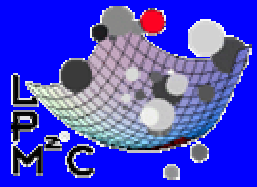


Localisation Forte d'Anderson 1958-2005

*Presque un demi siècle
et
toujours une frontière en optique.....*

Bart van Tiggelen

**Laboratoire de Physique et Modélisation des Milieux
Condensés
CNRS/Université of Grenoble, France**



Cliché, cliché !

- ☀ L'article d'Anderson's en 1958 a été « *often quoted but never read* »
- ☀ Il y a autant de définitions et d'approches qu'il y a de publications.
- ☀ La moitié des publications est

 - soit fausse
 - soit triviale
 - soit déjà contenue dans le papier d'Anderson en 1958
 - soit trop compliquée ou trop phénoménologique pour comprendre les expériences

Protect me from knowing what I don't need to know

Protect me even from knowing that there are things to know that I don't know

Protect from knowing that I decided not to know

about the things that I decided not to know about

Douglas Adams, *Mostly Harmless*, 1992



1958	Anderson	« vanishing of diffusion »
1960	Mott/Ioffe-Regel	$l \leq \frac{\lambda}{2\pi}$
1965	Mott	Minimum conductivity Variable range hopping
1972	Thouless	Sensitivity to BC: $g < 1$ (Thouless criterion)
1977	Anderson/Mott	Nobel Prize
1980	« gang of four »	Scaling theory $\frac{\partial \log(g)}{\partial \log(L)}$
1980	Götze, Vollhardt, Wölfle	Self-consistent theory
1982	Halperin, Pruisken	Scaling theory of Quantum Hall effect
>1982	Sharvin, Lagendijk, Maret, Maynard,...	Weak localization Mesoscopic physics!

1983	Fröhlich & Spencer	Mathematical proof for 3D Anderson model
1984	Anderson	25 years localization « <i>unrecognizable monster</i> »
1986	Anderson	« Theory of white paint »
1986	Kramer, Mackinnon, Economou, Soukoulis, Schreiber	Tight binding model
1987	Papanicolaou, Sheng	Prediction of Localization of Seismic Waves in layered Earth Crust
1987	Souillard	Localization of Gravitational Waves in Universe?
1988	John	Prediction of Localization of light in Photonic crystals
1990	Mello etal, Beenakker Altschuler	Random Matrix theory

<p>> 1991</p>	<p>Bell-labs Weaver Genack</p> <p>Exxon (Sheng etal)</p>	<p>2D localization microwaves 2D localization of ultrasound 3D localization of light Q1D localization microwaves 2D localization of bending waves</p>
<p>>1995</p>	<p>BEC community</p>	<p>Localization of light in BEC cold atoms gazes or the contrary?</p>
<p>1997</p>	<p>Wiersma, Lagendijk</p>	<p>3D localization of infrared light</p>
<p>> 1998</p>	<p>Cao, Wiersma etal, Soukoulis, Sebbah, ...</p>	<p>Random lasering from (pre) localized states</p>
<p>2000</p>	<p>Genack Beenakker, ...</p>	<p>Statistics in localized regime (exp) Idem (theo)</p>

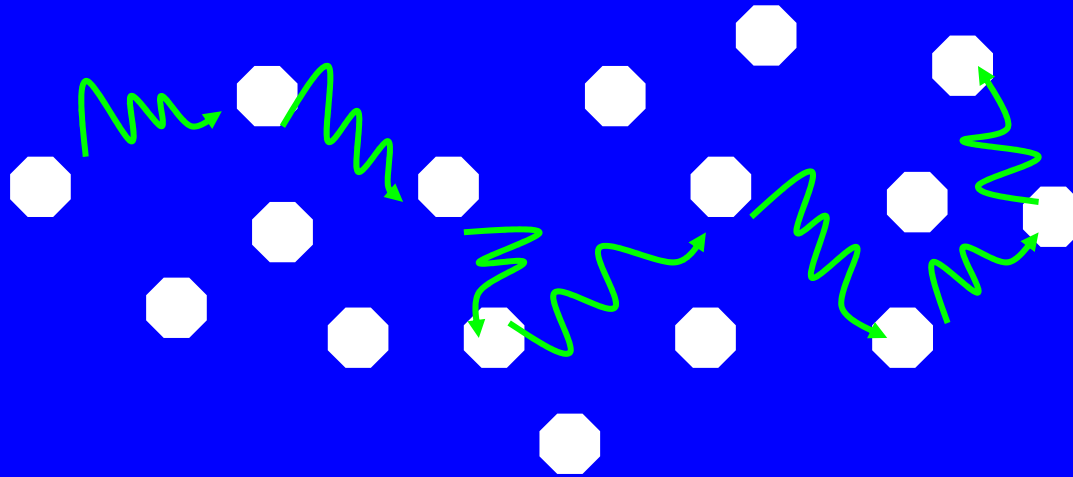
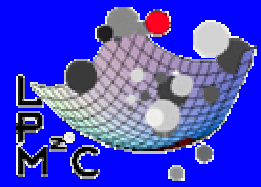
2005		Localization of seismic waves in the Earth crust
2006		Localization of light in BEC
2006		Localization of BEC in light speckle
2008	VIRGO	Non-observation of gravitational waves.....

