Frequency dispersion of photon-assisted shot noise in mesoscopic conductors

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We calculate the low-frequency current noise for ac-biased mesoscopic chaotic cavities and diffusive wires. Contrary to what happens for the admittance, the frequency dispersion in noise is determined not by the electric response time (the *RC* time of the circuit), but by the time that electrons need to diffuse through the structure (dwell time or diffusion time). We find that the derivative of the photon-assisted shot noise with respect to the external ac frequency displays a maximum at the Thouless energy scale of the conductor. This dispersion comes from the slow, charge-neutral fluctuations of the nonequilibrium electron distribution function inside the structure. Our theoretical predictions can be verified with present experimental technology.

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I. INTRODUCTION

Charge neutrality in mesoscopic devices is enforced by Coulomb interactions on a time scale (τ_{RC}) given by the product of the resistance and capacitance scales. The typical time (τ_D) for noninteracting electron motion is instead fixed by either the dwell or diffusion time, depending on the transparency of the interfaces between the system and the electrodes. The most common experimental situation is $\tau_{RC} \ll \tau_D$: electrons, which would normally slowly diffuse, are pushed to run by the electric fields that they are generating themselves by piling up charge. The consequence is that the typical response time of the device is $\tau^{-1}=\tau_{RC}^{-1}+\tau_D^{-1}\approx\tau_{RC}^{-1}$. This has been shown for the frequency dependence of both the admittance[1,2] and the noise[3–6] in mesoscopic chaotic cavities and diffusive wires. The inverse of the diffusion time appears instead as the relevant energy scale for the voltage or temperature dependence for both the conductance and the noise in superconducting–normal-metal hybrid systems.[7–13] Indeed, in this case the energy dependence is due to interference of electronic waves, which does not induce charge accumulation in the system. To our knowledge, in normal metallic structures a dispersion on the inverse diffusion time scale has been predicted so far only for the third moment of current fluctuations[5] and for the finite-frequency thermal noise response to an oscillating heating power.[14] An alternative and less investigated possibility is to study the low-frequency current noise (S) as a function of the frequency Ω of an external ac bias. The noise through a quantum point contact was calculated a decade ago[15] and later measured.[16,17] Since the quantum point contact is very short, electron diffusion does not introduce any additional time scale in the problem and the resulting frequency dispersion is simply linear.[15] More recently, the noise for an ac-biased chaotic cavity has been considered[18,19] in the limit of small fields, $eV/\hbar\Omega\ll 1$ (V is the amplitude of the ac bias and e is the electron change). The authors of Ref. 18 found that the noise disperse only on the τ scale—that is, the combination of the diffusion and the electric response time. This should be contrasted with the result of Ref. 20 where the noise in a diffusive wire was studied when the conductor was biased by a short voltage pulse. Even if the dispersion of the noise is not considered in that paper, the authors find that the ratio of the diffusion time to the duration of the pulse may affect the observed noise. More recently Shytov[21] for the same system and in the limit $eV/\hbar\Omega\gg 1$ found that the limiting values of S for $\Omega\tau_D\gg 1$ and $\Omega\tau_D\ll 1$ do not coincide. These facts indicate that an external-frequency dependence of the noise on the scale of the diffusion time can be present, but at the moment there exist no explicit predictions for this dispersion.

In this paper we obtain the complete external frequency dependence of the photon-assisted noise in chaotic cavities [the analytical expression (39) in the following] and in diffusive wires (numerically). We find that in general a dispersion on the scale of the diffusion time is present and, in particular, that $dS/d\Omega$ shows a clear maximum for $\Omega \approx 1/\tau_D$. We show that this dispersion comes from the slow, charge-neutral fluctuations of the nonequilibrium electron distribution function inside the structure. same frequency region the admittance does not disperse due to the enforcement of charge neutrality.

The plan of the paper is the following. In Sec. II we discuss photon-assisted transport through a chaotic quantum dot, first through a simple phenomenological model (Sec. II A), then using the microscopic Keldysh technique for the current (Sec. II B) and the noise (Sec. II C). In Sec. III we consider photon-assisted transport in diffusive wires. We start the discussion by considering Maxwell relaxation of charge (Sec. III A) and then show how to evaluate numerically the photoassisted noise through the diffusive wire (Sec. III B). Section IV gives our conclusions.

II. CHAOTIC QUANTUM DOT

A. Simple model of charge transport

We consider a chaotic cavity (see Fig. 1) connected to two metallic leads through two scatterers characterized by a set of transparencies $\{T_n^k\}$ where k takes the value 1 or 2. We define the conductances $G_k=G_Q\Sigma_n T_n^k$, the total conductance $G=G_1G_2/(G_1+G_2)$, and a dwell time $\tau_D=2\pi\hbar G_Q/(G_1+G_2)\delta$, where δ is the level spacing of the cavity and G_Q

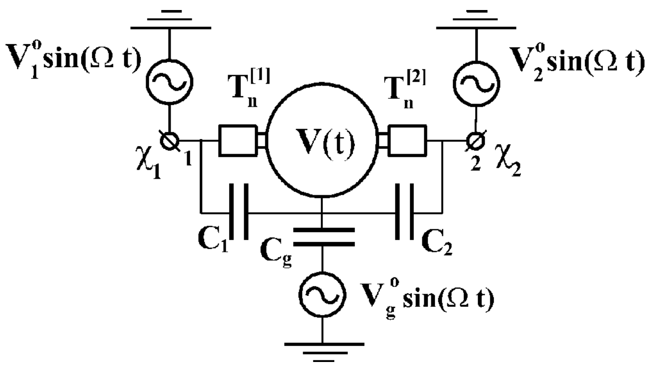

FIG. 1. Scheme of the system. A chaotic cavity connected to the electrodes through arbitrary coherent scatterers characterized by a set of transparencies $\{T_n\}$.

