)+f(\omega_1)f(\omega_2)\right)J_2(\epsilon,\omega_1,\omega_2,\omega_3).$$

$$J_2(\epsilon,\omega_1,\omega_2,\omega_3)=\int d\epsilon_1 d\epsilon_2 d\epsilon_3\delta(\epsilon_1+\epsilon_2+\epsilon_3-\epsilon)\times$$
$$\times\frac{1}{\pi^3}\frac{\Gamma(\omega_1,\epsilon_1)\Gamma(\omega_2,\epsilon_2)\Gamma(\omega_3,\epsilon_3)}{\left[\epsilon_1^2+\Gamma^2(\omega_1,\epsilon_1)\right]\left[\epsilon_2^2+\Gamma^2(\omega_2,\epsilon_2)\right]\left[\epsilon_3^2+\Gamma^2(\omega_3,\epsilon_3)\right]}$$

- $\Gamma(\omega,\epsilon)\approx\Gamma(\omega)$ depends weakly upon $\epsilon=\omega-uq$.
- Contributions to Γ come from a broad range of $\omega<\omega_i<T$.

$$\Gamma(\omega)=C_2\frac{\alpha|\lambda_-|}{u^3}T\omega^2\sqrt{\ln\frac{T}{|\omega|}}\sim\frac{T\omega^2}{J^2}\sqrt{\ln\frac{T}{|\omega|}}$$

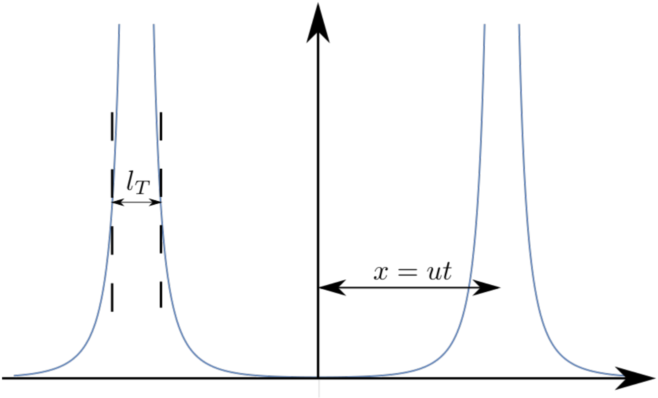
Spin correlation functions (linear Luttinger Luquid)

- Spin correlation function in terms of right and left bosons:

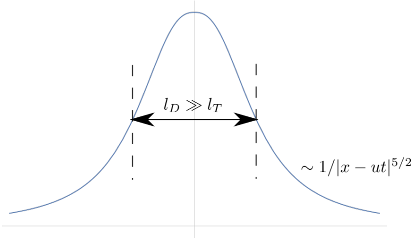
$$\left\langle \hat{S}_n^z(t)\hat{S}_0^z(0)\right\rangle = Ka^2(\langle R(x_n,t)R(0,0)\rangle + \langle L(x_n,t)L(0,0)\rangle)$$

- Bare bosonic correlation function, the only relevant length-scale $l_T = u/T$:

$$\langle R(x,t)R(0,0)\rangle = \frac{1}{4\pi^2}\cdot\frac{\pi^2 T^2}{u^2\sinh^2\frac{\pi T(x-ut)}{u}}$$



Spin correlation functions (magnetic field $h \gg T$)



- New length scale appears:

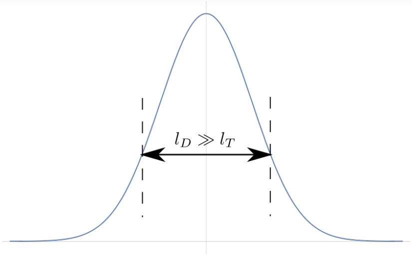
$$l_D(t) = (T/u)^{1/3}(C_2\alpha_1|t|)^{2/3} \propto |t|^{2/3}$$

- Correlation function:

$$\langle S_n^z(t)S_0^z(0)\rangle \approx \frac{\Gamma(5/3)}{2\pi^2}\cdot\frac{a^2}{l_T l_D}, \quad |n \mp ut/a| \ll l_D/a$$

$$\langle S_n^z(t)S_0^z(0)\rangle \approx \frac{3}{8\sqrt{2}\pi^{3/2}}\frac{l_D^{3/2}}{l_T a^{1/2}}\frac{1}{|n \mp ut/a|^{5/2}}, \quad |n \mp ut/a| \gg l_D/a$$

Spin correlation functions (zero magnetic field)



- Similar length scale:

