# Casimir Momentum?

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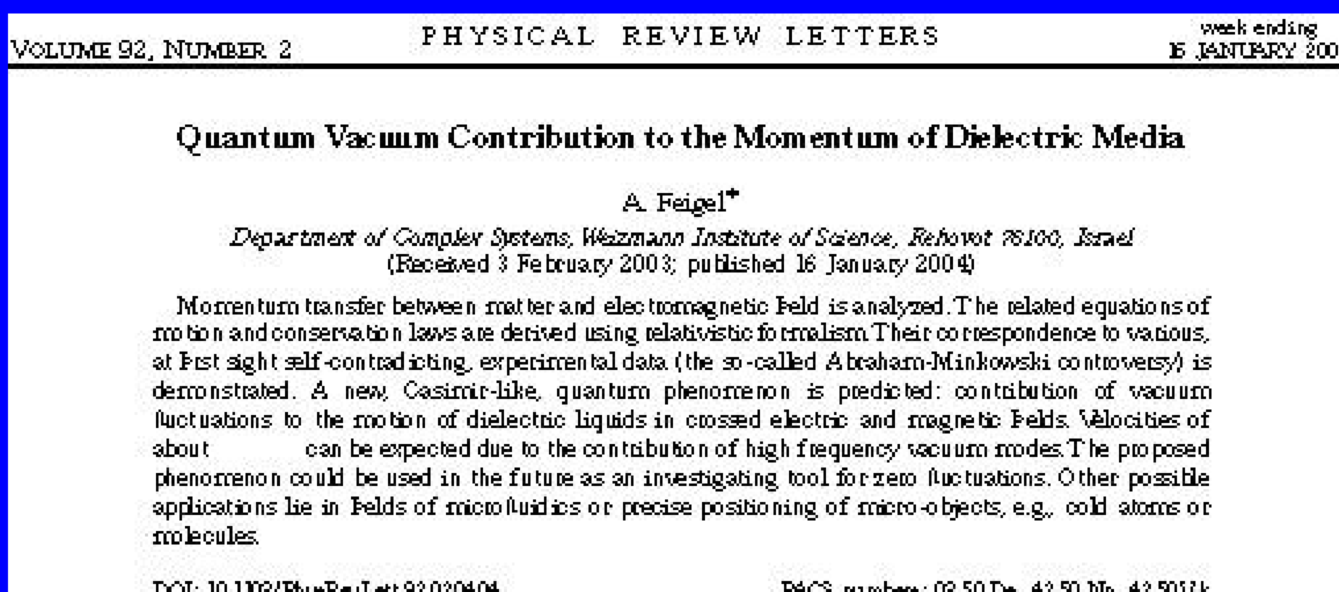
**Amsterdam, November 9, 2007**

# Momentum from Nothing

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

## Quantum Vacuum Contribution to the Momentum of Dielectric Media

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(Received 3 February 2003; published 16 January 2004)

Momentum transfer between matter and electromagnetic field is analyzed. The related equations of motion and conservation laws are derived using relativistic formalism. Their correspondence to various, at first sight self-contradicting, experimental data (the so-called Abraham-Minkowski controversy) is demonstrated. A new, Casimir-like, quantum phenomenon is predicted: contribution of vacuum fluctuations to the motion of dielectric liquids in crossed electric and magnetic fields. Velocities of about can be expected due to the contribution of high frequency vacuum modes. The proposed phenomenon could be used in the future as an investigating tool for zero fluctuations. Other possible applications lie in fields of microfluidics or precise positioning of micro-objects, e.g., cold atoms or molecules.

DOI: 10.1103/PhysRevLett.92.020404 PACS numbers: 03.50.De, 42.50.Nn, 42.50.Vk

# Feigel Theory:
# quantum vacuum contribution to momentum density

*cut-off in X-ray*

$$\langle 0|\mathbf{P}|0\rangle \propto \frac{1}{c_0}\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \times g\;\mathbf{E}_0\times\mathbf{B}_0$$

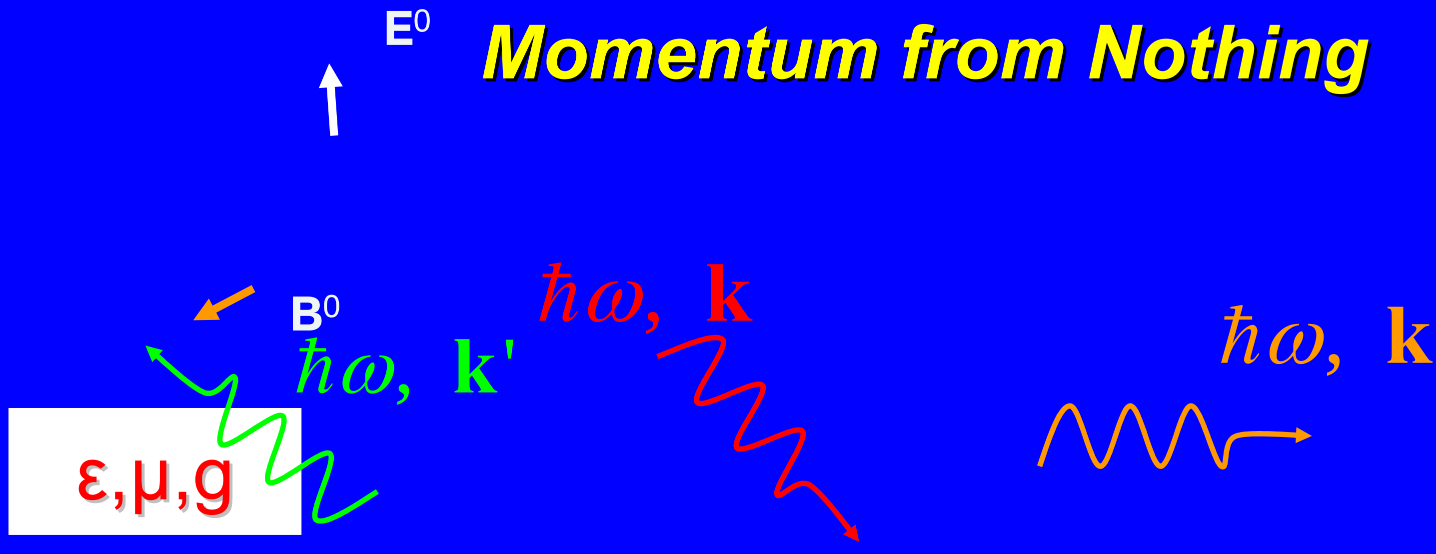
$$=\frac{2}{3}\frac{\hbar\omega_c^4}{\pi^3 c_0^4}\;g\,\mathbf{E}_0\times\mathbf{B}_0$$