$=e^2/(2\pi\hbar)$. We consider the case when the dwell time is much shorter than the inelastic scattering time due to the possible electron-electron or electron-phonon interaction. We also assume that $G \gg G_Q$, so that we can use semiclassical theory to describe the electron transport and neglect Coulomb blockade effects. The cavity is coupled capacitively to the leads through the capacitances C_k and to a gate through C_g. Three different time-dependent voltage biases are applied to the gate and the two contacts (V_g, V_1, and V_2). Clearly, the current depends only on two voltage differences; we keep the three voltages to simplify the notation. The potential difference between the two leads is harmonically oscillating at frequency Ω, as shown in Fig. 1. Since the cavity resistance is negligible with respect to the contact resistances, we assume a uniform electric potential inside.

One can then define the typical charge relaxation time as $\tau_{RC}=C_\Sigma/G$ where $C_\Sigma=C_1+C_2+C_g$. For the mesoscopic cavity its linear dimension L is much larger than the Fermi wavelength λ_F. Therefore the time τ_{RC} is always much shorter than the dwell time, since the ratio $\tau_{RC}/\tau_D \approx \delta/E_C \ll 1$. Here $E_C=e^2/C_\Sigma$ is the Coulomb energy and $d=2$ or 3 is the dimensionality of the cavity ($d=2$ for a quantum dot formed in a two-dimensional electron gas in a semiconductor nanostructure and $d=3$ for a metallic grain). Indeed, the mean-level spacing reads $\delta=1/(\nu_d L^d)$, where ν_d is the density of states ($\nu_3 \sim m k_F$ and $\nu_2 \sim m$). The charging energy can be estimated as $E_C \sim e^2/L$. Therefore we find

$$\delta/E_C \sim r_s\left(\frac{\lambda_F}{L}\right)^{d-1} \ll 1, \tag{1}$$

where $r_s=1/(k_F a_B)\sim 1$ is the conventional electron gas parameter, $a_B=\hbar^2/(me^2)$ being the Bohr radius.

We start the discussion by considering an elementary model of charge transport, where diffusion and electric drift are described classically. The charge Q in the cavity is related to the electric potential V of the cavity itself by the relation

$$Q=\sum_{k=1,2,g} C_k(V-V_k). \tag{2}$$

On the other hand, the continuity equation for charge Q reads

$$\partial_t Q + I = 0, \tag{3}$$

where the electron current leaking from the cavity can be phenomenologically described as

$$I = Q/\tau_D + \sum_{k=1,2} G_k(V-V_k). \tag{4}$$

The first term of Eq. (4) stems from the charge diffusion while the second is the standard Ohm's law. For any given time-dependent voltages $V_k(t)$ the system of equations (2) and (3) can be conveniently solved in terms of $V(\omega)$, the Fourier transform of the cavity potential. We find

$$V(\omega)=\frac{\tau/\tau_D}{1-i\omega\tau}\sum_{k=1,2,g} V_k(\omega)\times\left[\alpha_k\frac{\tau_D}{\tau_{RC}}+(1-i\omega\tau_D)\frac{C_k}{C_\Sigma}\right], \tag{5}$$

where we defined $\alpha_k=G_k/G_\Sigma$ for $k=1$ or 2, $G_\Sigma=G_1+G_2$, and $\alpha_g=0$. When $\tau_{RC}\ll\tau_D$, the response time $\tau=\tau_{RC}\tau_D/(\tau_{RC}+\tau_D)$ is simply given by τ_{RC}. For $\tau_{RC}\ll\tau_D$ and $\Omega\tau\ll 1$ one thus finds the very simple result

$$V(\omega)=\sum_k \alpha_k V_k(\omega), \tag{6}$$

and

$$I_1(\omega)=-I_2(\omega)=G_\Sigma\alpha_1\alpha_2(V_1-V_2). \tag{7}$$

Charge neutrality is perfectly enforced for low-frequency drive. In the following we will mainly consider this experimentally relevant low-driving-frequency limit $\Omega\tau_{RC}\ll 1$, which will enable us to use the $\omega \ll 1/\tau_{RC}$ response for $V(\omega)$. The method developed in the following is not limited by this condition, the extension to the case where ω is not negligible with respect to τ_{RC} is straightforward.

The admittance matrix at finite frequency can be evaluated employing the same model. One finds that its frequency dependence is similar to the cavity potential frequency dependence. This simple approach, however, fails to describe current fluctuations in the system—i.e., noise—which is the primary interest of our study. To achieve this goal, we resort to a general microscopic description of current fluctuations in mesoscopic conductors that allows access to the full counting statistics of charge transfer.[22] Within this method the low-frequency current noise can be evaluated as the second moment of the number of transferred charges over the long time interval $t\gg\max\{\hbar/eV, 1/\Omega\}$.

B. Keldysh action approach to electron transport

Nonequilibrium transport through a chaotic cavity can be described by a functional integral approach using a Keldysh contour.[23–25] One can define a generating function Υ that depends on two counting fields χ_1 and χ_2, such that the current and all higher moments averaged over a long time t_0 can be obtained by differentiating Υ (Refs. 22 and 26):

$$I_k(\chi_1,\chi_2) = ie\frac{\partial \Upsilon}{\partial \chi_k}, \quad S_k(\chi_1,\chi_2) = e^2\frac{\partial^2 \Upsilon}{\partial \chi_k^2}, \tag{8}$$

where I_k and S_k are the low-frequency current and noise, respectively. The generating function can be expressed through the following functional integral:

$$t_0\Upsilon(\chi_1,\chi_2) = -\ln\left[\int \mathcal{D}\check{\mathcal{G}} \int \mathcal{D}\varphi e^{i\mathcal{S}[\check{\mathcal{G}},\varphi]}\right], \tag{9}$$

where the action $\mathcal{S}$ has the form

$$\mathcal{S}[\check{\mathcal{G}},\varphi] = \mathcal{S}_{\text{con}} - 2\pi\delta^{-1}\,\text{Tr}(i\partial_t\check{\mathcal{G}}) + \mathcal{S}_\varphi. \tag{10}$$

