The Multiple Facets of Kardar-Parisi-Zhang Universality Class

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1. Introduction

My main research interest is universality in out-of-equilibrium systems. During my thesis I worked on both classical and quantum systems trying to build a relation between the two. To do so I studied the behavior of the fluctuations of the phase in a driven-dissipative Bose-Einstein condensate of Exciton-Polaritons (EP) via direct numerical simulations and analytical field-theoretical studies. Furthermore I studied the universal properties of Kardar-Parisi-Zhang (KPZ) equation in the presence of long-range temporal correlations, using non-perturbative renormalization group (NPRG) formalism.

2. Emergence of KPZ Physics in EP [1]

EP – EP are bosonic quasi-particles stemming out from the interaction between an exciton in quantum well and a confined photon; in the early 2000s it was first shown that EP can condensate in a steady state regime, when pumping overcomes the losses due to the unavoidable imperfections of the cavity [2]. Here the concept of condensation, which might seem ill-defined in a system in which the total number of particles is not conserved, is related to the macroscopic occupation of the lower EP band. The usual way to describe the dynamics of this condensate is a two-system model, in which the excitonic (high-energy) part acts as a reservoir for the (low-energy) condensate. When the two have well separated time-scale one can adiabatically integrate out the reservoir ending up with and effective classical-field description for the condensate. The resulting equation strongly resembles the complex Ginzburg-Landau equation (cGLE) usually studied in classical chaotic systems and is usually referred to as generalized Gros-Pitaevskii equation (gGPE) [3]. In the stationary regime when fluctuations of the density can be neglected, it is possible to show that the dynamics of the phase of the condensate described by gGPE follows the well-known KPZ equation.

KPZ Universality – KPZ was first introduced in the late eighties to describe self-critical growing surfaces and became a paradigmatic model for classical out-of-equilibrium systems due to its strongly non-perturbative features. Several discrete and continuous models in classical statistical mechanics, such as directed polymers in random media (DPRM), randomly stirred fluids and Burgers' turbulence, fall in KPZ universality class. By exploiting the analogy between DPRM and integrable models in quantum mechanics it was shown in the late 2000s that KPZ equation in one spatial dimension possesses three different universality sub-classes. To each subclass is associated a different probability distribution for the value of the fluctuations on the surface. It turned out that these distributions are surprisingly linked to the distributions of highest eigenvalues of random matrices in the Gaussian ensemble [4].

State of the Art – The mapping between gGPE and KPZ was first highlighted theoretically in the early nineties. The recent (both theoretical and experimental) progresses on EP physics renewed the interest on this mapping, questioning the possibility of having KPZ universal features in a strongly out-of-equilibrium quantum system. A first attempt in this direction was done in 2015 by direct simulation of gGPE using ad-hoc chosen parameters. The authors were able to find a good agreement between the two-point phase correlation and the theoretical KPZ exponents. However when experimental parameters were put into the gGPE the conclusion was negative [5]. This failure using the gGPE as a model for EP triggered the question if natural imperfections in the experimental system could reverse the negative answer. Furthermore this numerical analysis was only focused on the KPZ exponents while it is a well-known fact in the mathematics and statistical physics community that KPZ universality possesses a lot of interesting features beyond the set of scaling exponents.

Our Contribution – To go further in the understanding of the behavior of the phase of an EP condensate we modified the usual phenomenological description by adding more realistic terms in the model. For instance, a usual important effect is due to a momentum-dependent loss rate in the cavity due to contributions of the excitonic part of the spectrum. This reflects into an imaginary diffusion term in the gGPE which turns out to be crucial in the long-time and -space dynamics of the phase. Indeed using this term together with real experimental parameters and a refined dispersion for the EP we showed that not only the exponents are in perfect agreement with the KPZ predictions but also we were able to reproduce with very high accuracy some of the theoretical predictions for the

KPZ sub-classes. In particular we showed that the phase distribution in the growing regime follows the Tracy Widom (TW) GOE distribution, usually associated with flat initial conditions. Moreover we detected the emergence of stationary effects due to the finiteness of the sample; this reflects in a cross-over between the TW GOE distribution to the Baik-Rains (BR) distribution, usually associated to KPZ equation with Brownian initial conditions. The experimental proof of this mapping is an on-going work in collaboration with groups in Paris and Grenoble. To push forward the analytical mapping we introduced other experimental relevant effects, such as confinement, disorder and interaction with phonons, in the Lindblad master equation describing the microscopic dynamics of EP. Using Schwinger-Keldysh field-theoretical formalism it possible to show that the mapping with KPZ in the long-time and -distance regime gets modified in a non-trivial way but is still holding. Furthermore we checked these theoretical predictions via direct simulations of the gGPE with confinement and static disorder.