Localization of Waves

Review « Les Houches »,

<http://lpm2c.grenoble.cnrs.fr/Themes/tiggelen/cv.html>

Diffusion Multiple de la Lumière



$$\mathbf{x}_{\text{photon}}(n) = \mathbf{x}_1 + \mathbf{x}_2 + \dots + \mathbf{x}_n$$

$$P(\mathbf{x}_i) = \rho \frac{d\sigma}{d\Omega}(\mathbf{x}_{i-1} \rightarrow \mathbf{x}_i) \times \frac{1}{x_i^2} \exp(-x_i / \ell)$$

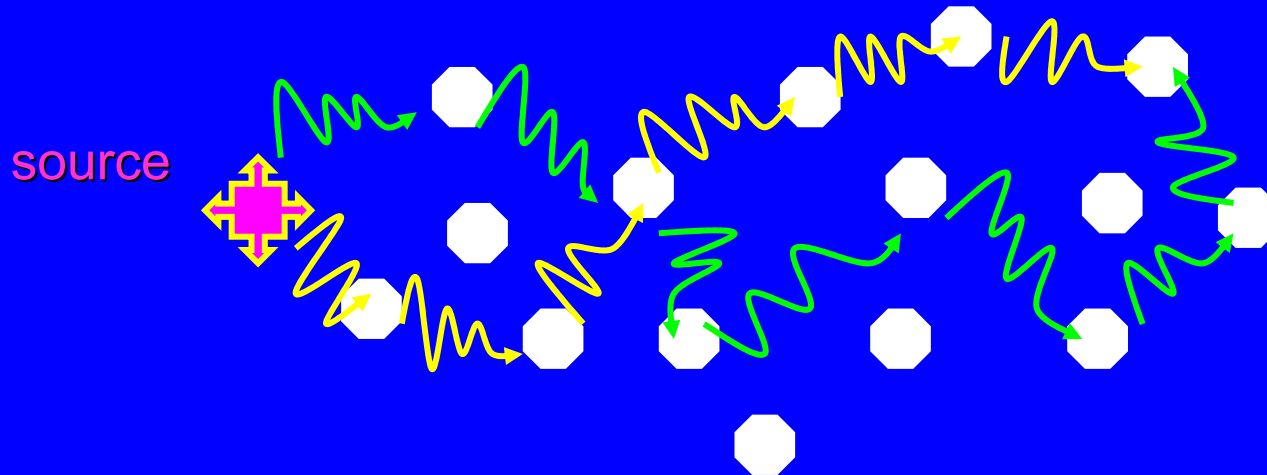
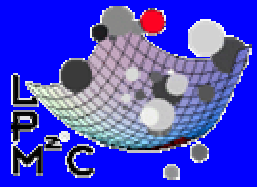
$$\int d^3 \mathbf{x} P(\mathbf{x}) = 1 \quad \Rightarrow$$

$$\ell = \frac{1}{\rho \sigma} = \frac{1}{\rho \int d^2 \Omega \frac{d\sigma}{d\Omega}}$$

