Casimir momentum in crossed electromagnetic fields.
QED correction to Abraham force?

LPMMC laboratoire de physique et de modélisation des milieux condensés

ANR

PHOTONIMPULS

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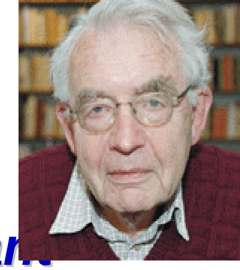
Casimir Workshop ; Leiden , March 2012

Casimir energy



1. *Isotropic radiation with power spectrum ω^3 is Lorentz-invariant (Einstein, 1917);*

$$\frac{1}{c_0^3}\int d\omega \int d\hat{k}\, \omega^2 \left[\frac{1}{2}\hbar\omega, \frac{1}{2}\hbar\mathbf{k}\right] = [\rho_{casi}, 0]$$

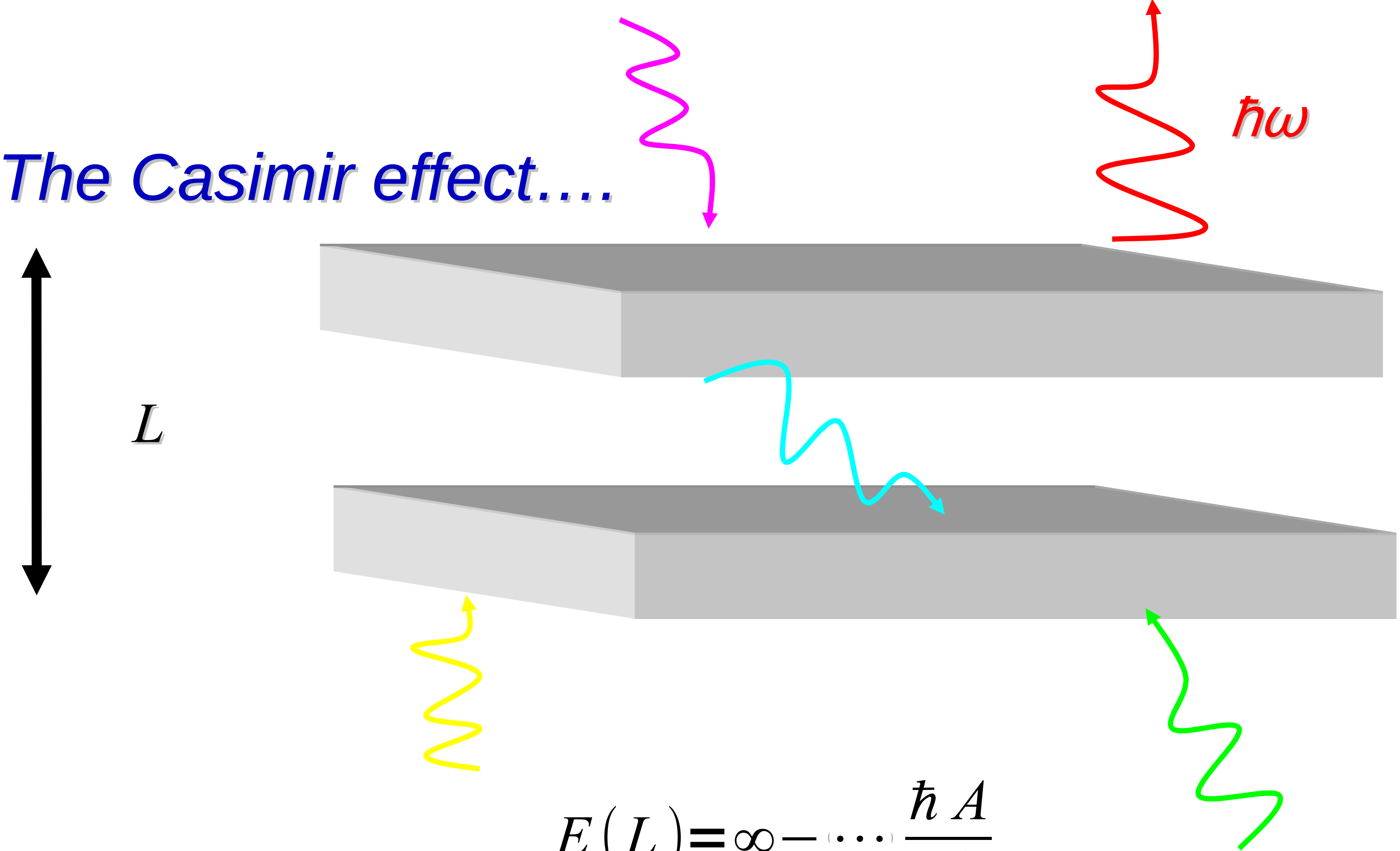
is an invariant four vector

1. *Van der Waals force $1/r^6$ (London, 1930)*
2. *Relation to Cosmological constant (Pauli, 1934, Davies, 1984)*
3. *Casimir Polder Force $1/r^7$ (1947)*
4. *Attraction between metallic plates (Casimir, 1948)*
5. *Lifshitz theory for dielectric media (Lifshitz, 1956, Dzyalonishiniskii 1961)*

1. *Observation of Casimir effect (Sparnaay, 1958, Lamoureux (5%), 1997), Chan etal, (1%), 2001)*

1. *Stability of the electron (Casimir, 1956, Boyer, 1968)*
2. *Unruh effect & Hawking radiation (Hawking 1974, Unruh 1976)*
3. *Bag model for Hadrons (Jaffe etal, 1974)*
4. *Sign of the Cosmological constant (Weinberg, … 1983)*
5. *Sonoluminescence (Schwinger, 1993, Eberlein, 1996)*
6. *Quantum friction and sheering the quantum vacuum (Pendry, 1998)*
7. *Casimir momentum in magneto-electric media (Feigel, 2004)*

The Casimir effect….



$$E(L) = \infty - \cdots \frac{\hbar A}{L^3}$$

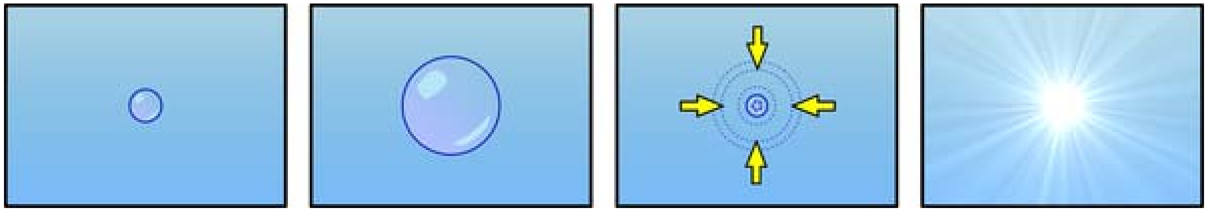
$$F(L) = -\frac{\partial E}{\partial L} = -3(\cdots)\frac{\hbar A}{L^4}\hat{L}$$

Negative pressure

No momentum exchange between matter and radiation

UV catastrophe in sonoluminescence

(> 1934)



Schwinger (1993)

$$\Delta E(\text{bubble}) = \int d^3r \left\{ \int d^3k \, \frac{1}{2}\hbar\omega_k(\text{bubble in water}) - \int d^3k \, \frac{1}{2}\hbar\omega_k(\text{water no bubble}) \right\}$$

$$\approx \frac{\hbar a^3 \, \omega_c^4}{c_0^3}\left(1-\frac{1}{\sqrt{\varepsilon}}\right) \approx 10\,\text{MeV}$$

cut-off in the UV ?