Here $\mathcal{S}_{\text{con}}$ describes the contacts[11,27]

$$\mathcal{S}_{\text{con}} = \frac{1}{2i}\sum_{n,k}\text{Tr}\ln\left[1 + \frac{1}{4}T_n^{[k]}(\{\check{\mathcal{G}}_k, e^{-i\check{\varphi}}\check{\mathcal{G}}e^{i\check{\varphi}}\} - 2)\right]$$

and $\mathcal{S}_\varphi$ describes the capacitors

$$\mathcal{S}_\varphi = \int_{-\infty}^{+\infty}\sum_{k=1,2,g}\frac{2C_k}{e^2}(\dot{\varphi}_+ - \dot{\varphi}_k)\dot{\varphi}_- dt.$$

The functional integral is performed over two fields $\check{\mathcal{G}}(t_1,t_2)$ and $\phi_\pm(t)$. The first is a 2×2 matrix and at mean-field level coincides with the semiclassical Keldysh Green's function for electrons in the cavity. It is constrained by the condition

$$\sum_l \int_{-\infty}^{+\infty} dt \check{\mathcal{G}}_{il}(t_1,t)\check{\mathcal{G}}_{lj}(t,t_2) = \delta_{ij}\delta(t_1-t_2). \tag{11}$$

The trace and product operation in Eq. (10) includes summation over Keldysh indices and integration over time. In this compact notation the normalization constrain reads $\check{\mathcal{G}}^2=1$. The second field is the pair of real functions $\{\varphi_+(t),\varphi_-(t)\}$. They result from the Hubbard-Stratonovich decoupling of the Coulomb interaction term, and their time derivatives $\dot{\varphi}_\pm(t)/e = V_\pm(t)$ present the "classical" and "quantum" fluctuating electrostatic potentials in the cavity. These fields appear in the action also in matrix form: $\check{\varphi}=\varphi_+(t)+\varphi_-(t)\check{\sigma}_x$, with $\check{\sigma}_x$ the Pauli matrix in Keldysh space.

The external time-dependent voltage drive $V_k(t)$ and the counting fields enter $\mathcal{S}$ through the Green function of the two leads $\check{\mathcal{G}}_k$:

$$\check{\mathcal{G}}_k = e^{i\sigma_x\chi_k/2}\check{\mathcal{G}}_{0k}e^{-i\sigma_x\chi_k/2} \tag{12}$$

and

$$\check{\mathcal{G}}_{0k}(t_1,t_2) = \begin{pmatrix} \delta(t_1-t_2) & 2F^{(k)}(t_1,t_2) \\ 0 & -\delta(t_1-t_2) \end{pmatrix}. \tag{13}$$

Here

$$F^{(k)}(t_1,t_2) = e^{-i\varphi_k(t_1)}F_{eq}(t_1-t_2)e^{i\varphi_k(t_2)} \tag{14}$$

and

$$F_{eq}(t) = \int\frac{d\varepsilon}{2\pi}e^{i\varepsilon t/\hbar}\tanh\left(\frac{\varepsilon}{2T}\right) = \frac{T}{i\sinh(\pi T t/\hbar)}, \tag{15}$$

where T is the temperature of the leads ($k_B=1$). The phase $\varphi_k(t)$ is related to voltage in contact k by the gauge relation $\hbar\dot{\varphi}_k(t)=eV_k(t)$, and the function $F^{(k)}(t_1,t_2)$ is connected to the nonequilibrium distribution function in the kth contact, as $f^{(k)}(t_1,t_2)=[1-F^{(k)}(t_1,t_2)]/2$. Note that $f^{(k)}(t_1,t_2)$ is a gauge-dependent quantity.

One should now calculate the current and noise starting from expression (9). We are considering the limit of large contacts, $G_Q/G \ll 1$, such that mesoscopic fluctuations are negligible. In this limit the electronic transport is then well described by the saddle point approximation. At this level of approximation we are also treating the interaction at the mean-field level. It means that we fully account for the classical charge relaxation (screening), but disregard other, more subtle effects, related to the Coulomb interaction, like (i) inelastic electron-electron collisions, defining the inelastic time scale, and (ii) logarithmic renormalization of transmission coefficients $T_n^{[k]}$ due to virtual elastic collisions. In zero-dimensional metallic systems, which we consider, the impact of effects (i) and (ii) on the current and noise is small by the same parameter $G_Q/G \ll 1$. If needed, these effects can be taken into account by considering Gaussian fluctuations of the phases $\varphi_\pm$ around the saddle point solution (see, for instance, Refs. 4, 25, and 28).

The saddle point condition on Eq. (10) leads to two set of equations:

$$\frac{\delta\mathcal{S}}{\delta\check{\mathcal{G}}} = 0 \quad \text{and} \quad \frac{\delta\mathcal{S}}{\delta\varphi_\pm} = 0, \tag{16}$$

where the first derivative has to be performed maintaining the normalization constraint (11). The first is a kinetic equation for the electron distribution function inside the cavity, and the second is the Poisson equation for the charge distribution. It is instructive, first, to relate the least-action principle to the elementary charge model discussed above and to find the distribution function inside the cavity. Minimization of action (10) with respect to $\check{\mathcal{G}}$ gives the following equation that relates $\check{\mathcal{G}}$ to the current through the interfaces:

$$\sum_k \check{\mathcal{I}}_k + \hbar\,\tau_D(\partial_{t_1}+\partial_{t_2})\check{\mathcal{G}} = 0, \tag{17}$$

where

$$\check{\mathcal{I}}_k = \frac{G_Q}{G_\Sigma}\sum_n \frac{T_n^{[k]}[e^{i\check{\varphi}}\check{\mathcal{G}}_k e^{-i\check{\varphi}},\check{\mathcal{G}}]}{4+T_n^{[k]}(\{e^{i\check{\varphi}}\check{\mathcal{G}}_k e^{-i\check{\varphi}},\check{\mathcal{G}}\}-2)} \tag{18}$$

are the spectral currents found in Ref. 29. One can readily verify by performing a derivative with respect to χ_k that at the mean-field level the χ_k-dependent current is a trace of expression (18):

$$I_k(\chi_1,\chi_2) = ie\frac{\partial \Upsilon}{\partial \chi_k} = \frac{eG_\Sigma}{G_Q}\,\mathrm{Tr}\{\check{\sigma}_x \check{\mathcal{I}}_k(t,t)\}/2. \tag{19}$$