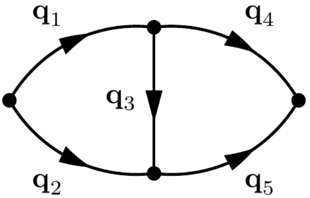
$$l_D(t) = (C_1 \alpha \lambda_- T|t|/u)^{1/2} \propto |t|^{1/2}$$

- Correlation function:

$$\langle S_n^z(t) S_0^z(0) \rangle = \frac{1}{4\pi^{3/2}} \frac{a^2}{l_T l_D} \frac{1}{\ln^{1/4}(l_D/l_T)}, \quad |n \mp ut/a| \ll l_D/a$$

$$\langle S_n^z(t) S_0^z(0) \rangle = \frac{1}{4\pi^{3/2}} \frac{a^2}{l_T l_D} \ln^{-1/4}\left(\frac{|na \mp ut|}{l_T}\right) \times$$
$$\times \exp\left(-\frac{1}{4} \frac{(na \mp ut)^2}{l_D^2} \ln^{-1/4}\left(\frac{|na \mp ut|}{l_T}\right)\right), \quad |n \mp ut/a| \gg l_D/a$$

Higher order corrections (beyond self-consistency) for $h \neq 0$



$$\delta\Gamma(\omega) = \frac{8\alpha_1^4 T^2}{(2\pi)^6 u^8}\omega^2 \int \frac{d\omega_i}{2\pi} \frac{1}{(\Gamma_1+\Gamma_2)(\Gamma_4+\Gamma_5)} \left(\frac{\omega_2\omega_4}{\Gamma_1+\Gamma_3+\Gamma_5} + \frac{\omega_1\omega_5}{\Gamma_2+\Gamma_3+\Gamma_4} \right)$$

- An estimate of the result:

$$\delta\Gamma(\omega) \sim \frac{|\alpha_1|}{u^2} T^{1/2}|\omega|^{3/2} \sim \Gamma(\omega)$$

- Correction is of the same order as the self-consistent result itself!

Applicability range of the self-consistent result and noisy KPZ equation

- M. Arzamasovs, F. Bovo, and D. M. Gangardt, Phys.Rev.Lett. 2014, in the context of One-dimensional superfluids: hydrodynamic description of bosonic chiral modes in the form of the Kardar-Paris-Zhang (or noisy Burgers) equation

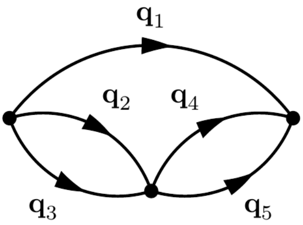
$$(\partial_t - u\partial_x)R = \frac{1}{2m}(\partial_x R)^2 + D\partial_x^2 R + \chi(t,x)$$

- Solution of the KPZ equation was provided in the paper by M. Prahofer and H. Spohn, J. Stat. Phys 115, 255 (2004)
- It was stated by Arzamasovs, F. Bovo and Gangardt, that hydrodynamic approximation is applicable for the quantum problem at very low frequencies $\omega \leq \omega^*$ only, where

$$\omega^* \propto T^7$$

- Such an estimate does not seem to be self-consistent. Actual boundary for the applicability of the 3/2 scaling is unknown at present.

Higher order corrections (beyond self-consistency) for symmetric case $h = 0$



- Contribution to self-energy:

$$\Sigma^{(4)}_{ret}(\omega = uq) \simeq \frac{432\alpha^3\lambda_-^3 T^3}{(2\pi)^5 u^8}\omega \int \frac{d\omega_i}{2\pi} \frac{\omega_2 + \omega_3}{(\Gamma_1 + \Gamma_2 + \Gamma_3)(\Gamma_1 + \Gamma_4 + \Gamma_5)}$$

- Surprisingly, it is purely real:

$$\Sigma^{(4)}_{ret}(\omega = uq) = C_1' \frac{\alpha\lambda_-}{u^2} \frac{T\omega}{\ln \frac{T}{|\omega|}}$$

- Higher-order diagrams may provide additional contributions to the decay rate, but they are small as some inverse powers of $\ln \frac{T}{\omega}$

Conclusions

- Thermal conductivity of the XXZ chain with weak random-field disorder diverges as T^{3-2K} as $T \to 0$ in the range of couplings corresponding to $3/2 < K < 5/2$
- At larger K nonlinearity of the spectrum should be taken into account and forward-scattering disorder may become relevant
- In the **clean** model decay rate of Bosonic quasiparticles is calculated for low-frequency excitations with $\omega \ll T$
- Dynamic spin-spin correlation function for the clean model is calculated

Future plans: interplay between disorder and spectrum nonlinearity.