Conservation locale
du flux

Longueur d'extinction

Diffusion Multiple de la Lumière



$\rho(\mathbf{r}, t)$ densité d'énergie

$$\partial_t \rho(\mathbf{r}, t) - D \nabla^2 \rho(\mathbf{r}, t) = S \delta(t) \delta(\mathbf{r} - \mathbf{r}_s)$$

$$\int d\mathbf{r} \rho(\mathbf{r}, t) = S \quad \text{Conservation globale du flux}$$

$$\langle \mathbf{r}^2(t) \rangle = \frac{\langle \rho(\mathbf{r}, t) \mathbf{r}^2 \rangle}{\langle \rho(\mathbf{r}, t) \rangle} = 6D t$$

$$D = \frac{1}{3} v_E \ell^*$$

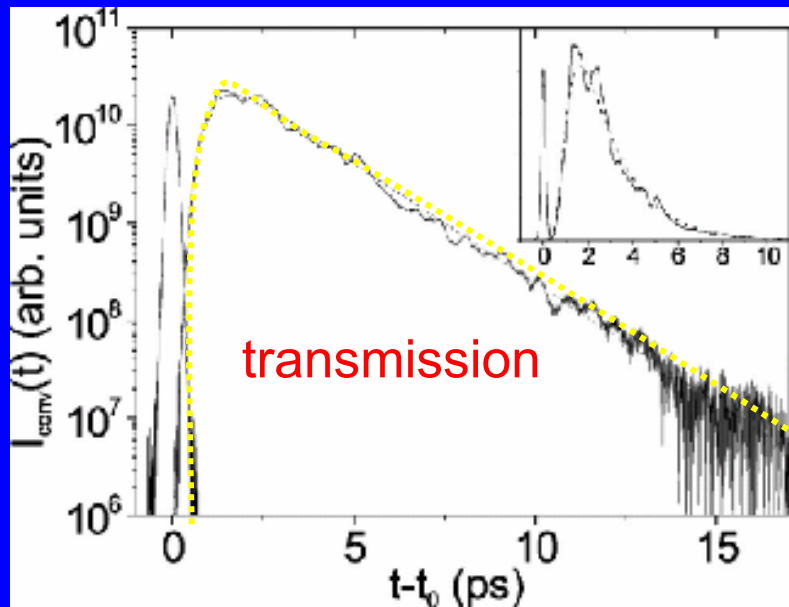
Constante de diffusion

La diffusion des photons, ça marche! **GaP poreux**