Dimensional regularisation?

Identity of the van der Waals Force and the Casimir Effect and the Irrelevance of These Phenomena to Sonoluminescence

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(Received 12 October 1998)

$$\Delta E(\text{bubble}) = \frac{23}{1536\pi}\,\frac{(\varepsilon-1)^2 c_0}{a}$$

$$\approx 0.001\text{ eV}$$

« Momentum from Nothing »

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Quantum Vacuum Contribution to the Momentum of Dielectric Media

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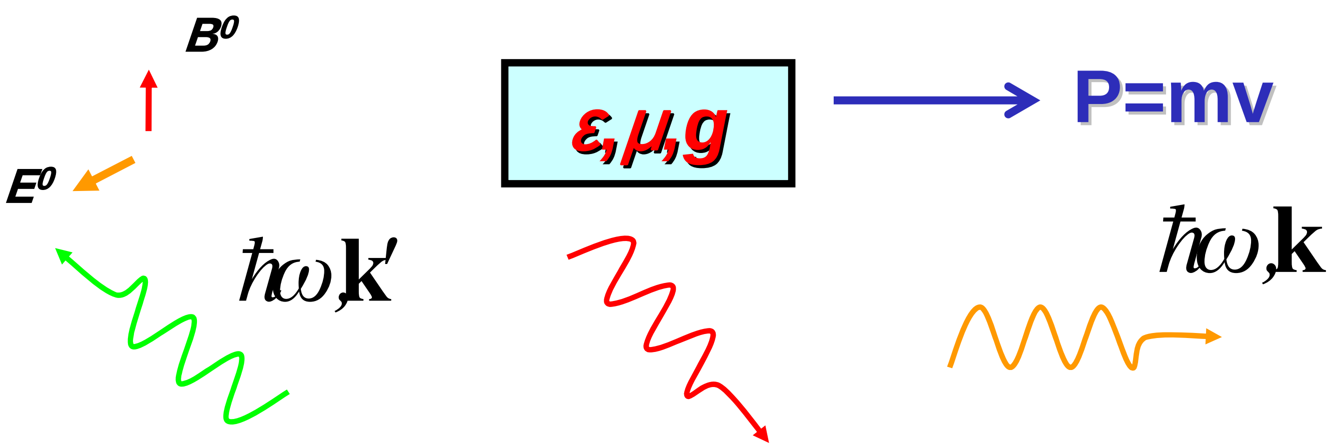
(Received 3 February 2003; published 16 January 2004)

Momentum transfer between matter and electromagnetic field is analyzed. The related equations of motion and conservation laws are derived using relativistic formalism. Their correspondence to various, at first sight self-contradicting, experimental data (the so-called Abraham-Minkowski controversy) is demonstrated. A new, Casimir-like, quantum phenomenon is predicted: contribution of vacuum fluctuations to the motion of dielectric liquids in crossed electric and magnetic fields. Velocities of about 50 nm/s can be expected due to the contribution of high frequency vacuum modes. The proposed phenomenon could be used in the future as an investigating tool for zero fluctuations. Other possible applications lie in fields of microfluidics or precise positioning of micro-objects, e.g., cold atoms or molecules.

DOI: 10.1103/PhysRevLett.92.020404 PACS numbers: 03.50.De, 42.50.Nn, 42.50.Vk



Bi-anisotropic Media

$$\mathbf{D}(\omega) = \varepsilon(\omega)\mathbf{E}(\omega) + \mathbf{g}(\omega)\cdot\mathbf{B}(\omega)$$

$$\mathbf{H}(\omega) = \mathbf{g}^T(\omega)\cdot\mathbf{E}(\omega) + \mu(\omega)^{-1}\mathbf{B}(\omega)$$

Fresnel dispersion law

$$\det\left(\varepsilon\frac{\omega^2}{c_0^2} - k^2 + \mathbf{kk} - \frac{\omega}{c_0}\mathbf{g}\cdot(\boldsymbol{\varepsilon}\cdot\mathbf{k}) + \frac{\omega}{c_0}(\boldsymbol{\varepsilon}\cdot\mathbf{k})\cdot\mathbf{g}^*\right) = 0$$

$$g_{ij}(\omega) = i\omega\, g\delta_{ij}$$

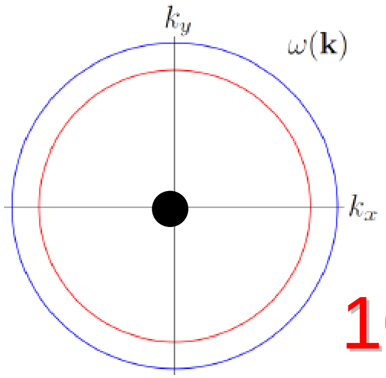
Rotatory power

$$g_{ij}(\omega) = (1-\varepsilon)\,\varepsilon_{ijl}\frac{v_l}{c_0}$$

Fizeau effect

$$g_{ij}(\omega) = g\left(E_i^0 B_j^0 - B_i^0 E_j^0\right)$$

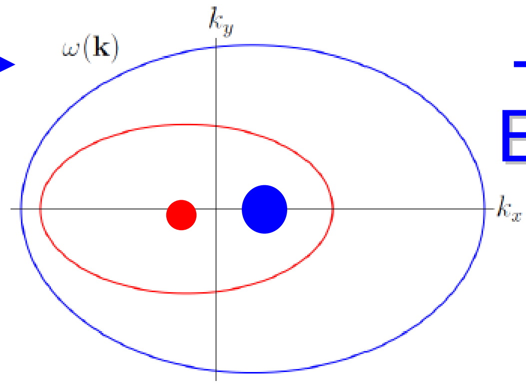
Magneto-electric birefringence



10^{-2}

v

10^{-8}



$E_0 \times B_0$

10^{-15}

phenomenological continuum theory

$$\partial_t\left(\rho\,\mathbf{v}+\varepsilon_0\mathbf{E}\times\mathbf{B}\right)=-\nabla\cdot\mathbf{T}^0$$

$$T^0_{ij}=\frac{1}{8\pi}\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)\delta_{ij}-\frac{1}{4\pi}\left(\varepsilon_0E_iE_j+\frac{1}{\mu_0}B_iB_j\right)$$

