Correlation of multiply scattered elastic waves and possible applications to seismology.

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Abstract:
The earth crust is a disordered medium with random impedance contrast where seismic waves are multiply scattered. At present, direct waves are well understood and have permitted us to establish a good model of the earth (PREM). However, direct waves constitute only a small part of seismic record compared to coda waves (multiple scattering). We intend to extract the information that coda waves contain.

A. Time correlation of field and weak change monitoring

The following small scale experiment (ultrasounds in concrete) shows how coda waves might allow us to monitor weak changes in the internal structure of the earth (temperature or pressure). Such a technique could be useful to keep a watch on volcanoes and faults, or could interest companies for concrete structure monitoring.

1. Physical principle

A thermal change $\Delta\theta = \theta - \theta_0$ of the medium results in a change in the wave velocities:

$$c_P = c_P^0 + \Delta c_P$$
$$c_S = c_S^0 + \Delta c_S$$

At first order it corresponds to a temporal dilatation of the impulse response:

$$h_\theta(t) = h_{\theta_0}(t(1+\epsilon))$$

with ϵ the dilation rate. It is related to a delay around time t given by $\epsilon = \frac{\delta}{t} \simeq \frac{\Delta c}{c} \simeq \chi\Delta\theta$. It is a good approximation as long as the wave forms are weakly distorted.

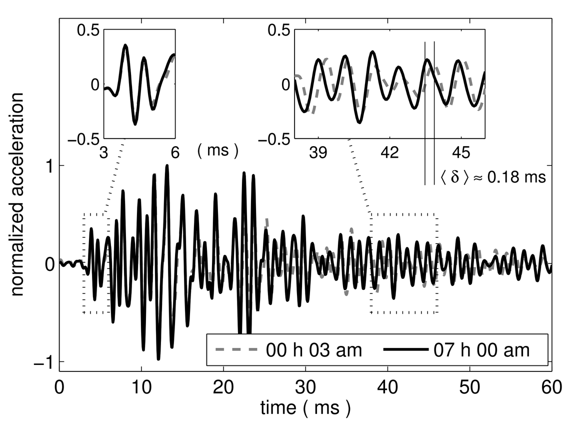

Figure 1: Example of two dynamic responses at two different times.

2. Experimental setup and data processing

The experiment is conducted on the second floor of a concrete structure.

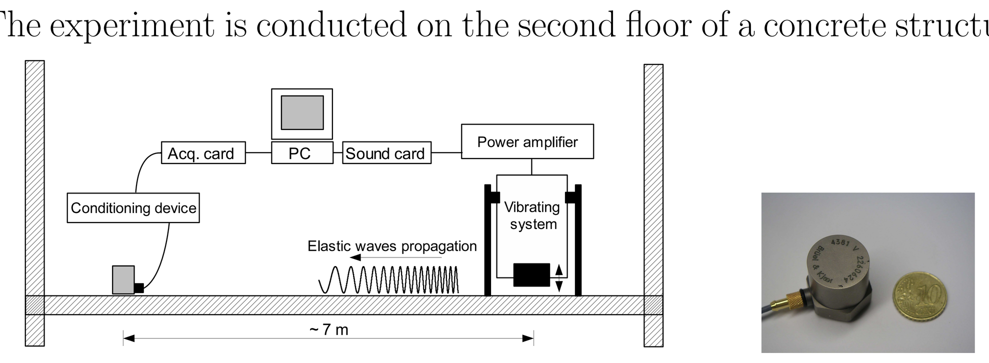

Figure 2: Experimental setup. A vibrating source sends a sweep (from 20Hz to 2kHz) in the concrete structure. The seismic acceleration is sensed 7m away by an accelerometer. [1]