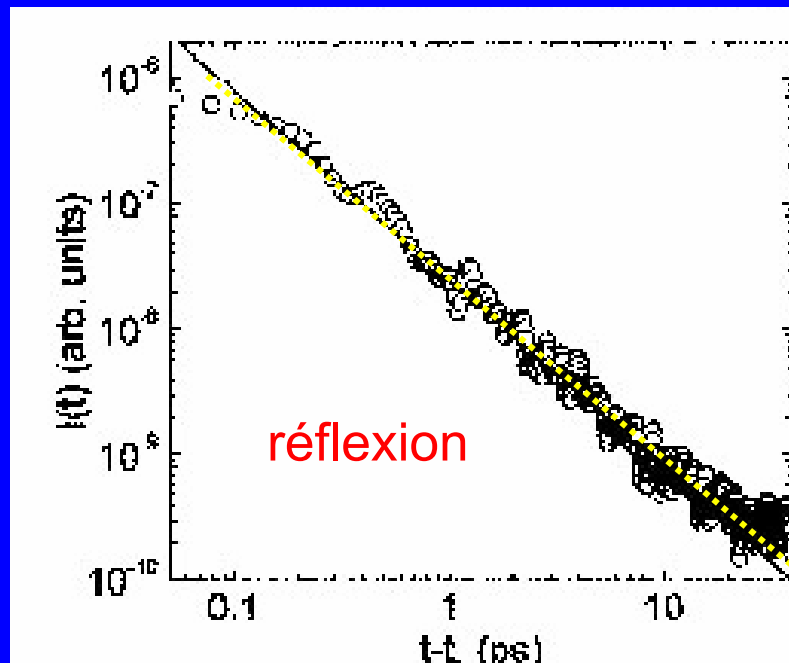
$$L = 20 \mu m$$

$$\lambda = 739 nm$$

Lagendijk et al, PRE 2003



Équation de diffusion

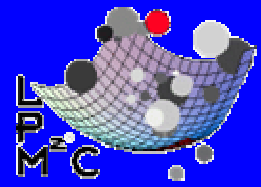


$1/t^{3/2}$

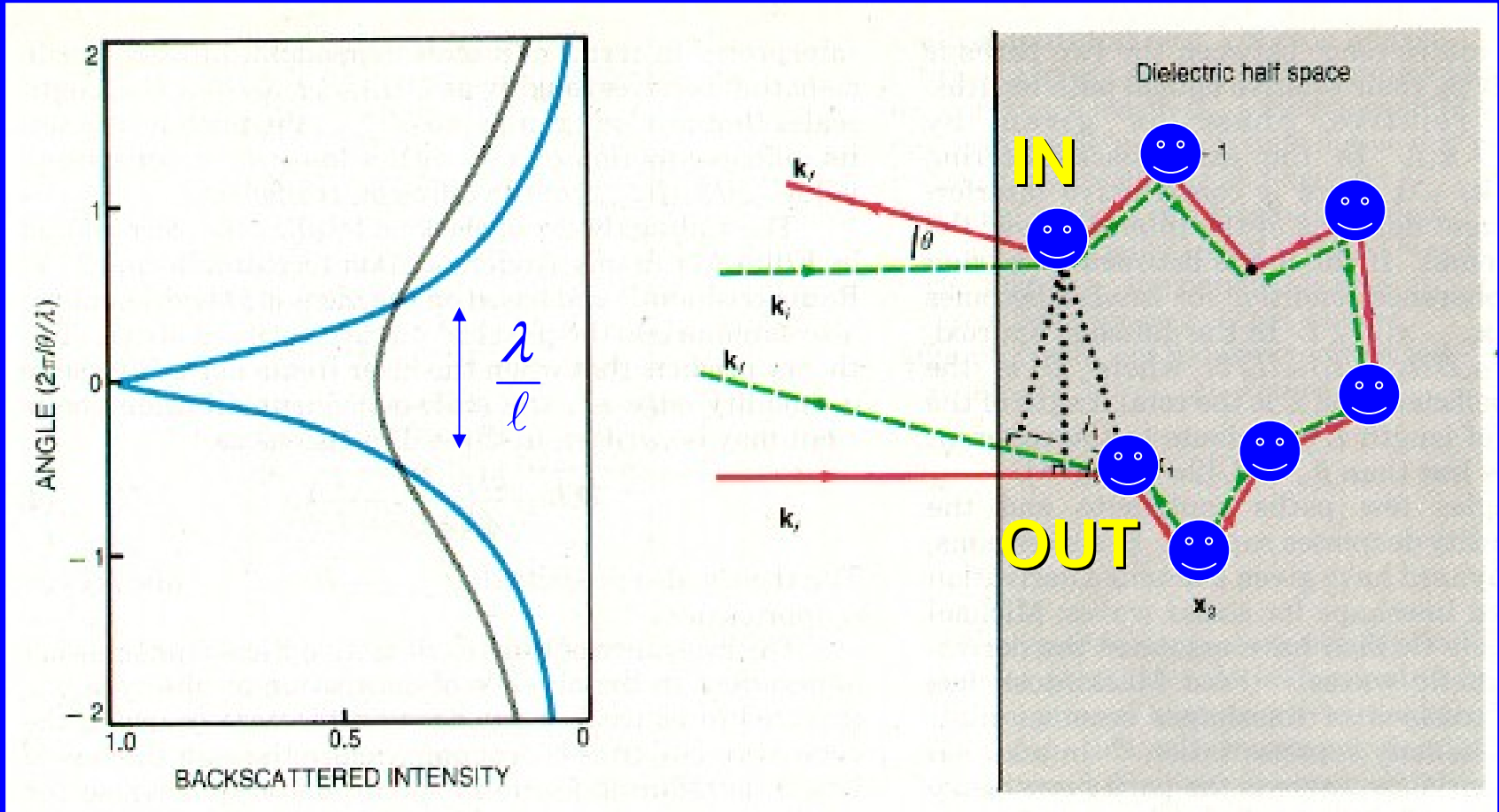
$$D = 23 m^2 / s$$

$$\ell^* = 250 nm \quad (k\ell^* = 2.1)$$

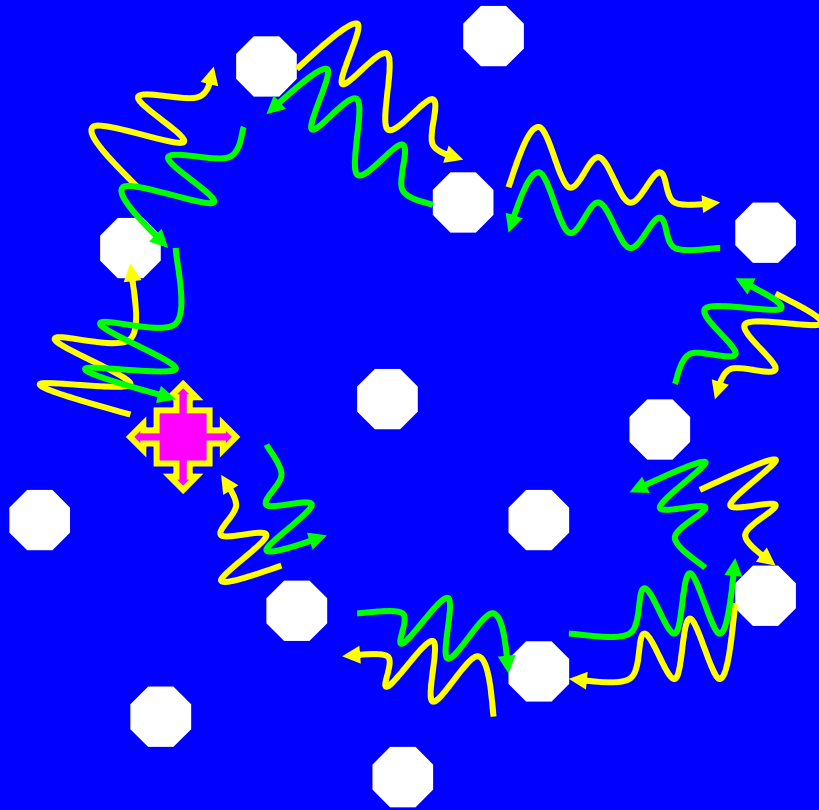
Coherent Backscattering



Maret & Wolf, Maynard, PRL, 1985
 Van Albada & Lagendijk, PRL, 1985



$$\text{CBS} \propto \int_{\text{in, out}} G(\text{IN} \rightarrow \text{OUT}) \exp[i\mathbf{k} \cdot (\mathbf{r}_{\text{in}} - \mathbf{r}_{\text{out}})]$$