Observed in X-ray

$$\left\langle 0\left|\frac{\mathbf{E}\times\mathbf{B}}{4\pi c_0}\right|0\right\rangle \propto \begin{cases} \dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \times g(\omega)\mathbf{E}_0\times\mathbf{B}_0=\frac{2}{3}\frac{\hbar\omega_c^4}{\pi^3\,c_0^4}\,g\,\mathbf{E}_0\times\mathbf{B}_0 \\ \dfrac{1}{c_0}\displaystyle\int d^3\mathbf{k}\,\frac{1}{2}\hbar\omega_k \times\left[\varepsilon(\omega)-1\right]\frac{\mathbf{v}}{c_0}=\rho_{casi}\mathbf{v} \end{cases}$$

Photonic momentum in dielectric media?

→ classical « Abraham » contribution already controversial

UV catastrophe of vacuum energy ?

Lorentz invariance of quantum vacuum?

Inertia of quantum vacuum?

The Abraham Force

Macroscopic Maxwell →

$$\partial_t \mathbf{G}_M + \nabla\cdot\mathbf{T} = -\mathbf{f}$$

$$\mathbf{G}_M = \mathbf{D}\times\mathbf{B}$$

$$\mathbf{f} = -E^2\nabla\varepsilon - H^2\nabla\mu$$

Minkowski

The Abraham Force

$$\mathbf{G}_A = \varepsilon_0 \mu_0 \mathbf{E} \times \mathbf{H} = \frac{1}{c_0} \mathbf{S}$$

$$\partial_t \mathbf{G}_A + \nabla \cdot \mathbf{T} = -\mathbf{f} - \varepsilon_0 (\varepsilon_r - 1/\mu_r) \partial_t (\mathbf{E} \times \mathbf{B})$$ Abraham

$$\mathbf{G}_N = \varepsilon_0 \mathbf{E} \times \mathbf{B} = \mathbf{G}_0$$

$$\partial_t \mathbf{G}_A + \nabla \cdot \mathbf{T} = -\mathbf{f} - \varepsilon_0 (\varepsilon_r - 1) \partial_t (\mathbf{E} \times \mathbf{B})$$ Nelson

$$\ldots = -\mathbf{f} - \frac{2}{5} [\partial_t \mathbf{E}_0(t)] \times \mathbf{B}_0(t) - \mathbf{E}_0(t) \times [\partial_t \mathbf{B}_0(t)]$$

Peierls

exp $$\mathbf{F} = (\varepsilon - 1) V \partial_t \mathbf{E}_0(t) \times \mathbf{B}_0(t)$$ Walker & Walker, Nature 1976

theo

Abraham momentum = kinetic momentum,
Minkowski momentum = conjugate momentum

Barnett (PRL 2010):

Nelson momentum = pseudo momentum

Nelson (PRA 1991)

The Abraham Force (our version)

Macroscopic Maxwell →

$$\partial_t \mathbf{G} + \nabla \cdot \mathbf{T} = -\mathbf{f}$$

$$\mathbf{G} = \mathbf{D} \times \mathbf{B}$$

$$\mathbf{f} = -E^2 \nabla \varepsilon - H^2 \nabla \mu$$

Maxwell-Lorentz force on induced polarization and current →

$$\partial_t \rho \mathbf{v} + \nabla \cdot \mathbf{U} = \mathbf{f} + \partial_t (\mathbf{P} \times \mathbf{B})$$

+

Microscopic Maxwell →

$$\partial_t \left(\rho \mathbf{v} + \varepsilon_0 \mathbf{E} \times \mathbf{B} \right) + \nabla \cdot \mathbf{T}_0 = 0$$

The Abraham Force (our version)

Macroscopic Maxwell ⟶

$$\partial_t \mathbf{G} + \nabla \cdot \mathbf{T} = -\mathbf{f} - \partial_t (\mathbf{P} \times \mathbf{B})$$

$$\mathbf{G} = \mathbf{D} \times \mathbf{B}$$

$$\mathbf{f} = -E^2 \nabla \varepsilon - H^2 \nabla \mu$$

Maxwell-Lorentz force on induced polarization and current ⟶

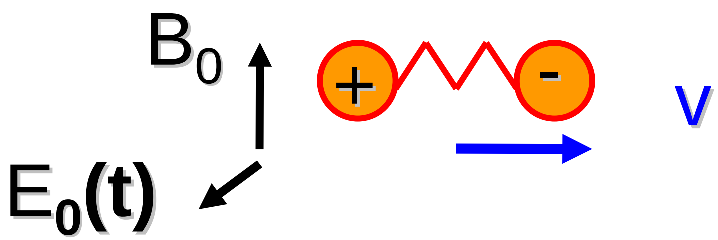
$$\partial_t \rho \mathbf{v} + \nabla \cdot \mathbf{U} = \mathbf{f} + \partial_t (\mathbf{P} \times \mathbf{B})$$

+

Microscopic Maxwell ⟶

$$\partial_t (\rho \mathbf{v} + \varepsilon_0 \mathbf{E} \times \mathbf{B}) + \nabla \cdot \mathbf{T}_0 = 0$$

Classical Abraham momentum in crossed EM fields



$$\mathbf{r}_{1,2} = \mathbf{R} \pm \frac{1}{2}(\mathbf{x})$$

$$m\ddot{\mathbf{r}}_1 = +q\mathbf{E}(t) + q\dot{\mathbf{r}}_1 \times \mathbf{B} + \mathbf{f}(r_{12})$$

$$m\ddot{\mathbf{r}}_2 = -q\mathbf{E}(t) - q\dot{\mathbf{r}}_2 \times \mathbf{B} - \mathbf{f}(r_{12})$$

$$2\mathrm{m}\dot{\mathbf{R}} + q\,\mathbf{x} \times \mathbf{B} = \mathrm{constant} = 0$$

$$m\ddot{\mathbf{x}} = 2\mathrm{q}\mathbf{E}(t) + 2\mathrm{q}\,\dot{\mathbf{R}} \times \mathbf{B} - m\omega_0^2\,\mathbf{x} \approx 0$$

$$2\mathrm{m}\dot{\mathbf{R}} = \frac{q^2/m}{\omega_0^2}\mathbf{E}_0(t) \times \mathbf{B}_0$$