We obtain an estimate of the impulse response $h_\theta(t)$ by convolution of the signals $r_\theta(t)$ with the sweep $s(t)$:

$$h_\theta(t) = \int r_\theta(t+\tau)s(\tau)d\tau$$

The delay δ at time t is defined as the time τ of the maximum of the correlation function for a time window t_W around time t:

$$C_{\Delta\theta,t}(\tau) = \frac{\int_{t-t_W/2}^{t+t_W/2} h_{\theta_0}(\nu)h_\theta(\nu+\tau)d\nu}{\sqrt{\int_{t-t_W/2}^{t+t_W/2} h_{\theta_0}(\nu)^2 d\nu \int_{t-t_W/2}^{t+t_W/2} h_\theta(\nu)^2 d\nu}}$$

3. Results

The experiment was conducted in summertime when outside temperature was increasing from $17°C$ (night) to $32°C$ (1p.m.). This resulted in noticeable variations of the concrete temperature: from $26°C$ to $30°C$.

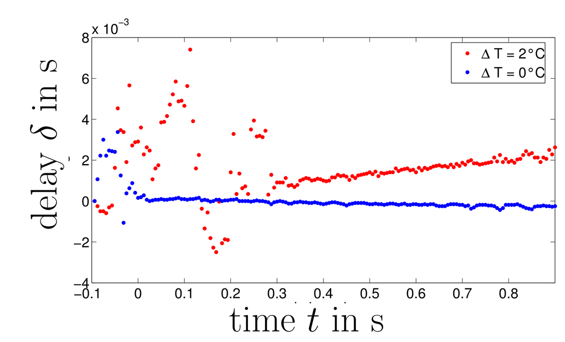

Figure 3: Delay δ increases linearly with t for multiply scattered and/or reverberated coda waves. The more scattered, the more precision we get for ϵ. From the delay estimation we calculate that the thermal dilation rate of concrete velocities is $0.15\%/°C$.

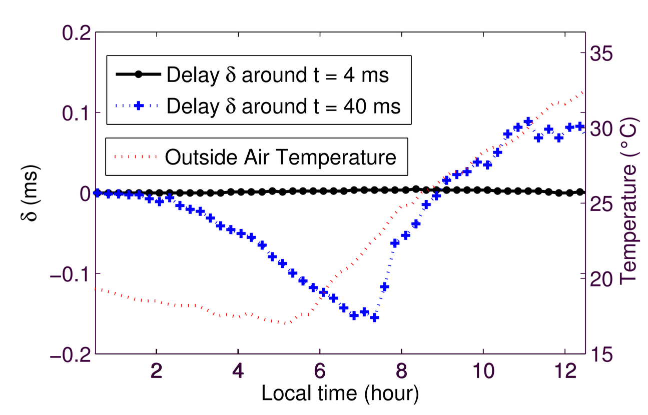

Figure 4: Delay δ calculated around $t = 4ms$ (direct waves) and around $40ms$ (coda waves) compared to outside air temperature. The velocity increases at night and decreases with solar warming. [1]

4. Conclusion

This experiment takes advantage of multiply scattered and/or reflected waves. They have traveled along much longer paths than direct waves and are, therefore, much more sensitive to weak variations of the medium. This idea is new but inspired by works from Poupinet et al. [2] R. Snieder (CWI) [3] and J.Page (DAWS) [4].

The present experiment measures the velocity change with temperature. In future work one could also hope to correct for this thermal effect to monitor even weaker changes like stress loading.

B. Spatial correlation of phase

We concentrate on a theoretical study of the infomation contained in the phase. In seismology one can mesure directly the field and not only the intensity (like in optics). The large wavelength ($\lambda \sim km$) allows us to obtain the phase and its spatial derivatives without to much difficulties with data processing. The advantages are: additional information and a data processing without a need for calibration.

1. Spatial field correlation

In the presence of multiple scattering the spatial field correlation function is related to the Green function between two points [5]:

$$C(x'-x'') = \langle \Psi(x',\omega)\Psi(x'',\omega)^* >\sim \Im m G(x',x'',\omega)$$

Behaviour:

$$C(x) = C(x'-x'') = \langle \Psi(x')\Psi(x'')^* \rangle = \begin{cases} J_0(kx)\exp(-x/2\ell) & (2D) \\ \mathrm{sinc}(kx)\exp(-x/2\ell) & (3D) \end{cases}$$

- the exponential decay is due to dephasing from scattering;
- the oscillations on the scale of the wavelength λ are due to a superposition of plane waves incident with arbitrary directions but with equal amplitude.