1. *P,C,T = OK*
2. *What about Lorentz invariance?*
3. *What is the momentum of photons in matter?*
4. *Beyond what frequency are we supposed to cut vacuum fluctuations, if at all?*
5. *Is there also a classical contribution ?*

# Casimir energy

1. Isotropic radiation with power spectrum $\omega^3$ is Lorentz-invariant (Einstein, 1917); $\int_0^\infty d\omega \int_{4\pi} d\Omega \, \tfrac{1}{2}\hbar\omega^3$ Lorentz scalar
2. Van der Waals force $1/r^6$ (London, 1930)
3. Relation to Cosmological constant (Dirac, 1934, Davies, 1984)
4. Casimir Polder Force $1/r^7$ (1947)
5. Attraction between metallic plates (Casimir, 1948)
6. Lifshitz theory for dielectric media (Lifshitz, 1956, Dzyalonishiniskii 1961)
7. Observation of Casimir effect (Sparnaay, 1958, Lamoureux (5%), 1997), Chan etal, (1%), 2001)
8. Stability of the electron (Casimir, 1956, Boyer, 1968)
9. Unruh effect & Hawking radiation (Hawking 1974, Unruh 1976)
10. Bag model for Hadrons (Jaffe etal, 1974)
11. Sign of the Cosmological constant (Weinberg, … 1983)
12. Sonoluminescence (Schwinger, 1993, Eberlein, 1996)
13. Casimir momentum in magneto-electric media (Feigel, 2004)