The second set of equations are obtained by derivation with respect to $\varphi_\pm$:

$$\frac{\delta}{\delta \varphi_\pm(t)}(\mathcal{S}_{\mathrm{con}} + \mathcal{S}_\varphi) = 0. \tag{20}$$

We begin by discussing the physical current that is simply I_k for $\chi_1 = \chi_2 = 0$. In this case Eq. (18) has a solution of the form (13) and depends on a single function $F(t_1,t_2)$. The derivative with respect to φ_+ in Eq. (20) gives the condition $\dot{\varphi}_- = 0$, which is solved by $\varphi_- = 0$; i.e., only the classical voltage fluctuations are left. It is convenient now to introduce new time variables $t_- = t_2 - t_1$ and $t_+ = (t_1 + t_2)/2$. Then Eq. (17) takes the form of a kinetic equation

$$\tau_D \frac{\partial F}{\partial t_+} + F = \sum_{k=1,2} \alpha_k e^{i\varphi_+(t_1)} F^{(k)} e^{-i\varphi_+(t_2)}. \tag{21}$$

The distribution function $F(t_+,t_-)$ is a periodic function of t_+, which can be represented as the Fourier series

$$F(t_+,t_-) = \sum_n e^{in\Omega t_+} F_n(t_-), \tag{22}$$

and its explicit expression in terms of the Fourier component of $F^{(k)}$ comes from the solution of the kinetic equation (21):

$$F_n(t_-) = \frac{\sum_k \alpha_k F_n^{(k)}(t_-)}{1 + in\Omega\tau_D}. \tag{23}$$

Here

$$F_n^{(k)}(t) = F_{eq}(t) J_n[2A_k \sin(\Omega t/2)], \tag{24}$$

where $A_k = e(V_0^{(k)} - V_0)/(\hbar\Omega)$; the equilibrium distribution function $F_{eq}(t)$ is given by Eq. (15); J_n are the Bessel functions; $V_0^{(k)}$ are the amplitudes of the ac fields in the leads, $V_k(t) = V_0^{(k)} \sin(\Omega t)$; and V_0 is the amplitude of the electric potential inside the cavity, $V(t) = \dot{\phi}_+(t)/e = V_0 \sin(\Omega t)$.

In order to make a connection with the classical description given by Eq. (3) we relate the distribution function F to the charge in the cavity:

$$Q(t_+) = -(2\pi e/\delta) F(t_+, t_- \to 0). \tag{25}$$

The kinetic equation (21) for $t_- \to 0$ reduces then to the continuity equation (3) for the total charge. This can be shown by exploiting the identity

$$\lim_{t_- \to 0} e^{i\varphi_+(t_1)} F_{eq}(t_1 - t_2) e^{-i\varphi_+(t_2)} = \frac{1}{\pi}\dot{\varphi}_+(t_+). \tag{26}$$

The equation of motion for the charge is given by the derivative with respect to φ_- in Eq. (20). It is composed of two terms. The first reads

$$\frac{\delta \mathcal{S}_{\mathrm{con}}}{\delta \varphi_-(t)} = \frac{G_\Sigma}{G_Q}\sum_{k=1,2} \mathrm{Tr}[\check{\mathcal{I}}_k(t,t)\check{\sigma}_x]$$
$$= -2\pi\delta^{-1}\,\mathrm{Tr}[\partial_t \check{\mathcal{G}}(t,t)\check{\sigma}_x] = \frac{2}{e}\dot{Q}(t), \tag{27}$$

where we have firstly used Eq. (17) and then the definition of charge (25). The second reads

$$\frac{\delta \mathcal{S}_\phi}{\delta \phi_-(t)} = -2\sum_{k=1,2,g} \frac{C_k}{e}(\dot{V} - \dot{V}_k). \tag{28}$$

Combining Eqs. (27) and (28) we see that the saddle point equation (20) fixes the charge $Q(t)$ in accordance with relation (3). Note that when the voltage time dependence (6) is enforced, $Q(t)$ vanishes identically, as can be verified by calculating the limit $t_- \to 0$ in Eq. (23). On the other hand, the energy distribution function $\widetilde{F}(t_+,\varepsilon) = \int dt_- \widetilde{F}(t_-,t_+) e^{i\varepsilon t_-/\hbar}$ varies periodically in time and its dependence on the ac driving frequency Ω is on the scale of the inverse diffusion time τ_D^{-1}. When an electron enters the cavity, after a very short time $\sim \tau_{RC}$, the charge rearranges to keep the cavity neutral; the distribution function, instead, will relax on a much longer time given by τ_D. As it will be shown in the following subsection, zero-frequency noise probes the electronic distribution function [see Eq. (38) in the following] and it does depend on the frequency Ω on the same scale.