No controversy exists in microscopic description
Consistent with Abrahams and Nelson version

he UV catastrophe is real in macroscopic 'description

Free electron (electric dipole)

$$\varepsilon(\omega)-1=-\frac{\omega_P^2}{\omega^2} \qquad \rho_{casi}=\frac{\hbar}{c_0^3}\int_0^\infty d\omega\,\omega^3\frac{\omega_p^2}{\omega^2}=\infty$$

magnetic dipole

$$g_{ME}(\omega)=-\frac{e^4 m_\Delta^2}{\omega_0^2 m^3 M^2}\Big[-\frac{\omega^2+\omega_0^2}{(\omega_0^2-\omega^2)^2}(\mathbf{E}_i^0\mathbf{B}_j^0-(\mathbf{E}^0\cdot\mathbf{B}^0)\delta_{ij})$$

$$+\frac{1}{\omega_0^2-\omega^2}(\mathbf{E}_i^0\mathbf{B}_j^0-\frac{1}{4}\mathbf{E}_j^0\mathbf{B}_i^0-\frac{1}{4}(\mathbf{E}^0\cdot\mathbf{B}^0)\delta_{ij})\Big].$$

Electric quadrupole

$$P_{casi}=\frac{\hbar}{c_0^3}\int dr\int_0^\infty d\omega\,\omega^3 g(\omega)E_0\times B_0=\infty$$

Rizzo etal, 2003-2009, Babington & BAvT, EPJD 2011

$$P_{casi}=\frac{\hbar c_0 g\left(\varepsilon(0)-1\right)}{a}E_0\times B_0?$$

Dimensional regularization for object of size a?
BAvT EPJD 2009

Casimir momentum, if infinite, is Lorentz invariant

$$L(\mathbf{E},\mathbf{B}) = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + 2\nu(\mathbf{E}^2 - \mathbf{B}^2)^2 + \frac{\nu}{2}(\mathbf{E}\cdot\mathbf{B})^2$$

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}(\omega)$$
$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}(\omega)$$

← Bi-anisotropic Lorentz-invariant vacuum

Fluctuation-Dissipation

$$\langle 0 | E_i(\mathbf{r},\omega) E^*_j(\mathbf{r}',\omega') | 0 \rangle = -2\hbar\omega^2 \,\mathrm{Im}\, G_{ij}(\mathbf{r},\mathbf{r}',\omega) \times 2\pi\delta(\omega-\omega')$$

$$\langle 0 | \frac{c_0}{4\pi} \mathbf{E}^* \times \mathbf{H} | 0 \rangle = 0$$

Zero energy flow

$$\langle 0 | \frac{\mathbf{E}^* \times \mathbf{B}}{4\pi c_0} | 0 \rangle = -\frac{4}{3}\nu K \; \mathbf{E}_0 \times \mathbf{B}_0$$

infinite momentum density

$$K = \frac{1}{(2\pi)^3} \tfrac{1}{2}\hbar \int_0^\infty d\omega \int_{4\pi} d\Omega \, \rho_0(\omega,\Omega)$$

Lorentz scalar

Ex: Helium

E0=450 V/mm; B0=1 T

$$\alpha(0)=0.22\ 10^{-40}\,\mathrm{Cm^2/V}\ \ (16.6\mathrm{a}_0^3)$$
$$\rho=0.17\ \mathrm{kg/m^3}\,(\mathrm{room}\ T)$$
$$g=0.017\ 10^{-22}\,\mathrm{m/VT}$$

(SI units)

$$v_{\mathrm{abr}}=\frac{\varepsilon_0\alpha(0)EB}{2\mathrm{m}_p}\approx 0.3\,\mathrm{nm/sec}$$

Classical abraham force

$$F_{\mathrm{abr}}\approx 7\cdot 10^{-32}\ N$$
$$N_{at}F_{\mathrm{abr}}\propto 10^{-13}\ N$$

$$v_{\mathrm{Feigel}}=\frac{\pi}{4}\frac{h}{\rho\lambda_c^4}gEB\approx 0.02\ \mathrm{nm/sec}$$

Semi-classical QED with cut-off 0.1 nm (Feigel)

$$v_{QED}\propto v_{\mathrm{abr}}\times(\quad\alpha)^2\approx 0.0002\ \mathrm{nm/sec}$$

Rigorous QED (Kawka, 2010,2012)

Ex: Helium

E0=450 V/mm; B0=1 T

$$\alpha(0)=0.22\ 10^{-40}\mathrm{Cm^2/V}\ \ (16.6a_0^3)$$

$$\rho=0.17\ \mathrm{kg/m^3}\ (\mathrm{room}\ T)$$

$$g=0.017\ 10^{-22}\mathrm{m/VT}$$

(SI units)

$$v_{\mathrm{abr}}=\frac{\varepsilon_0\alpha(0)EB}{2\mathrm{m}_p}\approx 0.3\ \mathrm{nm/sec}$$

Classical Abraham Force

$$F_{\mathrm{abr}}\approx 7\cdot 10^{-32}\ N$$

$$N_{at}F_{\mathrm{abr}}\propto 10^{-13}\ N$$

$$v_{\mathrm{Feigel}}=\frac{\pi}{4}\frac{h}{\rho\lambda_c^4}gEB\approx 0.02\ \mathrm{nm/sec}$$

Semi-classical QED with cut-off 0.1 nm (Feigel)

$$v_{QED}\propto v_{\mathrm{abr}}\times(\text{Z?}\alpha)^2\approx 0.001\ \mathrm{nm/sec}$$

Rigorous QED (Kawka, 2010,2012)

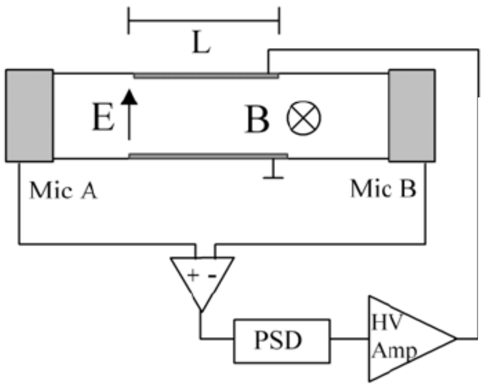
$$\frac{dp}{dt} = \alpha(0)\frac{dE}{dt} \times B$$