# *Momentum from Nothing*

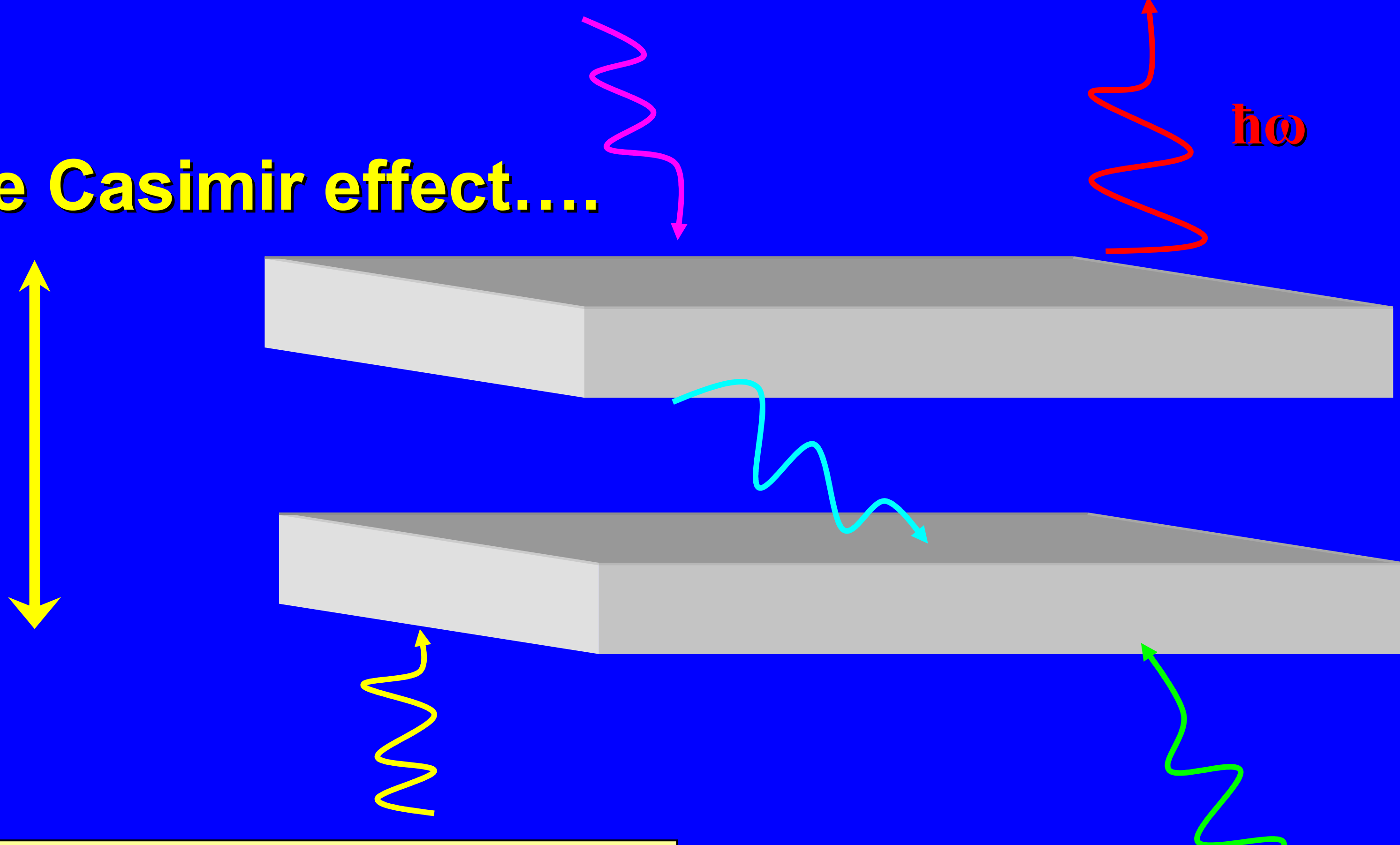



1. *It is wrong*
2. *It has been done before*
3. *It is not even wrong*
4. *It is fantastic!*

# *Momentum from Nothing*


1. *It is wrong*
2. *It has been done before*
3. *It is not even wrong*
4. *It is fantastic!*
5. *It is not important ,*
   *otherwise I would have found it first…*

# The Casimir effect….




$$E(L) = \infty - 0.0137..\frac{\hbar c_0 A}{L^3}$$

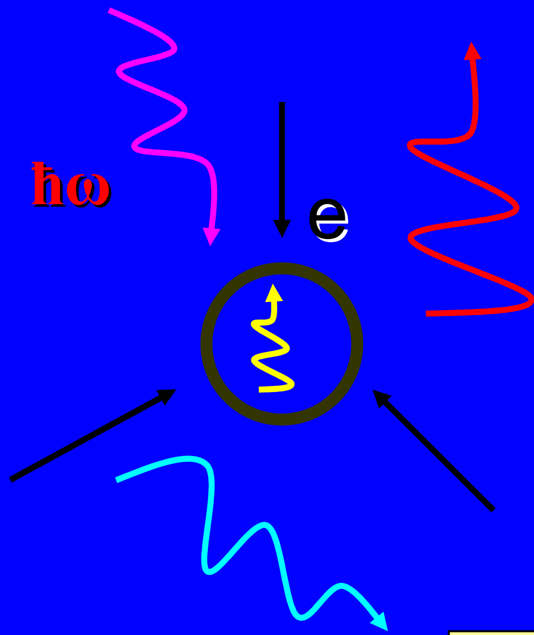
$$\mathbf{F}(L) = -\frac{\partial E}{\partial \mathbf{L}} = -3(\cdots)\frac{\hbar c_0 A}{L^4}\hat{\mathbf{L}}$$

***No momentum exchange between matter and radiation***

***Unambiguously determined negative pressure***

# A brilliant idea…. (1956)

## Does Casimir force stabilize the electron ?




$$F_{\text{Coulomb}} = +\frac{e^2}{2r_e^2}$$

$$F_{\text{Casimir}} = -(\ldots)\frac{\hbar c_0 A}{r_e^4} = -\alpha\ \frac{\hbar c_0}{2r_e^2}$$

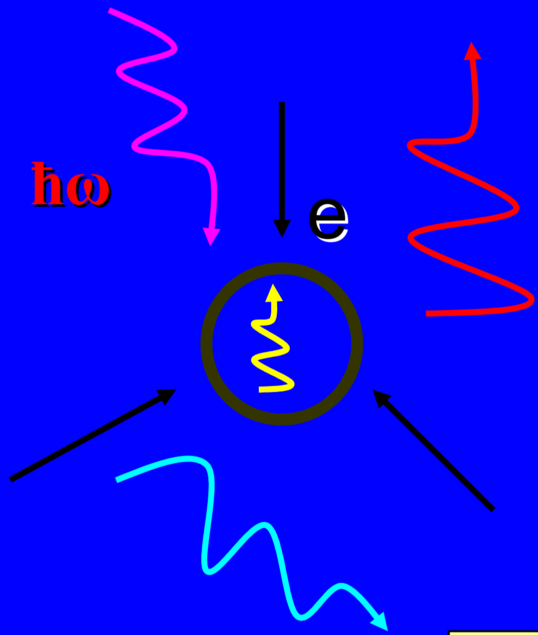
repulsive

attractive

$$\frac{e^2}{\hbar c_0} = \alpha = 0.0073..$$

# A brilliant idea…. (1951)

## Does Casimir force stabilize the electron ?




$$F_{\mathrm{Coulomb}} = +\frac{e^2}{2r_e^2}$$

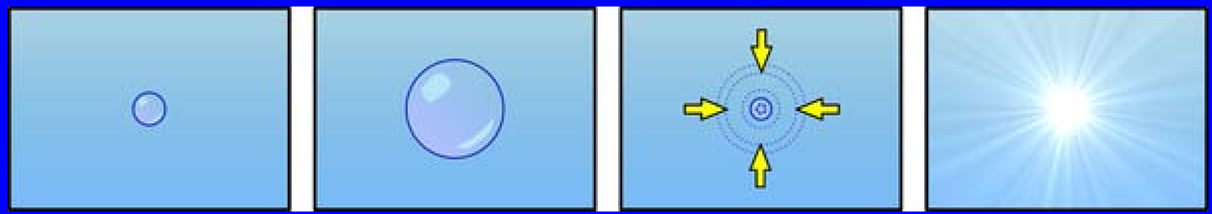
repulsive

$$F_{\mathrm{Casimir}} = -\frac{\partial E}{\partial a} = +0.092\frac{\hbar c_0}{2r_e^2}$$

Boyer (1968) also repulsive !

