Effect of disorder close to the superfluid transition in a two-dimensional Bose gas

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We experimentally study the effect of disorder on trapped quasi two-dimensional (2D) ^{87}Rb clouds in the vicinity of the Berezinskii-Kosterlitz-Thouless (BKT) phase transition. The disorder correlation length is of the order of the Bose gas characteristic length scales (thermal de Broglie wavelength, healing length) and disorder thus modifies the physics at a microscopic level. We analyze the coherence properties of the cloud through measurements of the momentum distributions, for two disorder strengths, as a function of its degeneracy. For moderate disorder the emergence of coherence remains steep but is shifted to a lower entropy. In contrast, for strong disorder, the growth of coherence is hindered. Such studies of dirty atomic Bose gases are relevant to the understanding of superfluid-insulator transitions occurring in several condensed matter systems.

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Together with band structure and interactions, disorder is a key ingredient for the understanding of transport in condensed matter physics [1]. At low temperature, it affects the conductivity of a metal and it even induces phase transitions to insulating states [2]. A striking example is Anderson localization [3], which has recently been observed in 3D ultracold gases [4].

Disorder is especially relevant in 2D systems, such as Si-MOSFET [5], or thin metal-alloy films [6], in which quantum phase transitions to insulating phases have been observed. Moreover, in high-Tc superconductors, doping intrinsically introduces inhomogeneities in the CuO-planes [7]. Understanding the complex interplay between disorder and interactions in these systems remains a major challenge.

Whereas the above mentioned electronic systems are fermionic, superconductivity originates from the bosonic nature of Cooper pairs. As long as disorder does not break the Cooper pairs, the problem is reduced to a study of dirty bosons [8, 9]. It has mainly been studied numerically in the framework of the disordered 2D Bose-Hubbard model. Disorder may either favor or disfavor superfluidity [10], and the occurrence of a Bose glass, an isolating, gapless, compressible phase has been predicted [8].

In the context of ultra-cold atoms, the properties of disordered *trapped* Bose gases have been studied both in 1D [11–13] and 3D [14]. In 2D Bose gases, disorder will affect the physics of the Berezinskii-Kosterlitz-Thouless (BKT) superfluid phase transition [15, 16], which is associated with the pairing of thermal vortices. In a continuous system, the effect of disorder is expected to depend on the correlation length of the disorder σ, which has to be compared to the characteristic length scales of the cloud such as the thermal de Broglie wavelength λ_{dB} and the healing length ξ, *i.e.* the vortex core size [17]. For slowly varying disorder ($\sigma \gg \xi, \lambda_{\mathrm{dB}}$), the physics can be locally described by the homogeneous BKT transition, and disorder causes a percolation transition of superfluid islands. In contrast, for a microscopically correlated disorder ($\sigma \lesssim \xi, \lambda_{\mathrm{dB}}$), the very nature of the phase transition may be affected by strong enough disorder, and the interplay between BKT physics and disorder is more involved.

In this letter, we present an experimental study of the effect of microscopically correlated disorder on the coherence properties of a 2D ultracold atomic gas near the BKT superfluid transition. As in [18], the coherence properties are probed by the study of the momentum distribution, which is the Fourier transform of the first order correlation function g_1 [19]. We observe that ramping up the disorder is an adiabatic process and results in a suppression of the low momentum peak, *i.e.* a decrease of coherence. In particular, for a moderate disorder, we measure a shift of the emergence of coherence towards low entropy. For strong disorder, the growth of coherence is hindered both as a function of entropy and temperature. This behavior is compatible with the existence of a Bose glass phase with only short-range coherence.