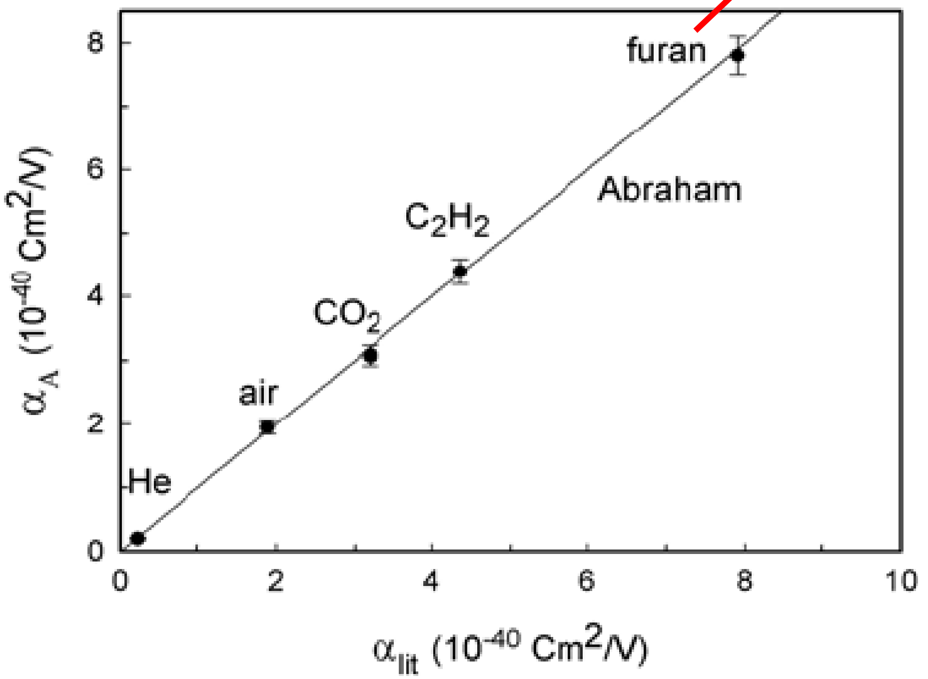
Abraham force

Acoustic pressure

$$P(\omega) = P_0 + \alpha(0) \times E \times B \times \omega \times \cos\omega t \times n \times L$$

V= 8 nm/sec+- 0.8
Feigel correction: 2 nm/se





E=450 V/mm;
B=1 T;
f= 7.6 kHz

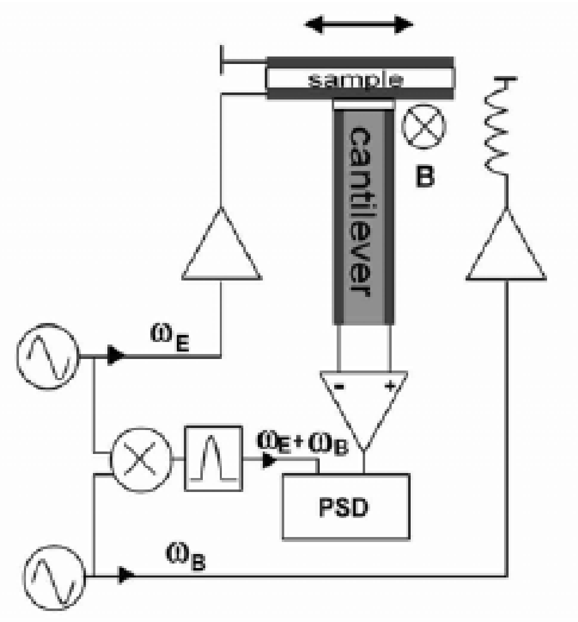
P/(EB)

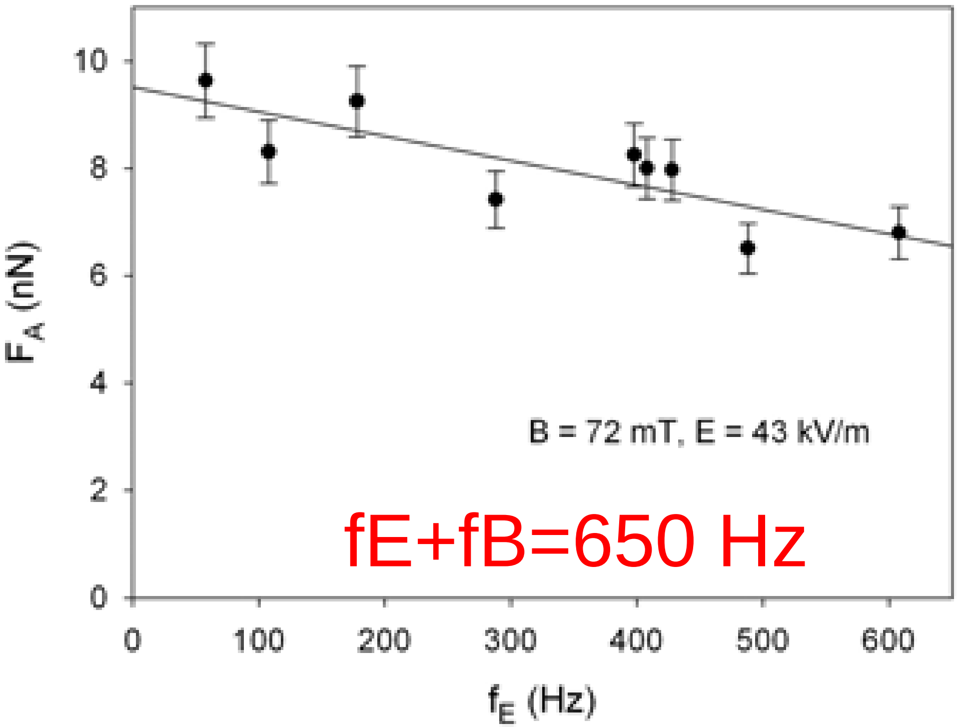
α(0)

Rikken / Van Tiggelen, PRL 2011

$$\frac{dp}{dt} = \varepsilon_0 \left(\varepsilon_r - 1\right) \frac{d}{dt} \left(E \times B\right)$$

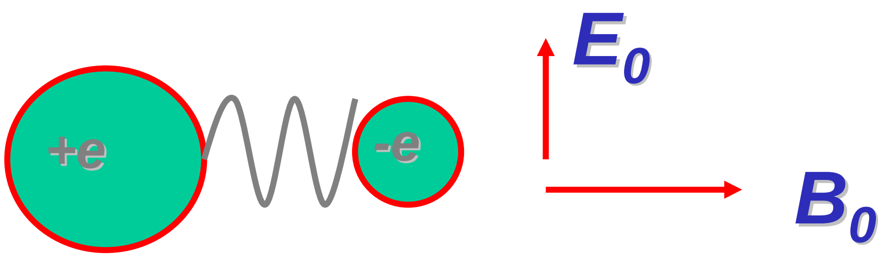
$\varepsilon_r = 1.7 \ 10^5$ *Y5V ceramic*





Rikken / Van Tiggelen, submitted 2012

Casimir momentum: 1/6
QED of atom in crossed fields



$$\mathbf{A}_0 = \frac{1}{2}\mathbf{B}_0 \times \mathbf{r} \qquad \phi = -\mathbf{E}_0 \cdot \mathbf{r}$$