# *Sonoluminescence*

***(> 1934)***



Schwinger (1993)

$$\Delta E(\text{bubble}) = \int d^3\mathbf{r}\left\{\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_{\mathbf{k}}(\text{bubble in water}) - \int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_{\mathbf{k}}(\text{water no bubble})\right\}$$

$$\approx \frac{\hbar a^3 \omega_c^4}{c_0^3}\left(1-\frac{1}{\sqrt{\varepsilon}}\right) \approx 10\,\text{MeV}$$

**cut-off in the UV ?**

**Ignore divergencies ?**

**Identity of the van der Waals Force and the Casimir Effect and the Irrelevance of These Phenomena to Sonoluminescence**

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(Received 12 October 1998)

$$\Delta E(\text{bubble}) = \frac{23}{1536\pi}\,\frac{(\varepsilon-1)^2 c_0}{a} \approx 0.001\,\text{eV}$$

Where is the infinite Casmir energy ?

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c_0^4} < T_{\mu\nu} >$$

Cosmological constant related to vacuum fluctuations ?

Dirac (1934)

Birell & Davies (1984)

Why is Λ close to zero, yet slightly postive?

# The magical mystery world of regularization

$$\int \frac{d^d\mathbf{k}}{(2\pi)^d}(x^2+k^2)^{-p/2} = \frac{x^{d-p}\Gamma\left(\frac{p-d}{2}\right)}{(4\pi)^d\,\Gamma\left(\frac{p}{2}\right)} \quad (p>d)$$

$$E(\text{bubble}) \approx {}^4\!/_3\,\pi\, a^3 \int d^3\mathbf{k}\,\frac{1}{2}\hbar k c_0\left(1-\frac{1}{\sqrt{\varepsilon}}\right) \xrightarrow[\substack{x=0\\ d=3\\ p=-1}]{} 0$$

**(+ ∞)**

**Sonoluminescence?**

$$\int_{x<1} d^d\mathbf{x}\int_{y<1} d^d\mathbf{y}\,\frac{1}{|\mathbf{x}-\mathbf{y}|^\gamma} = \frac{\pi^{d-1/2}\,2^{d-\gamma}\,\Gamma\left(\frac{d-\gamma+1}{2}\right)}{(d-\gamma)\Gamma(d/2)\Gamma(d+1-\gamma/2)} \quad (d>\gamma/2)$$

$$\int_{x<a} d^3\mathbf{x}\int_{y<a} d^3\mathbf{y}\,\frac{-23\alpha^2N^2}{4\pi|\mathbf{x}-\mathbf{y}|^7} \xrightarrow[\substack{d=3\\ \gamma=7}]{} +\frac{23(\varepsilon-1)^2}{1536\,a}$$

**(- ∞)**

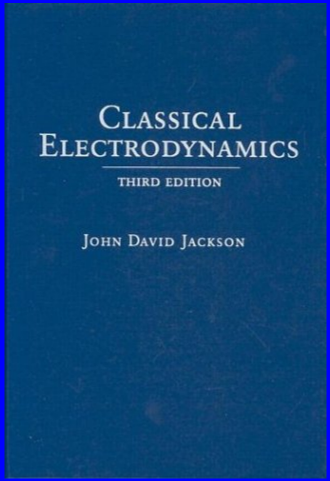
**Casimir energy of N dipoles distributed in a sphere**
**Milton, 1998**

# Classical Electrodynamics of bi-anisotropic media:

$$\partial_t \mathbf{D} = c_0 \nabla \times \mathbf{H} \qquad \nabla \cdot \mathbf{D} = 0$$

$$\partial_t \mathbf{B} = -c_0 \nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

**Macroscopic Maxwell equations**




$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} + \boldsymbol{\chi} \cdot \mathbf{B}$$

$$\mathbf{H} = \boldsymbol{\chi}^* \cdot \mathbf{E} + \mathbf{B}$$

**Constitutive equations**

$$\rho(\mathbf{r}) = \sum_a m_a \delta(\mathbf{r} - \mathbf{r}_a) \qquad \mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \approx \mathbf{v}_a = \frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t}$$

**Microscopic Matter**

**Newton -Lorentz equation**

$$m_a \frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t} = q_a \left( \mathbf{E} + \frac{\mathbf{v}_a}{c_0} \times \mathbf{B} \right)$$