We prepare a 2D Bose gas as in [18]. More precisely, 2D trapping is obtained by a combination of a blue detuned TEM_{01} beam, which confines the gas in an horizontal plane, and a red detuned Gaussian beam for the in-plane confinement. The trap oscillation frequencies are $\omega_x/2\pi = 8\,\mathrm{Hz}$, $\omega_y/2\pi = 15\,\mathrm{Hz}$, $\omega_z/2\pi = 1.5\,\mathrm{kHz}$. The atom number N is varied between 2×10^4 and 6×10^4 in order to change the degeneracy of the gas across the BKT transition. The temperature, measured from a fit to the wings of the momentum distribution [18], remains constant at $64.5\pm2.0\,\mathrm{nK}$. At this temperature, $\sim 70\%$ of the atoms are in the ground state of the vertical harmonic oscillator. The dimensionless 2D interaction strength is $\tilde{g} = \sqrt{8\pi}a_\mathrm{s}/a_\mathrm{z} = 0.096$, where $a_\mathrm{s} = 5.3\,\mathrm{nm}$ is the 3D scattering length, $a_\mathrm{z} = \sqrt{\hbar/m\omega_z}$ the vertical harmonic oscillator characteristic length, m the atom mass, and $\hbar$ the reduced Planck constant.

The disorder potential is a speckle pattern produced by a 532 nm laser beam, which passes through a diffusive plate and is focused on the atoms. The repulsive disorder potential is characterized by its mean value $\overline{V}$ (equal to its standard deviation) and its correlation lengths, inversely proportional to the numerical aperture of the optical system [20]. Given the intensity of the beam and its transverse waist radius of 1 mm, the maximum value of $\overline{V}$ felt by the atoms is $\overline{V}_\mathrm{max} = \mathrm{k_B} \times 60(10)\,\mathrm{nK}$. As the beam is tilted by 30°, the in-plane disorder is effectively anisotropic [21]. The correlation lengths of the disorder are such that $\sigma_x/2 = \sigma_y = 0.5\,\mu\mathrm{m}$ (half-width at $1/\sqrt{e}$). These correlation lengths are of the order of both the thermal de Broglie wavelength $\lambda_\mathrm{dB} = \sqrt{2\pi\hbar^2/mk_\mathrm{B}T} \approx 0.73\,\mu\mathrm{m}$ and the healing length (at the BKT transition) $\xi = \lambda_\mathrm{dB}/\sqrt{D_c\tilde{g}} \approx 0.82\,\mu\mathrm{m}$, where $D_c \approx \log(380.3/\tilde{g}) \approx 8.3$ is the BKT critical phase space density [22]. However, σ_x and σ_y are small compared to the Thomas-Fermi radii of the cloud at the BKT transition $l_x = \frac{\hbar\sqrt{2\tilde{g}D_c}}{m\omega_x\lambda_\mathrm{dB}}{=}25\,\mu\mathrm{m}$, $l_y = (\omega_x/\omega_y)l_x = 13\,\mu\mathrm{m}$, a necessary condition for self-averaging measurements. In our experiment, we use a single realization of the speckle pattern.

In the experimental sequence, the disorder potential is slowly ramped in 250 ms after the preparation of the 2D gas. Finally, after a holding time of 250 ms, all trapping potentials are switched off and the atom cloud expands in 3D during a free fall of 83.5 ms. The column density of the gas along z is then measured by fluorescence imaging from the top. As explained in [18], the measured spatial distribution in the x, y plane reflects the in-trap momentum distribution. Since, the momentum distributions appear to be cylindrically symmetric, we perform an azimuthal averaging [23] to obtain the momentum profiles $n(k)$ as a function of the wavenumber k.