Coulomb Gauge

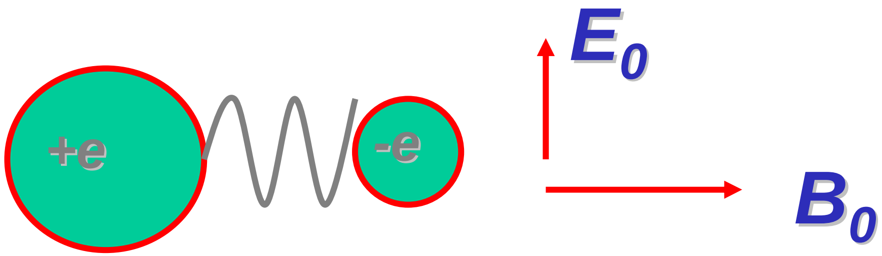
$$H = \frac{1}{2m_1}\left(\mathbf{p}_1 - e\mathbf{A}_0(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_1)\right)^2 + \frac{1}{2m_2}\left(\mathbf{p}_2 + e\mathbf{A}_0(\mathbf{r}_2) + e\mathbf{A}(\mathbf{r}_2)\right)^2$$

$$+ e\mathbf{E}_0 \cdot \mathbf{r}_{21} + V(r_{12})$$

$$+ \sum_i \hbar\omega_i \left(a_i^* a_i + 1/2\right)$$

Casimir momentum: 2/6

QED of harmonic oscillator in crossed fields



Conjugate momenta ≠ kinetic momentum

$$\mathbf{p}_1 = m_1\mathbf{v}_1 + e\mathbf{A}_0(\mathbf{r}_1)$$
$$\mathbf{p}_2 = m_1\mathbf{v}_2 - e\mathbf{A}_0(\mathbf{r}_2)$$

Pseudo momentum is conserved

$$\hat{\mathbf{K}} = \mathbf{p}_1 + \mathbf{p}_2 + \frac{1}{2}e\mathbf{B}_0 \times \mathbf{r}_{21} = \mathbf{P}_{kin} + e\mathbf{B}_0 \times \mathbf{r}$$

$$[\mathbf{K}, H] = 0$$

Coulomb Gauge

Ground state changes due to coupling with quantum vacuum

$$|\tilde{\Psi}_0\rangle = \left[1 - \frac{1}{2}{\sum_{i\mathbf{Q}n}}' \frac{|W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}|^2}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2}\right] |0\mathbf{Q}_0\{0\}\rangle$$

$$+ {\sum_{i\mathbf{Q}n}}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n}} |i\mathbf{Q}n\rangle$$

$$+ {\sum_{i\mathbf{Q}n}}' {\sum_{i'\mathbf{Q}'n'}}' \frac{W_{i\mathbf{Q}n,i'\mathbf{Q}'n'} W_{i'\mathbf{Q}'n',0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})(E_{0\mathbf{Q}_0 0} - E_{i'\mathbf{Q}'n'})} |i\mathbf{Q}n\rangle$$

$$- W_{0\mathbf{Q}_0 0,0\mathbf{Q}_0 0} {\sum_{i\mathbf{Q}n}}' \frac{W_{i\mathbf{Q}n,0\mathbf{Q}_0 0}}{(E_{0\mathbf{Q}_0 0} - E_{i\mathbf{Q}n})^2} \rangle |i\mathbf{Q}n\rangle$$

$\delta M \mathbf{v} +$
$\langle \psi_0 | e\mathbf{A} | \psi_0 \rangle$

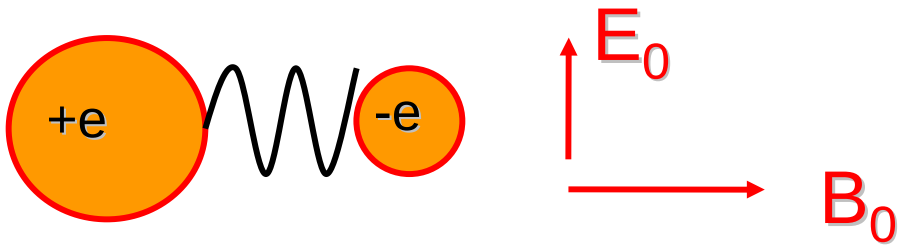
$0.84\alpha^3$

$\alpha(\omega = 0, \mu + \delta\mu)$

$\langle 0 | \mathbf{A} | 0 \rangle = 0$

Casimir momentum: 3/6

QED of hydrogen atom in crossed fields



$$\hat{K} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + e\mathbf{A}(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_2) + e\mathbf{B}_0 \times \mathbf{r}_{21} + \sum_i \hbar\mathbf{k}_i\left(a_i^* a_i + 12\right)$$

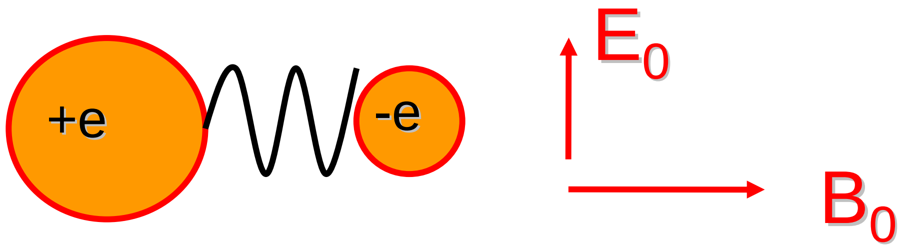
No multipole approximation in $\mathbf{A}(\mathbf{r}) \propto \sum_{gk} \mathbf{g}_k \exp(i\mathbf{kr})\, a_{\mathbf{gk}} + c.c$

$$\langle \Psi_0 | \mathbf{K} | \Psi_0 \rangle = M\mathbf{v} + \delta M\,\mathbf{v} + \frac{8}{3}\frac{E_0}{c_0^2}\mathbf{v}$$

$$+\varepsilon_0 \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 + \varepsilon_0\,\delta\mu\,\partial_\mu \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 + \mathbf{K}_1 + \mathbf{K}_2$$

$$\delta M = \delta(m_1 + m_2) \quad \delta\mu = \delta\left(\frac{m_1 m_2}{m_1 + m_2}\right) \quad \delta m_i = \frac{4}{3\pi}\alpha\hbar\int_0^\infty dk\,\frac{\hbar k}{\hbar^2 k^2/2\mathrm{m}_i + \hbar kc}$$