# Bi-anisotropic Media

$$\mathbf{D} = \varepsilon\mathbf{E} + \boldsymbol{\chi}\cdot\mathbf{B}$$

$$\mathbf{H} = \boldsymbol{\chi}^{*}\cdot\mathbf{E} + \mathbf{B}$$

**Fresnel dispersion law** $\left(\Phi_{\mathbf{p}}\right)_{inm} = i\varepsilon_{nml}p_l$

$$\det\left(\varepsilon\frac{\omega^2}{c_0^2} - p^2 + \mathbf{pp} + i\frac{\omega}{c_0}\boldsymbol{\chi}\cdot\mathbf{\Phi_p} - i\frac{\omega}{c_0}\mathbf{\Phi_p}\cdot\boldsymbol{\chi}^*\right) = 0$$

$$\chi_{ij}(\omega) = i\omega\, g\,\delta_{ij}$$

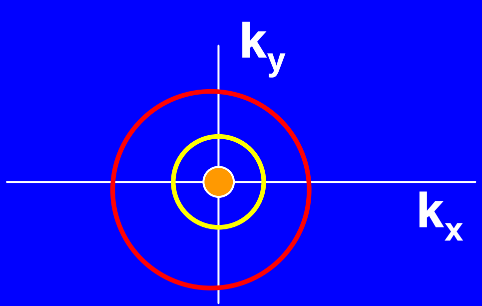
**Rotatory power**

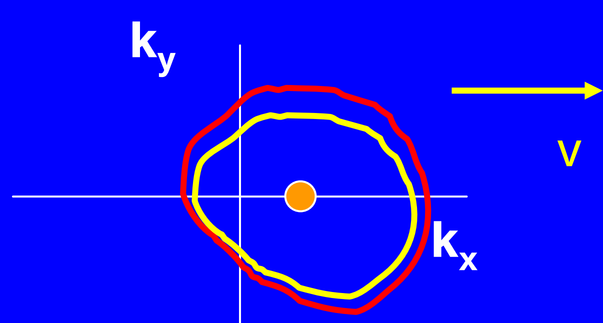
***Fizeau effect***

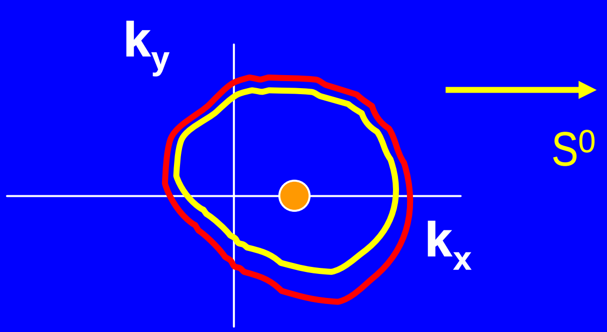
$$\chi_{ij}(\omega) = (1-\varepsilon)\varepsilon_{ijl}\frac{\mathrm{v}_l}{c_0}$$

$$\chi_{ij}(\omega) = g\left(E_i^0 B_j^0 - B_i^0 E_j^0\right)$$

**Magneto-electric birefringence**



# Momentum exchange radiation & Matter

**Macro-Maxwell**

$$\partial_t \mathbf{G} + \nabla \cdot \mathbf{T} = \mathbf{f}(r,t)$$

$$\mathbf{G} = \frac{1}{4\pi c_0}\mathbf{D}\times\mathbf{B} \qquad T_{ij} = E\delta_{ij} - \frac{1}{4\pi c_0}\left(H_i B_j + E_i D_j\right)$$

**Minkowski version of Energy/momentum conservation**

**Newton-Lorentz**

$$\partial_t\left(\rho\,\mathbf{v} - \mathbf{P}\times\mathbf{B}\right) + \nabla\cdot\mathbf{U} = -\mathbf{f}(r,t)$$

$$U_{ij} = \rho v_i v_{\mathbf{i}} + E_i P_j - M_i B_j - \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{P} - \mathbf{M}\cdot\mathbf{B}\right)\delta_{ij}$$

**« pseudo momentum » conservation**

$$f_j\,(r,t) = \frac{1}{8\pi}\left[\mathbf{E}\cdot\left(\partial_j \boldsymbol{\varepsilon}\right)\cdot\mathbf{E} + \mathbf{H}\cdot\left(\partial_j \boldsymbol{\mu}^{-1}\right)\cdot\mathbf{H} + 2\mathbf{E}\cdot\left(\partial_j \boldsymbol{\chi}\right)\cdot\mathbf{H}\right] + \text{free charges}$$

**EM force density**

$$\partial_t\left(\rho\,\mathbf{v} + \frac{1}{4\pi c_0}\mathbf{E}\times\mathbf{B}\right) + \nabla\cdot\mathbf{T}^0 = 0$$

$$T_{ij}^0 = \frac{1}{8\pi}\left(\mathbf{E}^2 + \mathbf{B}^2\right)\delta_{ij} - \frac{1}{4\pi}\left(E_i E_j + B_i B_j\right)$$

# No inelastic recoil from zero-point motion

$$\frac{d}{dt}\int d^3\mathbf{r}\left(\rho\,\mathbf{v}\;+\frac{1}{4\pi c_0}\mathbf{E}\times\mathbf{B}\right) = -\lim_{r\to\infty}\oint d\mathbf{S}\cdot\mathbf{T}_0 = \frac{d\mathbf{p}}{dt}_{\text{recoil}}$$

$$\propto \frac{\hbar}{c_0}\int d\omega\int d^2\mathbf{k}_{in}\int d^2\mathbf{k}_{out}\,\frac{d\sigma}{d\Omega}\left(\mathbf{k}_{in}\to\mathbf{k}_{out}\right)\left(\hat{\mathbf{k}}_{in}-\hat{\mathbf{k}}_{out}\right)$$