C. Photon-assisted shot noise

In order to obtain the noise we need to calculate the first derivative of the counting-field-dependent current $S_k(t) = -i\,\partial I_k(t)/\partial \chi$ for $\chi_1 = \chi_2 = 0$. To obtain the noise, it is thus sufficient to calculate the linear correction in χ to $\check{\mathcal{G}}$,

$$\check{\mathcal{G}} = \check{\mathcal{G}}_0 - 2i\chi\check{\mathcal{G}}_1 + \cdots, \tag{29}$$

and substitute this expression into Eq. (18).[11,12] The term $\check{\mathcal{G}}_0$ is given by Eq. (13) with the distribution function F given by Eqs. (22) and (23). In order to fulfill the normalization condition (11) the correction $\check{\mathcal{G}}_1$ should anticommute with $\check{\mathcal{G}}_0$. This condition can be fulfilled automatically by using the parametrization proposed in Ref. 23:

$$i\chi\check{\mathcal{G}}_1 = \begin{pmatrix} 1 & F \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & W \\ W' & 0 \end{pmatrix}\begin{pmatrix} 1 & F \\ 0 & -1 \end{pmatrix}. \tag{30}$$

For the boundary conditions (12) the phase $\check{\phi}(t)$ on a cavity acquires the χ-dependent correction as well:

$$\check{\phi}(t) = \phi_+(t) + \chi\{\theta(t) + \theta'(t)\check{\sigma}_x\} + \cdots. \tag{31}$$

By defining $\chi = \chi_1 - \chi_2$ we can calculate the noise as a derivative with respect to χ and we are free to choose $\chi_1 = \alpha_2\chi$ and $\chi_2 = -\alpha_1\chi$. In this way $W' = 0$ and $\theta' = 0$. Then from Eqs. (17) and (20) we obtain the system of coupled equations for W,

$$i(1+\tau_D\partial_{t_+})W = R - \chi\sum_{k=1,2}\alpha_k e^{i\phi_+(t_1)}F_k e^{-i\phi_+(t_2)}[\theta(t_1)-\theta(t_2)], \tag{32}$$

and θ,

$$\dot{Q}_1(t) = \chi C_\Sigma \ddot{\theta}(t)/e. \tag{33}$$

Here the operator R reads

$$R = \sum_k \chi_k\alpha_k[(1-\beta_k)(F\circ F + F_k\circ F_k) + \beta_k(F\circ F_k + F_k\circ F)], \tag{34}$$

and $Q_1(t)$ is the correction to the charge, given by

$$Q_1(t_+) = -(2\pi e/\delta)W(t_+, t_- \to 0), \tag{35}$$

where

$$\beta_k = G_Q/G_k\sum_n T_n^{(k)}(1-T_n^{(k)}) \tag{36}$$

are the Fano factors of the two junctions and with the circle $\circ$ we indicate a time convolution.

We now expand the χ-dependent current (18) up to linear in χ terms in order to evaluate the noise: $I_k(\chi)=I_k^{(a)}+I_k^{(b)}+I_k^{(c)}$, where

$$I_k^{(a)} = \frac{\pi G_k}{e}[\mathrm{Tr}(W) - \chi\dot{\theta}/\pi],$$

$$I_k^{(b)} = \frac{i\pi G_k}{2e}\chi_k\,\mathrm{Tr}\{2 - F\circ F_k - F_k\circ F\},$$

$$I_k^{(c)} = -\frac{i\pi G_k}{2e}\chi_k(1-\beta_k)\mathrm{Tr}\{(F-F_k)\circ(F-F_k)\}. \tag{37}$$

As we can see from this expansion one needs to know Tr W only—i.e., $W(t,t)$ averaged over one period in time—in order to obtain the low-frequency noise. Since the phases $\theta(t)$ and $\phi_+(t)$ oscillate periodically in time around zero mean value, it follows from the kinetic equation (32) and the relation (26), that Tr $W-\chi\dot{\theta}/\pi=-i$Tr R; thus, the actual time dependence of the phase $\theta(t)$ is not important for evaluation of the low-frequency noise. Substituting the expression for W into the current expansion (37) one can relate the noise S with distribution functions $f(t_1,t_2)=[1-\tilde{F}(t_1,t_2)]/2$ and $f_k(t_1,t_2)=[1-\tilde{F}_k(t_1,t_2)]/2$:

$$\begin{aligned}S &= GT(1-2\beta_1\alpha_2) + 4\pi G\alpha_2\beta_1\,\mathrm{Tr}\\ &\quad\times[f\circ(1-f_1) + f_1\circ(1-f)]\\ &\quad+ 2\pi G(1-2\alpha_2\beta_1)\mathrm{Tr}[f\circ(1-f)] + (1\leftrightarrow 2).\end{aligned} \tag{38}$$

Here the terms in the first and second lines are the thermal and partition noise due to contact resistances, while the third line represents the noise of the chaotic cavity with open contacts, which stems from the fluctuation of the distribution function inside the dot. Evaluating the traces we finally obtain

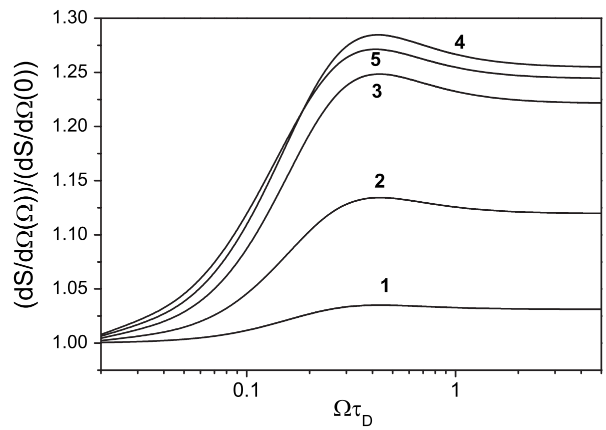

FIG. 2. Frequency dependence of a differential photon-assisted shot noise in the symmetric chaotic cavity ($G_1=G_2$) under fixed flux $A=e(V_1^0-V_2^0)/(\hbar\Omega)$. The magnitude of $dS/d\Omega$ is normalized to its value at $\Omega=0$. Curve (1) $A=1.0$, (2) $A=2.0$, (3) $A=3.0$, (4) $A=4.0$, and (5) $A=5.0$. For symmetric cavities the curves appear to be independent of the transmission distribution of the contacts.

$$\begin{aligned}S &= G\sum_{n,l,r}\frac{\hbar\Omega(l+r)\coth(\hbar\Omega(l+r)/2T)}{1+\Omega^2\tau_D^2 n^2}[\mathcal{F}\mathcal{J}(A_1,A_2)\\ &\quad+\mathcal{F}_1\mathcal{J}(A_1,A_1)+\mathcal{F}_2\mathcal{J}(A_2,A_2)] + 2GT(1-\beta_1\alpha_2-\beta_2\alpha_1),\end{aligned} \tag{39}$$

where

$$\mathcal{J}(A_1,A_2) = J_{n+l}(A_1)J_l(A_1)J_{r-n}(-A_2)J_r(-A_2),$$

$$\mathcal{F}_1 = (\alpha_1 + \beta_1\alpha_2 - \mathcal{F})/2,$$

$$\mathcal{F}_2 = (\alpha_2+\beta_2\alpha_1-\mathcal{F})/2,$$
$$\mathcal{F} = \alpha_1\alpha_2 + \beta_1\alpha_2^3 + \beta_2\alpha_1^3, \tag{40}$$

and the amplitudes are $A_1=\alpha_2 A$ and $A_2=-\alpha_1 A$, with $A=e(V_1^0-V_2^0)/\hbar\Omega$.