Casimir momentum: 4/6
QED of hydrogen atom in crossed fields



$$\hat{K} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + e\mathbf{A}(\mathbf{r}_1) - e\mathbf{A}(\mathbf{r}_2) + e\mathbf{B}_0 \times \mathbf{r}_{21} + \sum_i \hbar\mathbf{k}_i\left(a_i^* a_i + \frac{1}{2}\right)$$

$$-\frac{m_1^2\mathbf{v}_1^2}{2c_0^2}\mathbf{v}_1 + \text{idem } 2$$

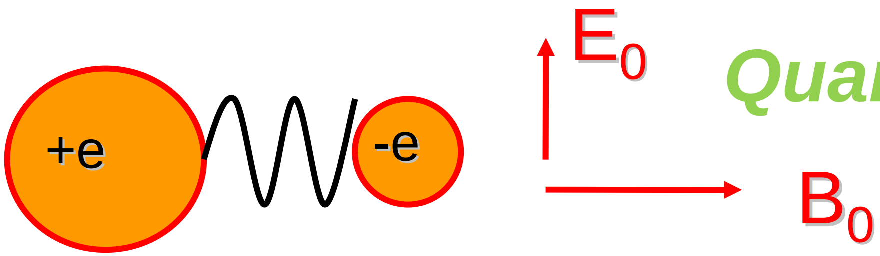
$$\langle\Psi_0|\mathbf{K}|\Psi_0\rangle = M\mathbf{v} + \delta M\,\mathbf{v} + \frac{8}{3}\frac{E_0}{c_0^2}\mathbf{v} - \frac{5}{3}\frac{E_0}{c_0^2}\mathbf{v}$$

$$+\varepsilon_0\,\alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 + \delta(\mu)\varepsilon_0\partial_\mu\,\alpha(0)\mathbf{B}_0 \times \mathbf{E}_0$$

$$+\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_R$$

$$\delta M = \delta(m_1 + m_2) \quad \delta\mu = \delta\left(\frac{m_1 m_2}{m_1 + m_2}\right) \quad \delta m_i = \frac{4}{3\pi}\alpha\hbar\int_0^\infty dk\,\frac{\hbar k}{\hbar^2 k^2/2\mathrm{m}_i + \hbar kc}$$

Casimir momentum: 5/6
QED of hydrogen in crossed fields



Quantum vacuum contribution:

$$\mathbf{K}_1 = -\mathbf{B}_0 \times \mathbf{E}_0 \frac{1}{3}\frac{e^2\hbar^2}{a_0 c_0^2 \mu^2}\sum_n \langle 0| \frac{e^2}{4\pi\varepsilon_0 r}\hat{\mathbf{r}}|n\rangle \cdot \frac{1}{(E_n - E_0)^2}\cdot \langle n|\mathbf{r}|0\rangle$$

$$= -\varepsilon_0 \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 \alpha^2 (0.208 + 0.0045) = -0.21\alpha^2\, \mathbf{K}_A$$

$$\mathbf{K}_2 = +\varepsilon_0 \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0\, \alpha^2 \frac{1}{27}\frac{e^2}{4\pi\varepsilon_0 a_0^2}\sum_n \langle 0|\hat{\mathbf{r}}|n\rangle \cdot \frac{1}{E_n - E_0}\cdot \langle n|\mathbf{r}|0\rangle$$

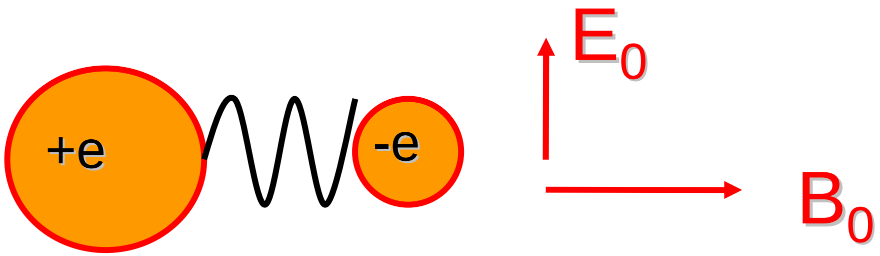
$$= +\varepsilon_0 \alpha(0)\mathbf{B}_0 \times \mathbf{E}_0 \alpha^2 (0.079 + 0.018) = +0.1\alpha^2\, \mathbf{K}_A$$

Discrete Rydberg states

Continuous spectrum assuming plane waves for electrons

Casimir momentum: 6/6

QED of hydrogen in crossed fields

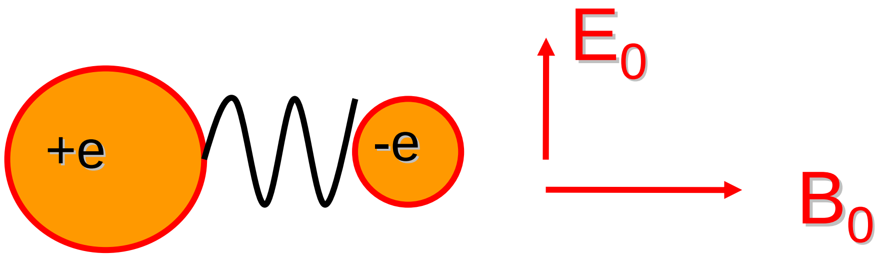


Relativistic contribution:

$$\mathbf{K}_R = -\frac{e^2}{2m_e M c_0^2}\langle 0_E \,|\, p^2(\mathbf{B}_0 \times \mathbf{x}) \,|\, 0_E \rangle$$

$$\propto \alpha^2 \frac{m_e}{M}\mathbf{K}_A$$

Casimir momentum: 6/6
QED of hydrogen in crossed fields



$$\langle \mathbf{K} \rangle = \langle \mathbf{K}_{\mathrm{kin}} \rangle + \frac{E_0}{c_0^2}\mathbf{v} + \left(1 - 0.1\,\alpha^2\right)\mathbf{K}_A + O(\alpha^3)$$

$$\mathbf{K}_A = \varepsilon_0 \alpha(0) \mathbf{B}_0 \times \mathbf{E}_0$$

Casimir momentum of H atom exists and slightly reduces the classical Abraham momentum