We study the effect of disorder on the momentum distribution for different initial conditions, both above and below the BKT transition. As in [18], we quantify the degeneracy of the non-disordered gas with the ratio N/N_c, where N_c is the critical atom number of an ideal Bose gas in our trap. The BKT phase transition happens at $N/N_c \approx 1.26$ for our parameters. In Fig. 1, we compare the momentum distribution without disorder to the results obtained after ramping up the disorder potential to $\overline{V} = \overline{V}_\mathrm{max}/2$ and $\overline{V} = \overline{V}_\mathrm{max}$. For $N/N_c = 1.06$ (Fig. 1c), the non-disordered gas is in the normal phase. In this case, the addition of the disorder has little effect, reducing slightly the low momentum population ($k < 2\,\mu\mathrm{m}^{-1}$). For $N/N_c = 1.32$ (Fig. 1b), the gas has just entered the superfluid phase in the absence of disorder and a low momentum peak is clearly present. In this case, the disorder has a strong effect. The population at very low momentum ($k < 0.5\,\mu\mathrm{m}^{-1}$) is strongly suppressed and the low momentum peak almost disappears. The profiles with disorder are then qualitatively similar to the one in the normal phase (Fig. 1c). For $N/N_c = 1.75$ (Fig. 1a), the non-disordered gas is deep in the superfluid phase with a large low momentum peak. In this case, the addition of the disorder, leads to a reduction of the height of the peak but not to its disappearance. In all our data, adding disorder always results in a reduction of the coherence of the Bose gas.

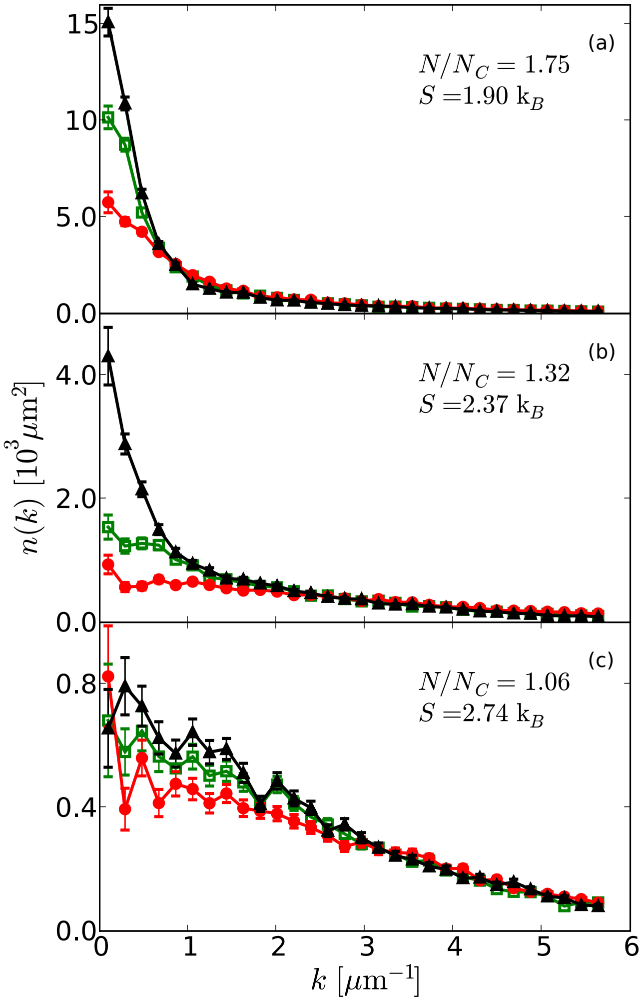

FIG. 1. Azimuthally averaged momentum distribution profiles for $N = 5.6\times10^4$ (a), $N = 3.8\times10^4$ (b) and $N = 2.9\times10^4$ (c). In each case, we show the influence of the disorder: No disorder $\overline{V} = 0$ (Black triangles), $\overline{V}_\mathrm{max}/2$ (green open squares), $\overline{V}_\mathrm{max}$ (red circles).