Expression (39) is the main result of this section. For $\Omega\tau_D\ll 1$ it reduces to the result by Lesovik and Levitov[15] with the effective Fano factor $\mathcal{F}$ appearing at the place of the quantum point contact Fano factor. The photon-assisted noise shot noise in a chaotic cavity in the limit of small $A\ll 1$ has been considered recently by Polianski, Samuelsson, and Büttiker.[19] They found that at A^2 order there is no frequency dispersion. One can recover this result, expanding the general expression (39) up to second order in A. For all other cases the frequency dispersion is present, as can be seen from Fig. 2, where $dS/d\Omega$ is shown as a function of Ω for small temperature $T\ll\hbar\Omega$. In particular, we find that $dS/d\Omega$ displays a maximum for $\Omega\sim 1/\tau_D$ reminiscent of the reentrant behavior in superconductors. Expression (39) allows also one to study the dependence of noise as a function of the flux, as shown in Fig. 3. We found that oscillations are present as in the quantum point contact results of Ref.15; the main difference is that the form depends now on the driving frequency. For $\Omega\tau_D\approx 1$ the oscillations are slightly reduced.

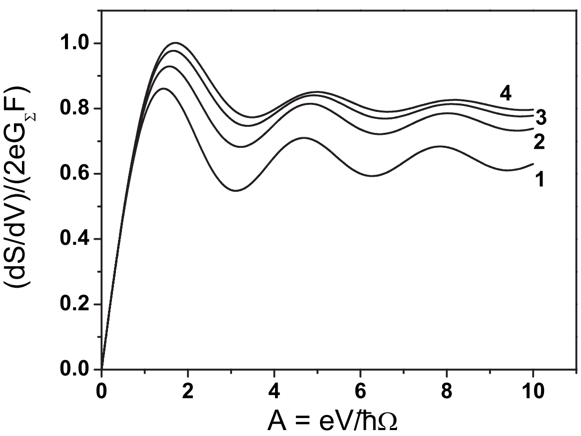

FIG. 3. Flux dependence of the differential photon-assisted shot noise in the symmetric chaotic cavity ($G_1=G_2$). Curve (1) $\Omega/E_{\rm Th}=0.0$, (2) $\Omega/E_{\rm Th}=0.25$, (3) $\Omega/E_{\rm Th}=0.5$, and (4) $\Omega/E_{\rm Th}=1.0$.

III. DIFFUSIVE WIRE

A. Classical charge relaxation

We pass now to discuss the photon-assisted noise in a diffusive wire. We characterize the wire by a length L_x in the direction of the current, a diffusion coefficient D, and a conductance $G\gg G_Q$. These parameters define the Thouless energy of the wire, $\epsilon_{\rm Th}=\hbar D/L^2$, which gives the typical diffusion time $\tau_D\sim\hbar/\epsilon_{\rm Th}$ through the structure. The wire is connected to two leads which are kept at oscillating voltage difference $eV(t)=eV_0\sin(\Omega t)$. In order to observe a strong nonequilibrium effect, we consider not too long samples, where the diffusion time τ_D is much shorter than the energy relaxation time τ_E in the system. At low temperature τ_E stems primary from the inelastic electron-electron collisions. It can be estimated as $\tau_E\sim(D/E)^{1/2}\nu$ for the case of quasi-one-dimensional geometry (ν being a density of states per spin) and $\tau_E\sim(G/G_Q)\hbar/E$ for the case of quasi-two-dimensional film, where $E=\max\{eV_0,\hbar\Omega\}$.

Following the Section II A it is instructive to start the discussion by considering the classical charge relaxation in the diffusive wire. It is described by the set of coupled equations of the classical electrodynamics in the diffusive media (see, e.g., Ref. 30):

$$\phi(\mathbf{r},t)=\int U(|\mathbf{r}-\mathbf{r}'|)\rho(\mathbf{r}',t)d\mathbf{r}', \qquad (41)$$

$$\dot\rho+\boldsymbol{\nabla}\cdot\mathbf{j}=0, \qquad (42)$$

$$\mathbf{j}=-\sigma\boldsymbol{\nabla}\phi-D\boldsymbol{\nabla}\rho. \qquad (43)$$

Here $\rho(\mathbf{r},t)$ and $\mathbf{j}(\mathbf{r},t)$ are the charge and current densities, $\phi(\mathbf{r},t)$ is the electrostatic potential, σ is the conductivity, and $U(|\mathbf{r}|)$ is the (possibly screened by nearby gates) Coulomb potential. The conductivity σ of the wire is frequency independent in the wide frequency range $\omega<1/\tau_{\rm tr}$, $\tau_{\rm tr}$ being a transport scattering time. It is related to the diffusion coefficient by the Einstein relation $\sigma=2e^2\nu D$ (the factor of 2 is the spin multiplicity). It is worth mentioning that Eqs. (41)–(43) are analogs of the previously discussed equations (2)–(4).