$$=0$$

$$\frac{d\sigma}{d\Omega}\left(\mathbf{k}_{in}\to\mathbf{k}_{out},+\boldsymbol{\chi}\right)\overset{T}{=}\frac{d\sigma}{d\Omega}\left(-\mathbf{k}_{out}\to-\mathbf{k}_{in},-\boldsymbol{\chi}\right)\overset{P}{=}\frac{d\sigma}{d\Omega}\left(\mathbf{k}_{out}\to\mathbf{k}_{in},+\boldsymbol{\chi}\right)$$

$$L(\mathbf{E},\mathbf{B},\mathbf{v}) = -\rho\sqrt{1-\mathbf{v}^2} + \frac{1}{2}(\mathbf{E}^2-\mathbf{B}^2) + 2\nu(\mathbf{E}^2-\mathbf{B}^2)^2 + \frac{\nu}{2}(\mathbf{E}\cdot\mathbf{B})^2$$

**Bi-anisotropic Lorentz-invariant vacuum**

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_{zp}$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_{zp}$$

**Fluctuation-Dissipation**

$$\langle 0 | E_i(\mathbf{r},\omega) E^*{}_j(\mathbf{r}',\omega') | 0 \rangle = -2\hbar\omega^2 \,\mathrm{Im}\, G_{ij}(\mathbf{r},\mathbf{r}',\omega) \times 2\pi\delta(\omega-\omega')$$

$$\langle 0 | \frac{\mathbf{E}^*\times\mathbf{H}}{4\pi} | 0 \rangle = 0$$

$$\langle 0 | \frac{\mathbf{E}^*\times\mathbf{B}}{4\pi} | 0 \rangle = -\frac{4}{3}\nu K \; \mathbf{E}_0\times\mathbf{B}_0$$

**Energy flow**

**Momentum density**

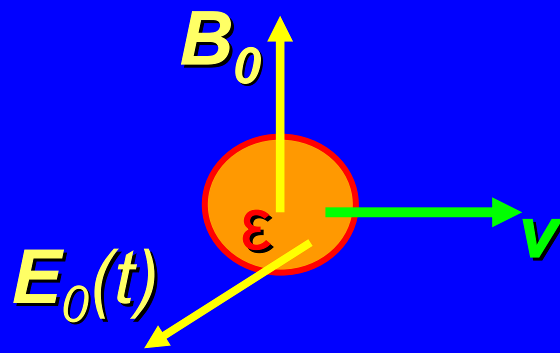
$\omega^3$

$$K = \lim_{\omega_c\to\infty} \frac{1}{(2\pi)^3} \tfrac{1}{2}\hbar \int_0^{\omega_c} d\omega \int_{4\pi} d\Omega \, \rho_0(\omega,\Omega)$$

**Lorentz scalar**

# Classical contribution

## I. Dielectric sphere




$$\rho \mathbf{v} - \mathbf{P}\times\mathbf{B} = \text{constant} \;= 0 \quad \Longrightarrow \quad m\mathbf{v}(t) = \alpha(0)\,\mathbf{E}_0(t)\times\mathbf{B}_0$$

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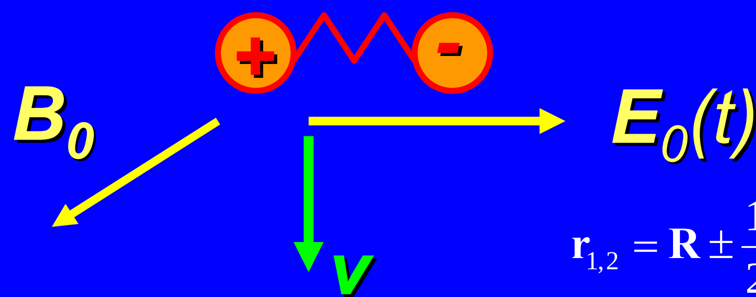
## II. Classical dipole




$$\mathbf{r}_{1,2} = \mathbf{R} \pm \frac{1}{2}(\mathbf{a}+\mathbf{x})$$

$$m\ddot{\mathbf{r}}_1 = +q\mathbf{E}(t) + q\dot{\mathbf{r}}_1\times\mathbf{B} + \mathbf{f}(r_{12})$$
$$m\ddot{\mathbf{r}}_2 = -q\mathbf{E}(t) - q\dot{\mathbf{r}}_2\times\mathbf{B} - \mathbf{f}(r_{12})$$