2. Phase derivative correlation

The wrapped phase Φ is defined as the complex phase of the wave function $\Psi = A\exp(i\Phi)$.

Using the hypothesis of gaussian statistics for the multiply scattered field, one can obtain [6]:

$$C_{\Phi'}(x>\lambda) \equiv \langle \Phi'(x')\Phi'(x'') \rangle \to \begin{cases} (k/\pi x)\exp(-x/\ell) & (2D) \\ (1/2x^2)\exp(-x/\ell) & (3D) \end{cases}$$

- No more oscillations on the scale of the wavelength.
- Possibility to retrieve the mean free path of the medium.

3. Unwrapped phase correlation

The unwrapped phase Φ_U is defined as the complex phase of the wave function $\Psi = A\exp(i\Phi)$ corrected for its discontinuous jumps of 2π.

$$C_{\Phi_U}(x) = \int_0^{-x/2} dx' \int_0^{x/2} dx'' C_{\Phi'}(x',x'')$$

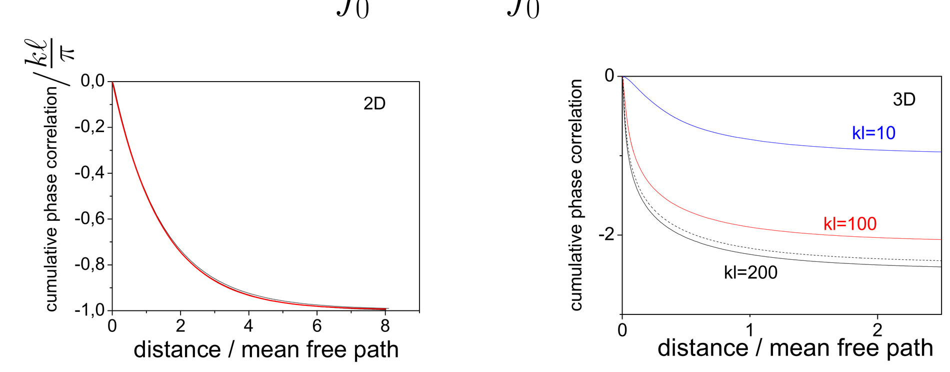

Figure 5: 2D: universal exponential decay towards $-k\ell/\pi$
3D: The asymptotic value varies only logarithmically with $k\ell$ [6]

4. Conclusion

The phase correlations reveal the mean free path ℓ. Experiments are in preparation.

References

[1] E. Larose, J. De Rosny, L. Margerin, D. Anache, P. Gouedard, M. Campillo, and B.A. van Tiggelen. Observation of multiple scattering of khz vibrations in a concrete structure and application to monitoring weak changes. *Phys. Rev. E*, 2006.

[2] G.Poupinet, W.L. Ellsworth, and J. Frechet. Monitoring velocity variations in the crust using earthquake doublets: An application to the calaveras fault, california. *J. Geophys. Res.*, 1984.

[3] R. Snieder, A. Grêt, R.C. Aster, and P.R. Kyle. Monitoring rapid temporal change in a volcano with coda wave interferometry. *Geophys. Res. Lett.*, 2005.

[4] M.L. Cowan, I.P. Jones, J.H. Page, and D.A. Weitz. Diffusing acoustic wave spectroscopy. *Physical Review E*, 65, June 2002.

[5] B. Shapiro. Large intensity fluctuations for wave propagation in random media. *Phys. Rev. Lett.*, 1986.

[6] B.A. van Tiggelen, D. Anache, and A. Ghysels. Role of mean free path in spatial phase correlation and nodal screening. *Europhys. Lett.*, 2006.

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