One can solve Eqs. (42) and (41) in terms of the Fourier transform $\rho_{\mathbf{q}}(t)$ of the charge excitation as

$$[\partial_t+D\mathbf{q}^2+\sigma\mathbf{q}^2U(\mathbf{q})]\rho_{\mathbf{q}}(t)=0, \qquad (44)$$

where $U(\mathbf{q})$ is the Coulomb potential form factor. Typically the short diffusive wire can be made of a film with size $L_x>L_y$ and thickness $d\ll\{L_x,L_y\}$, where the length scales $L_{x,y}$ are of the same order of magnitude, so that one effectively has a two-dimensional metal. In this situation for the nonscreened Coulomb interaction we put $U(\mathbf{q})=2\pi/|\mathbf{q}|$. It follows then from Eq. (44) that a charge fluctuation, spread over the system size with a wave vector $\mathbf{q}\sim2\pi/L_x$, will relax on the typical time scale $\tau^{-1}=\epsilon_{\rm Th}+\omega_{\rm M}$, where $\omega_{\rm M}\sim\sigma_\square/L_x$ is the Maxwell relaxation rate. Taking into account that $\nu\sim m(k_Fd)$, the definition for the sheet conductivity $\sigma_\square$ and $\epsilon_{\rm Th}$, one estimates for a good metal that

$$\epsilon_{\rm Th}/\omega_{\rm M}\sim\frac{\lambda_F^2}{L_xd}\ll1, \qquad (45)$$

with λ_F being the Fermi wavelength. Thus the charge relaxation time τ is set by the Maxwell time, $\tau\sim1/\omega_{\rm M}$, playing now the role of RC time, while the diffusion time τ_D drops out from the problem. Then at the low-driving-frequency limit $\Omega\tau\ll1$, the system is charge neutral, $\rho(t)=0$, and as follows from Eq. (43) the admittance $Y(\omega)$ is frequency independent, $Y(\omega)=G\equiv\sigma_\square(L_y/L_x)$. At the same time the electric field $\mathbf{E}=-\boldsymbol{\nabla}\phi$ is constant along the wire, so that

$$\phi(x,t)=V_0\sin(\Omega t)(L_x-x)/L_x, \qquad (46)$$

where x is the coordinate in the current's direction. This expression is the analog of the result (6) in the case of zero-dimensional chaotic quantum dots.

B. Photon-assisted shot noise in diffusive wire

The photon-assisted shot noise in the diffusive wire in the limit of large driving frequency, $\Omega\tau_D\gg1$, has been theoretically considered by Shytov.[21] In this limiting case the noise can be expressed in terms of the electron distribution functions averaged over time and can be analytically evaluated when $eV/\hbar\Omega\gg1$. To obtain the noise for arbitrary values of Ω we employ the same procedure used for a chaotic cavity, taking into account the spatial dependence of the Green's function $\check{\mathcal{G}}$.

We describe the wire by the following action:[31]

$$S[\mathcal{G},\phi]=2i\pi\nu\int d\mathbf{r}\,\mathrm{Tr}\left\{\frac{1}{4}D(\boldsymbol{\nabla}\hat{\mathcal{G}})^2-[\partial_t+ie\check{\phi}(z,t)]\hat{\mathcal{G}}\right\}+S_\phi, \qquad (47)$$

where $\check\phi=\phi^++\phi^-\check\sigma_x$ and $\phi^\pm$ are the "classical" and "quantum" components of the electrostatic potential. The term S_ϕ is the electromagnetic part of the action

$$S_\phi = 2\int dt \sum_{\mathbf{q}} \phi^+_{-\mathbf{q}}\left(\frac{1}{U(\mathbf{q})} + 2\nu e^2\right)\phi^-_{\mathbf{q}}. \quad (48)$$

The first term in this expression takes into account the electromagnetic energy, and the second one is the static compressibility of the electron gas. The action (47) has to be supplemented by the boundary conditions for the Green's function at the ends of the wire:

$$\check{\mathcal{G}}(t_1,t_2)\Big|_{x=L} = \begin{pmatrix} \delta(t_1-t_2) & 2F_{eq}(t_1-t_2) \\ 0 & -\delta(t_1-t_2) \end{pmatrix} \quad (49)$$

and

$$\check{\mathcal{G}}(t_1,t_2)\Big|_{x=0} = e^{i\check{\sigma}_x\chi/2}\begin{pmatrix} \delta(t_1-t_2) & 2F_L(t_1,t_2) \\ 0 & -\delta(t_1-t_2) \end{pmatrix}e^{-i\check{\sigma}_x\chi/2}. \quad (50)$$

Here the equilibrium distribution function $F_{eq}(t)$ is given by Eq. (15),

$$F_L(t_1,t_2) = e^{-i\varphi(t_1)}F_{eq}(t_1-t_2)e^{i\varphi(t_2)}, \quad (51)$$

and $\varphi(t)=(eV_0/\hbar\Omega)\cos(\Omega t)$.

Minimizing the action (47) with respect to $\mathcal{G}$ one obtains the Usadel equations

$$D\nabla_z(\check{\mathcal{G}}\nabla_z\check{\mathcal{G}}) + i[i\partial_t - e\check{\phi}(z,t),\check{\mathcal{G}}] = 0. \quad (52)$$

By considering the physical limit of this equation at $\chi=0$ and integrating it over the time $t_-=t_1-t_2$ one can rederive Eqs. (42) and (43) with charge and current densities given by

$$\rho(\mathbf{r},t) = -2e\nu\{\pi F(t_+,t_-\to 0,\mathbf{r}) + e\phi^+(\mathbf{r},t)\}, \quad (53)$$

$$\mathbf{j}(\mathbf{r},t) = \mathrm{Tr}\{\check{\sigma}_x\check{\mathbf{j}}(\mathbf{r},t,t)\}/2, \quad (54)$$

where we have defined the spectral matrix current

$$\check{\mathbf{j}}(\mathbf{r},t_1,t_2) = \frac{\pi\sigma}{e}(\hat{\mathcal{G}}\nabla\hat{\mathcal{G}}). \quad (55)$$

By the same token, if one takes the saddle point of the action (47) with respect to ϕ^-, the Poisson relation (41) is reproduced. Note that the appearance of the ϕ term in the definition of the charge density stems from the fact that the gauge div $\mathbf{A}=0$ is now used, while the preceding definition of charge (25) implied the gauge $\phi(t)=0$ inside the cavity.