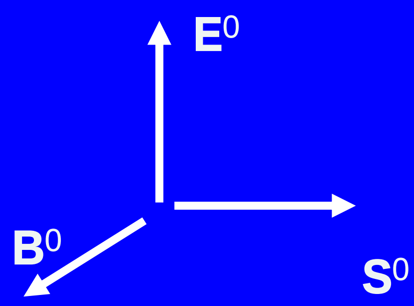
$\Longrightarrow$

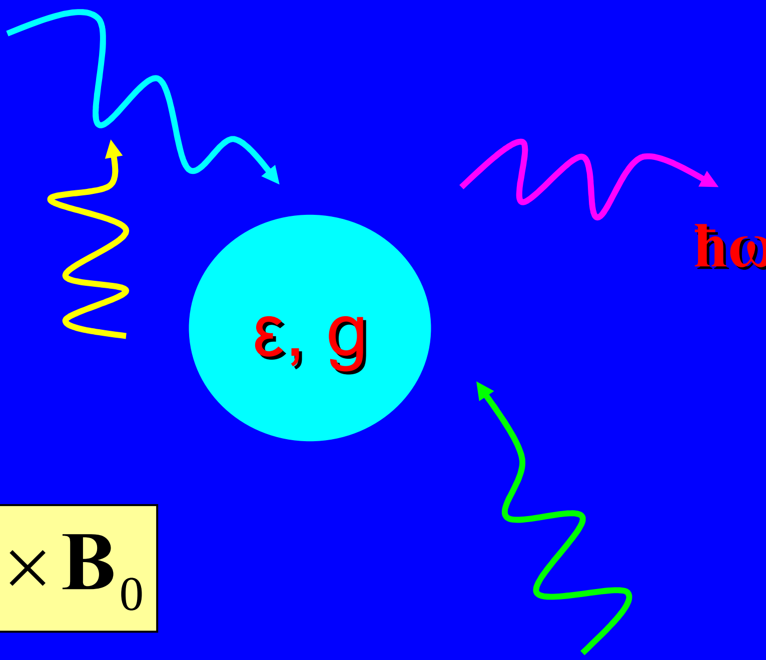
$$2m\dot{\mathbf{R}} + q\mathbf{x}\times\mathbf{B} = \text{constant} = 0$$
$$m\ddot{\mathbf{x}} = 2q\mathbf{E}(t) + 2q\dot{\mathbf{R}}\times\mathbf{B} - m\omega_0^2\,\mathbf{x} \approx 0$$

$$2m\dot{\mathbf{R}} = \frac{q^2/m}{\omega_c^2+\omega_0^2}\mathbf{E}(t)\times\mathbf{B} = \frac{\omega_0^2}{\omega_c^2+\omega_0^2}\alpha(0)\,\mathbf{E}(t)\times\mathbf{B}$$

**Drift velocity**

# Momentum of magneto-electric sphere




$$m\mathbf{v} =$$

classical

$$= (\varepsilon - 1)\, V \;\; \mathbf{E}_0 \times \mathbf{B}_0$$

Zero-point
1e Born

$$- g\, V \;\; \mathbf{E}_0 \times \mathbf{B}_0 \int d^3\mathbf{k}\, \hbar k = \infty$$

Zero-point
2e Born:

$$- \frac{7.54\ldots}{4\pi^4} \times \Lambda(a) \times \hbar(\varepsilon - 1)\, g \;\; \mathbf{E}_0 \times \mathbf{B}_0$$

$$\Lambda(a) \;=\; \int_{x<a} d^3\mathbf{x} \int_{y<a} d^3\mathbf{y}\, \frac{1}{|\mathbf{x}-\mathbf{y}|^7} \to \infty - \frac{\pi^2}{12\, a}$$

*cut-off in X-ray ?*

$$\langle 0 | m\,\mathbf{v} | 0 \rangle \propto (\varepsilon - 1)\frac{V}{c_0}\mathbf{E}_0 \times \mathbf{B}_0 + \frac{V}{c_0}\left( \int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \right) g\;\mathbf{E}_0 \times \mathbf{B}_0$$

$$-0.0158\frac{\hbar c_0}{a}(\varepsilon - 1) g \mathbf{E}_0 \times \mathbf{B}_0$$

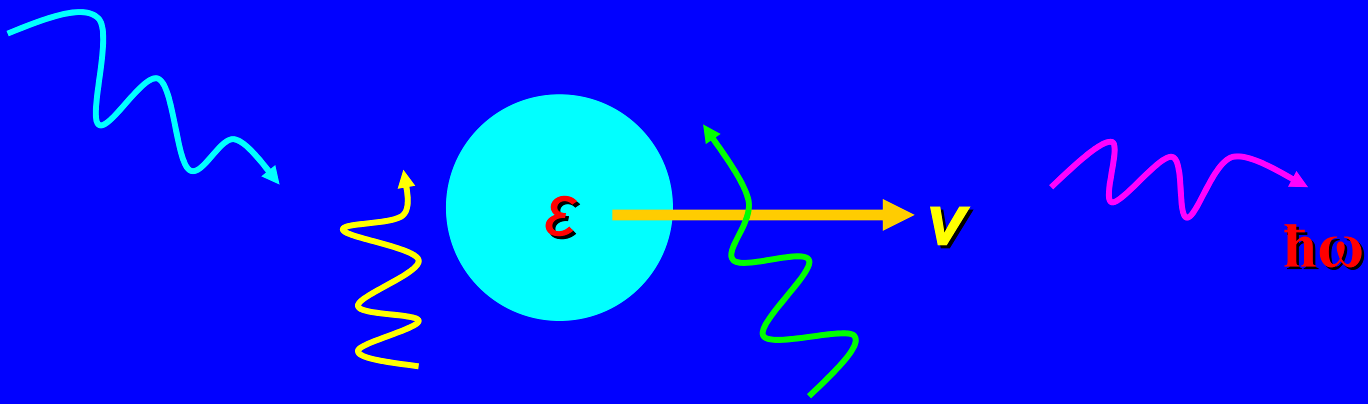
**Feigel theory: $gE_0B_0=10^{-11}$** **$v_{zp}$ = 50 nm/s**

**$FeGaO_3$** **$v_{zp}$ = 18 cm/s**

**Classical: $E_0=10^5$ V/m , $B_0$=20 T** **$v_{class}$= 0.5 μm/s**

**Dimensional regularization:** **$v=v_{class}$ +/- $10^{-3}$ nm/s**

# Casimir Mass ?




$$\langle 0|\mathbf{p}_{\mathrm{mat}}|0\rangle - 0.0158\,\frac{\hbar(\varepsilon-1)^2}{ac_0}\,\mathbf{v} = \mathrm{constant}$$