BaVT, Kawka, Rikken, submitted to EPJD

SUMMARY
Casimir momentum in crossed E,B

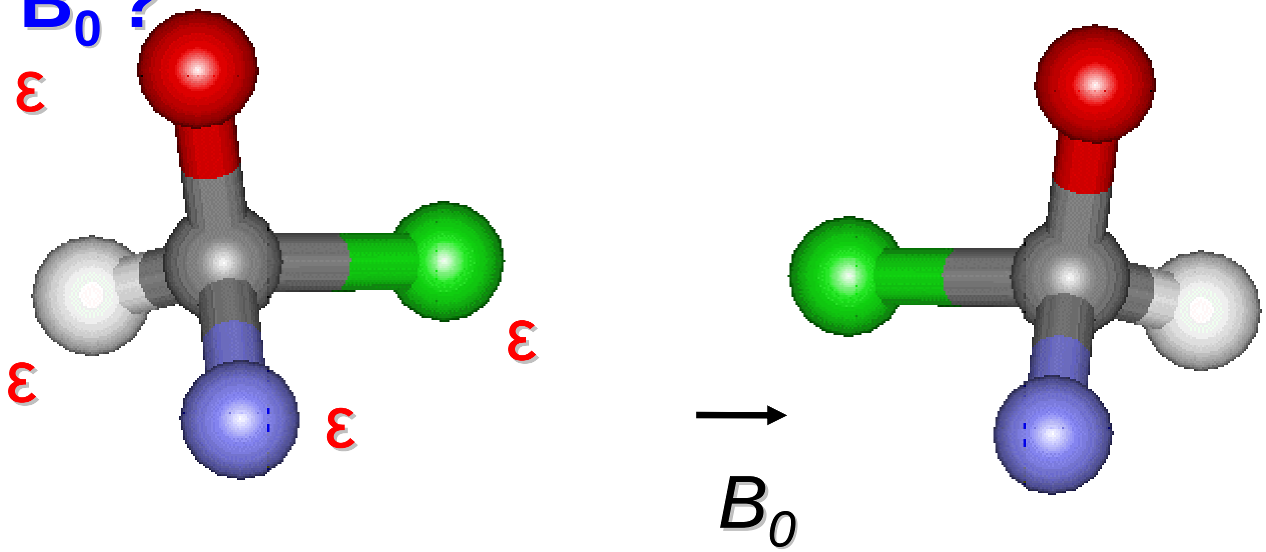
- Classical Abraham force , linear in E_0 and B_0 ,
  is observed for neutral atoms
  and for strong dielectrics
- QED contribution by Feigel is not observed
- UV divergencies disappear in mass renormalization or cancel.
  Need to go beyond multipole approximation
- Quantum vacuum contributes to Abraham momentum in order -(1/137)^2
  Will this be -(Z/137)^2 for Z > 1??

A Casimir momentum with only magnetic field?

$$\langle \mathbf{E} \times \mathbf{B} \rangle = g B_0 \ ?$$

- **Classically no equivalent Abraham version in charge neutral systems**
- **g must be a pseudo scalar**
  → medium must be chiral (on nanoscale)
- **Describe chirality microscopically, not phenomenologically**
  via «magneto-chiral « index of refraction ($\Delta n = g\, B_0 . k$)
- **Would separate enantiomers using magnetic fields**
  = Pasteurs dream !
- **Medium must be magnetic since $\langle E \times H \rangle = 0$**

Pasteur’s dream with a Casimir momentum P= g B_0 ?

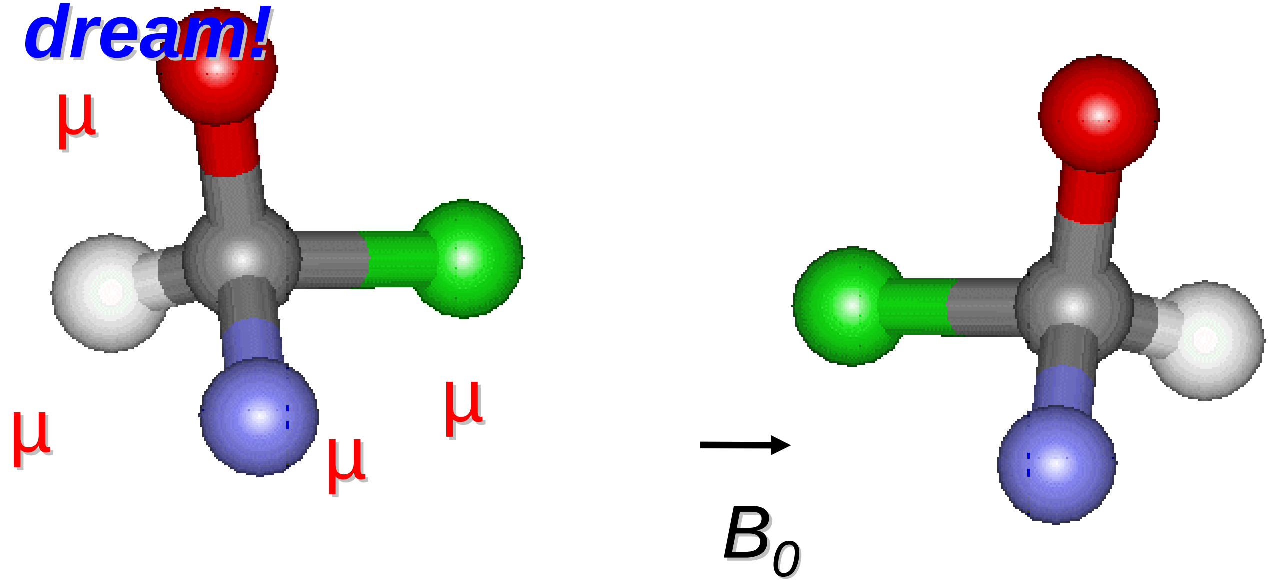


hiral geometry with electric polarizabilities with Zeeman splitting

$$\alpha(\omega,\sigma)=\frac{4\pi c_0^2}{\omega_0^2}\frac{\gamma}{\omega^2-\omega_0^2+i\sigma VB+i\gamma\omega_0}$$

$$\mathbf{B}=\mu_0\mathbf{H}\Rightarrow\langle 0|\int d\mathbf{r}\,\mathbf{E}\times\mathbf{B}|0\rangle\propto\langle 0|\int d\mathbf{r}\,\mathbf{E}\times\mathbf{H}|0\rangle=0$$

A Casimir momentum P= g B_0? Pasteur's dream!



iral geometry with magnetic polarizabilities with Zeeman splittin

$$\chi(\omega,\sigma) = \chi(0)\frac{\omega_0^2}{\omega^2 - \omega_0^2 + i\sigma VB + i\gamma\omega}$$

$$\langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{H} | 0 \rangle = 0$$

$$\langle 0 | \int d\mathbf{r}\, \mathbf{E}\times\mathbf{B} | 0 \rangle = g\mathbf{B}_0$$

$$g = \left(\frac{4\hbar c}{3e\mu_0}\right)(c\,\chi(0)\,\mu_0)^5\left(\frac{1.4\times 10^4}{L^{14}}\right)$$

Na Tetraeder L=10 nm → g/m = 1 nm/sec/T

Babington , BaVT, EPL 2011

Lorentz center
Casimir Physics
School and Workshop 2012
School: 5 - 9 March 2012
Workshop: 12 - 16 March 2012 Leiden, the Netherlands
Lorentz center
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Thank you !