3. KPZ with Long-Range Temporal Correlations [6]

Statement of the Problem – It is particularly important to focus on the case of EP with disorder. The associated KPZ equation has a noise correlator which in principle is non-local in space and time (if the disorder is not static). The long-range time correlation is of particular interest because it breaks one of the key symmetry of KPZ equation, that is the one under infinitesimal tilting of the surface. Such invariance is related to Galilean invariance (GI) of the Burgers equation describing the dynamics of the gradient of the KPZ field. This symmetry protects the renormalization of the microscopic non-linearity and fixes an exact relation between the static and dynamic exponent of KPZ universality class. A lot of perturbative studies have been performed in the last twenty years in order to understand the phase diagram of this long-range model, many of which with much different predictions even at the qualitative level [7, 8].

State of the Art – In particular it is not clear if the presence of an infinitesimal LR correlation is enough to forbid the GI to be dynamically restored and thus recover the usual KPZ fixed point. Dynamical renormalization group (DRG) analysis performed by Medina *et al*. suggests that a threshold for the exponent of the power-law temporally-correlated noise exists, below which usual KPZ fixed point is reached; on the contrary self-consistent studies done by Katzav and Schwartz found that any infinitesimal LR correlation spoils usual KPZ behavior. Further numerical and analytical studies didn't manage to solve the inquiry. All these results focused on the one dimensional case, whose scaling exponents are known exactly in the usual KPZ equation thanks to an accidental time reversal symmetry. In more than one dimension perturbation theory is known to fail for KPZ equation, because of the strong-coupling nature of the interacting fixed-point.

NPRG – These controversies in literature called for a clarification, which is in principle achievable by using NPRG approach. This technique relies on the existence of an exact renormalization group (RG) equation for the one-particle irreducible (1PI) generating functional [9]. This equation is integro-differential and in principle very hard to solve. A lot of research has been done on the subject since the early nineties and different approximation schemes and ansatz for the effective action exist. This non-perturbative approach allows to tackle intrinsically strong coupled problems such as fully developed turbulence and KPZ equation, and is the natural candidate to understand the physics of KPZ equation with long-range time correlation.

Our Contribution – To tackle the problem we employed the Martin-Siggia-Rose-Janssen-DeDominicis (MSRJD) field-theoretical representation of KPZ equation and used this action as ansatz for the NPRG. All the couplings were replaced by functions of the momentum and frequency and allowed to run under RG. The dynamical restoration of GI can be checked by allowing time and non-linearity to flow separately. This approximation is at second order in the response field and allows to keep the full momentum and frequency dependence of the correlators. By tailoring the initial (microscopic) value of the running function associated with the noise vertex we could study the influence of different types of temporal correlations in the noise. Using this technique we were also able to study the influence of short-range (SR) noises which are in general hard to study with perturbation theory even in one-dimension.
Our analysis showed that for any SR noise the usual KPZ fixed point is reached, with a typical RG time which depends of the microscopic value of the SR time correlation. For the LR regime we found that there is a threshold for the value of the exponent of the correlation; below this value GI is restored dynamically and the usual KPZ fixed point is found. Above this value, which can be calculated exactly for any dimension, a line of LR fixed point with continuously varying exponents emerges. This work validated the results first proposed by Medina using DRG. The higher dimensional case is an ongoing work.

4. Besides the Thesis and Beyond

Besides these two main projects I got the opportunity to discover other topics related to classical and quantum out-of-equilibrium systems. In particular I have been interested in the generalization of growth phenomena and hydrodynamics on Riemannian manifolds, active matter and passive scalars, and the behavior of growth phenomena constrained by mass conservation. In particular the NPRG study of the latter is a topic of current investigation, in collaboration with people in Paris. Concerning quantum systems I find very interesting the non-linear hydro-dynamical description in one dimension and its relation to universality and phase transition. Another question that I find really appealing is the investigation of the pre-thermal regime in quantum systems starting from out of equilibrium initial conditions.

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