*This would imply a change of mass:*

$$\Delta m = -0.0158\,\frac{\hbar(\varepsilon-1)^2}{ac_0}$$

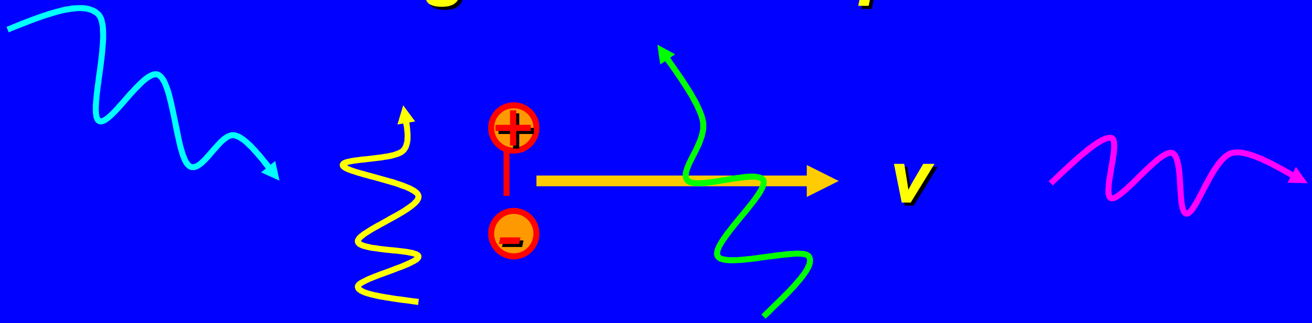
$$2\alpha = 0.0146..$$

*a= 1 μm: $10^{-9}$ $m_e$*

*a= 1 nm: $10^{-6}$ $m_e$*

*a= 1 $r_e$: 1 $m_e$……*

# *Moving electric dipole*




$$T_{\mathbf{kk}'ij}(\omega) = t_0(\omega)\delta_{ij} - \frac{t_0(\omega)}{\omega}\left\{\varepsilon_{inm}\varepsilon_{njl}k_m \mathrm{v}_l + \varepsilon_{inm}\varepsilon_{njl}\mathrm{v}_m k'_l\right\}$$

$$t_0(\omega) = \frac{-4\pi\Gamma\omega^2}{\omega_0^2 - \omega^2 - {}^2\!/\!_3\, i\omega\Gamma\omega_0^2 / c_0}$$

$$\mathbf{P}_{\mathrm{rad}}(\omega) = {}^1\!/\!_2 \frac{\hbar\omega}{c_0}\mathrm{Im}\frac{t_0(\omega)}{\alpha\omega^3 / c_0^3}\ \mathbf{v} \Rightarrow \mathbf{P}_{\mathrm{rad}} = -\frac{\hbar\omega_0}{c_0^2}\ \mathbf{v}$$

*This implies a change of mass:* $\Delta m = -\frac{\hbar\omega_0}{c_0^2}$ *1 eV=*$10^{-9}\, m_p$

# *Casimir momentum to shed new light on the controversial nature of Casimir energy*

*1992*

# Contribution of zero-point fluctuations

$$\langle 0 | E_i(\mathbf{r},\omega) E^*_j(\mathbf{r}',\omega') | 0 \rangle = -2\hbar\omega^2 \,\mathrm{Im}\, G_{ij}(\mathbf{r},\mathbf{r}',\omega) \times 2\pi\delta(\omega-\omega')$$

$$\int d^3\mathbf{r} \,\langle 0 | (\mathbf{E}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r}))_i | 0 \rangle = 0$$

$$\int d^3\mathbf{r} \,\langle 0 | (\mathbf{E}(\mathbf{r}) \times \mathbf{B}^*(\mathbf{r}))_i | 0 \rangle \propto \mathbf{E}_0 \times \mathbf{B}_0$$

$$= -\frac{\hbar}{\pi} \int_0^\infty d\omega\, \omega \sum_{\mathbf{k}} \left\{ k_i \,\mathrm{Im}\, G_{jj}(\mathbf{k},\mathbf{k},\omega) - k_j \,\mathrm{Im}\, G_{ij}(\mathbf{k},\mathbf{k},\omega) \right\}$$

$$\mathbf{G}(\omega) = \mathbf{G}_0(\omega) + \mathbf{G}_0(\omega)\cdot\mathbf{V}(\omega)\cdot\mathbf{G}_0(\omega) + \mathbf{G}_0(\omega)\cdot\mathbf{V}(\omega)\cdot\mathbf{G}_0(\omega)\cdot\mathbf{V}(\omega)\cdot\mathbf{G}_0(\omega) + \cdots$$

## Calculation of β………. (Polarization of photons)

$$\beta = \beta_1 + \ldots + \beta_{10}$$

$$\beta_1 = \int_0^\infty dx \int_0^\infty dy \; x^4 y^2 \frac{(x+2y)}{(x+y)^2} j_0(x)\, j_0(y) \quad \times e^{-\varepsilon(x+y)}$$

$$= -12 \int_0^{\pi/2} d\theta \; \cos(5\theta) \sin^4(\theta) \cos(\theta) = -0.589..$$

⋮

$$\beta = 7.54......$$