It should be noted that switching on the disorder preserves the entropy. When the disorder potential is ramped up to a mean value $\overline{V}_\mathrm{max}$ in 250 ms and then down in 250 ms, we find no heating compared to the non-disordered situation within our experimental preci-

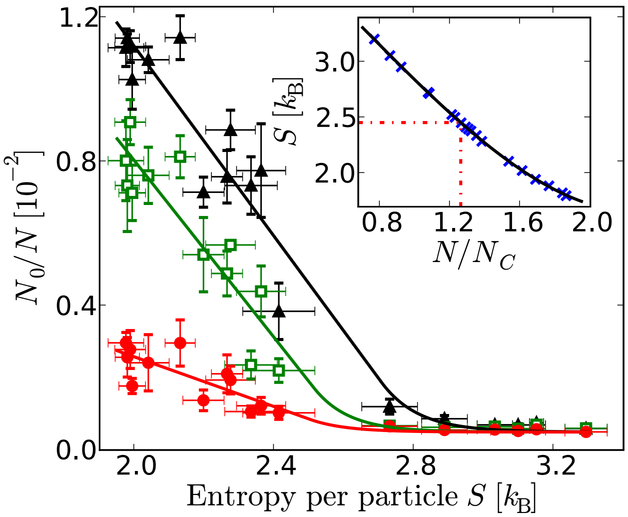

FIG. 2. Fraction of atoms N_0/N in the central pixel of the momentum distribution as a function of the average entropy per particle S: Non-disordered case $\overline{V} = 0$ (black triangles), $\overline{V} = \overline{V}_{\max}/2$ (green open squares), $\overline{V} = \overline{V}_{\max}$ (red circles). Each point results from the averaging of 5 experimental profiles and the error bars are statistical. Lines are guides to the eyes. Inset: Entropy per particle measured by quantum Monte-Carlo simulations at 64.5nK (blue cross) and fitted by a 3^{rd} order polynomial (black line). The dashed lines indicate the BKT transition.

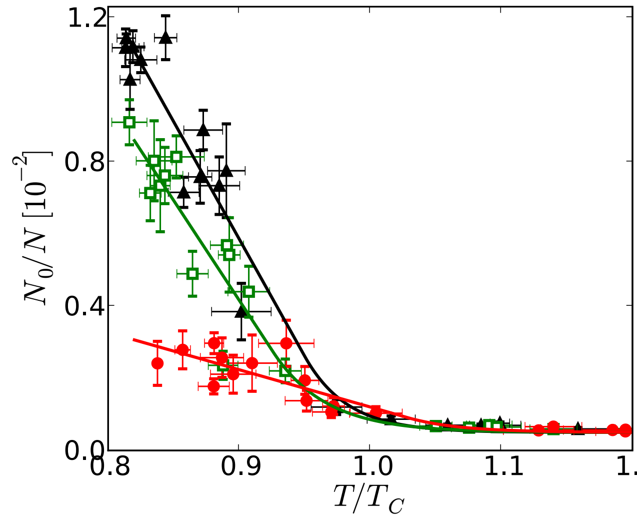

FIG. 3. Fraction of atoms N_0/N in the central pixel of the momentum distribution as a function of the normalized estimated temperature (see text) for disorder strengths, $\overline{V} = 0$ (black triangles), $\overline{V} = \overline{V}_{\max}/2$ (green open squares), and $\overline{V} = \overline{V}_{\max}$(red circles). Each point results from the averaging of 5 experimental profiles and the error bars are statistical. Lines are guides to the eyes.

sion (± 1 nK). We observe that the process is reversible. In the following, we thus assume that consecutive pictures with and without disorder correspond to the same entropy (Fig. 1).

Nevertheless, even in the absence of disorder, we do not have experimental access to the entropy. For finding the correspondence between entropy and N/N_c, we rely on quantum Monte-Carlo simulations of the non-disordered *in-situ* distribution [24], from which the entropy can be determined because of the scale invariance of the 2D Bose gas [25]. The calibration of the average entropy per particle S as a function of N/N_c for our experimental conditions is shown in the inset of Fig. 2.