To obtain the photon-assisted noise we have solved the Usadel equation (52) numerically, using the charge-neutrality condition (46). To accomplish this program technically we switch to the energy representation of (52). Since the driving is periodic, we can single out the t_+ dependence for any operator $\check{A}$:

$$\check{A}(t_1,t_2) = \sum_n \check{A}_n(t_-)e^{in\Omega_0 t_+}. \quad (56)$$

In the energy domain this implies that

$$\check{A}(\epsilon_1,\epsilon_2) = \sum_n \widetilde{A}_n(\epsilon_1)2\pi\delta(\epsilon_1-\epsilon_2+n\hbar\Omega), \quad (57)$$

where $\widetilde{A}_n(\epsilon-n\hbar\Omega/2)$ is the Fourier transform of $\check{A}_n(t_-)$:

$$\check{A}_n(\epsilon-n\hbar\Omega/2) = \int_{-\infty}^{+\infty} dt e^{i\epsilon t}\check{A}_n(t). \quad (58)$$

It is thus convenient to define the energy variable E, so that $\epsilon=E+k\hbar\Omega$, such that k is an integer and $-\hbar\Omega/2<E<\hbar\Omega/2$. One can then represent $\check{A}$ in a matrix form

$$\check{A}_{nm}(E) = \widetilde{A}_{m-n}(E+n\Omega). \quad (59)$$

In this way the time convolution between two operators becomes a simple matrix product. In the matrix representation the distribution function entering Eq. (49) for the right lead $(x=L)$ takes a diagonal form

$$F_R(E)_{nm} = \delta_{nm}\,\mathrm{sgn}(E+n\hbar\Omega).$$

For the left lead at zero temperature it reads instead

$$F_L(E)_{nm} = -\delta_{nm} + 2i^{n-m}\sum_{l=-\infty}^{m} J_l(-A)J_{m-n-l}(A),$$

where $A=eV_0/\hbar\Omega$. In practical calculations it is enough to restrict the matrix size to $|n|\le 3A$, since A gives a typical number of multiphoton processes involved.

Following the ideas of the circuit theory,[11,27] we solve Eq. (52) by representing the wire as a chain of $N-1\gg 1$ chaotic cavities (where $\check{\mathcal{G}}$ is uniform) with N identical tunnel barriers between them. This approach leads to a finite-difference version of the Usadel equation:

$$\left[\frac{1}{2}(\check{\mathcal{G}}_{k-1}+\check{\mathcal{G}}_{k+1}) + i\frac{\check{\epsilon}-\check{V}^k}{N(N-1)\epsilon_{\mathrm{Th}}},\check{\mathcal{G}}_k\right] = 0. \quad (60)$$

The operators $i\partial_t\equiv\check{\epsilon}$ and $\check{V}^k$ have matrix representations

$$(\check{\epsilon})_{n,n} = E+n\Omega,$$

$$\check{V}^k_{nm}(E) = ieV_0(N-k)(\delta_{n,m-1}-\delta_{n,m+1})/(2N). \quad (61)$$

Equation (60) can be efficiently solved by means of subsequent iterations. Then photon-assisted shot noise can be

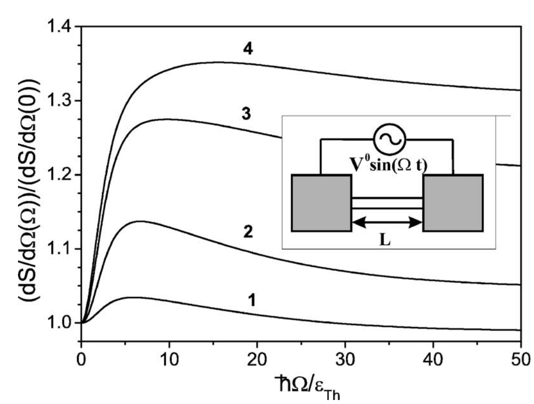

FIG. 4. Frequency dependence of the differential photon-noise for a diffusive wire for given values of $A=eV^0/\hbar\Omega$. Here $\epsilon_{\mathrm{Th}}=\hbar D/L^2$ and the numerical calculation has been performed with ten nodes. The magnitude of $dS/d\Omega$ is normalized to its value at $\Omega=0$. Curve (1) $A=1.0$, (2) $A=2.0$, (3) $A=3.0$, and (4) $A=4.0$.

obtained by differentiating the χ-dependent current, $S=-i\,\partial I_k/\partial\chi$ at $\chi=0$, where

$$I_k(\chi) = (G/G_Q)N\,\mathrm{Tr}\{[\check{\mathcal{G}}_k,\check{\mathcal{G}}_{k+1}]\check{\sigma}_x\}/8 \qquad (62)$$

can be taken at any one of the N barriers due to the current conservation.

Results for the differential noise $dS/d\Omega$ versus ac frequency are shown in Fig. 4. We find that diffusion due to impurities induces on the photon-assisted noise a similar frequency dependence as the transmission through a chaotic cavity. Again a maximum is present with the main difference that the energy scale is set by the Thouless energy instead of the inverse dwell time.

IV. CONCLUSIONS

In conclusion, we have shown that the frequency dispersion of the photon-assisted shot noise probes directly the diffusion time in mesoscopic conductors. Our predictions can be verified by an experiment analogous to that described, for instance, in Ref. 17, where a chaotic cavity can be formed between two quantum point contacts. By carefully choosing the transparencies of the cavity (or the length of a diffusive wire) one can match the Thouless energy ($\hbar/\tau_D$) in the range of frequencies that have been already investigated. A Thouless energy of 10 μeV, which is typically realized in mesoscopic conductors, corresponds to $\Omega/2\pi=2.4$ GHz which is a frequency readily accessible in experiments.

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