We now focus our analysis on the fraction of atoms N_0/N in the central pixel of the momentum distribution ($k < 0.2\,\mu\text{m}^{-1}$). This quantity is related to the fraction of atoms that are coherent on a length scale larger than $\sim 5\,\mu$m [18]. We plot N_0/N as a function of the entropy per particle S (Fig. 2). We find that, at fixed entropy, the coherence of the gas is reduced in the presence of disorder. For $\overline{V} = \overline{V}_{\max}/2$, the emergence of coherence happens at a lower entropy per particle compared to the non-disordered case (shift of $\sim 0.2 \times k_B$). For $\overline{V} = \overline{V}_{\max}$, we never reach a sufficiently low entropy to observe a large increase of coherence.

In order to connect to theoretical studies of the dirty boson problem, we analyze the effect of disorder at constant temperature rather than at constant entropy. This means that we have to determine the temperature from the experimental disordered profiles. In the absence of exact theoretical predictions for a 2D disordered gas, we assume that our non-disordered Hartree-Fock mean-field model [18] gives a reliable estimation of the temperature at large momenta even in the disordered case. We choose to fit the wings of the distribution for k $> 3.5\mu\text{m}^{-1}$ in order to keep a sufficiently high signal to noise ratio. Experimentally, we find that for $\overline{V} = \overline{V}_{\max}$ the temperature increases on average by 5.5 nK compared to the non-disordered case ($T = 64.5$ nK).

Figure 3 presents N_0/N as a function of the temperature normalized to T_c, the critical temperature for a non-interacting Bose gas for our trap parameters and the measured atom number. The results without disorder and with a disorder of amplitude $\overline{V}_{\max}/2$ are qualitatively similar and shifted by less than 1 nK. Such a small shift may correspond to a systematic error in our temperature determination with disorder. Within our accuracy, we can however conclude that for this amount of disorder the coherence properties are weakly affected. This is surprising, since $\overline{V}_{\max}/2 \approx 30$ nK is quite large compared to the mean-field chemical potential at the phase transition in the absence of disorder $\mu_c = \tilde{g}\hbar^2 D_c/m{\lambda_{\text{dB}}}^2 \approx k_B \times 8$ nK. The coherence of the gas thus appears robust to such a disorder.

However, for $\overline{V} = \overline{V}_{\max}$, the coherence increases much slower when T decreases. At our lowest temperatures, N_0/N does not reach the value of 0.0035 which corresponds to the superfluid transition in the absence of disorder [18]. In this case, although we do not measure the superfluid fraction, we can suspect that the system is not in a superfluid phase. Our findings are in qualitative agreement with the system being in a Bose glass phase with short-range coherence [8]. Actually, for such a high amount of disorder, the existence of a superfluid is not necessarily expected even at zero temperature because of a disorder-driven superfluid to insulator quantum phase transition [26].

In conclusion, we have experimentally studied the effect of a correlated disorder on the coherence properties of a 2D trapped Bose gas. We observe that, at fixed entropy, disorder always reduces the coherence of the gas. For moderate disorder, we quantitatively measure a shift of the emergence of coherence towards low entropy. Next, we have analyzed our results at fixed temperature, which has to be measured in the absence of an accurate disordered gas theory. We find that moderate disorder weakly modifies the coherence properties, whereas disorder above a certain threshold suppresses the coherence growth. Whether our observed suppression of coherence is related to a shift of the superfluid transition and/or to the physics of the disorder-induced quantum phase transition remains to be elucidated. A complete mapping of the phase diagram of the dirty Bose gas is accessible to future ultracold atom experiments. The mechanism of the disorder action may also be addressed, *e.g.* the pinning of thermally excited vortices [27]. Further, similar studies on strongly interacting fermions add the possibility to elucidate the physics of disorder-induced breaking of bosonic pair.

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N.B.: In the last stage of the redaction of our paper, we have learned about a related work focussing on the effect of disorder on the coherence of a 2D gas in the deep superfluid regime [28].

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