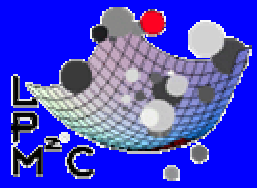
Réciprocité!

Fraction d'énergie qui rentre à la source
au temps t par interférence constructive

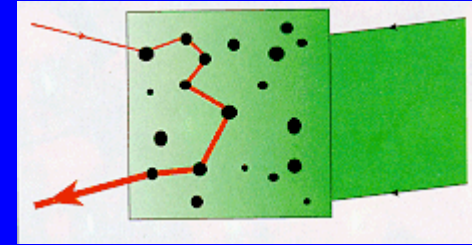
$$v_E dt \left(\frac{\lambda}{2\pi} \right)^{d-1} \times \rho(\mathbf{r} = 0, t) = \begin{cases} = \infty & d = 1, 2 \\ \propto \frac{1}{(kl)^2} & d = 3 \end{cases}$$

Random laser

Milieu désordonné avec du gain

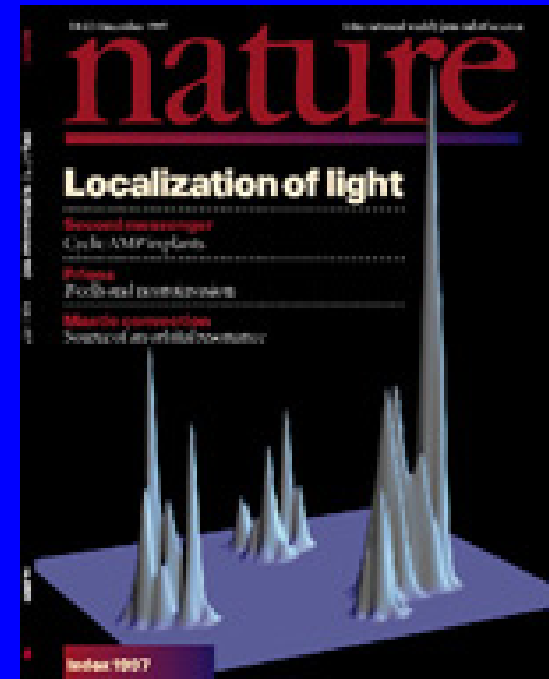


$$kl > 1$$



(photo: Diederik Wiersma)

$$kl < 1$$



C'est quoi alors, la localisation forte??

$D=0 \longrightarrow \langle R^2 \rangle < t$ Absence of diffusion

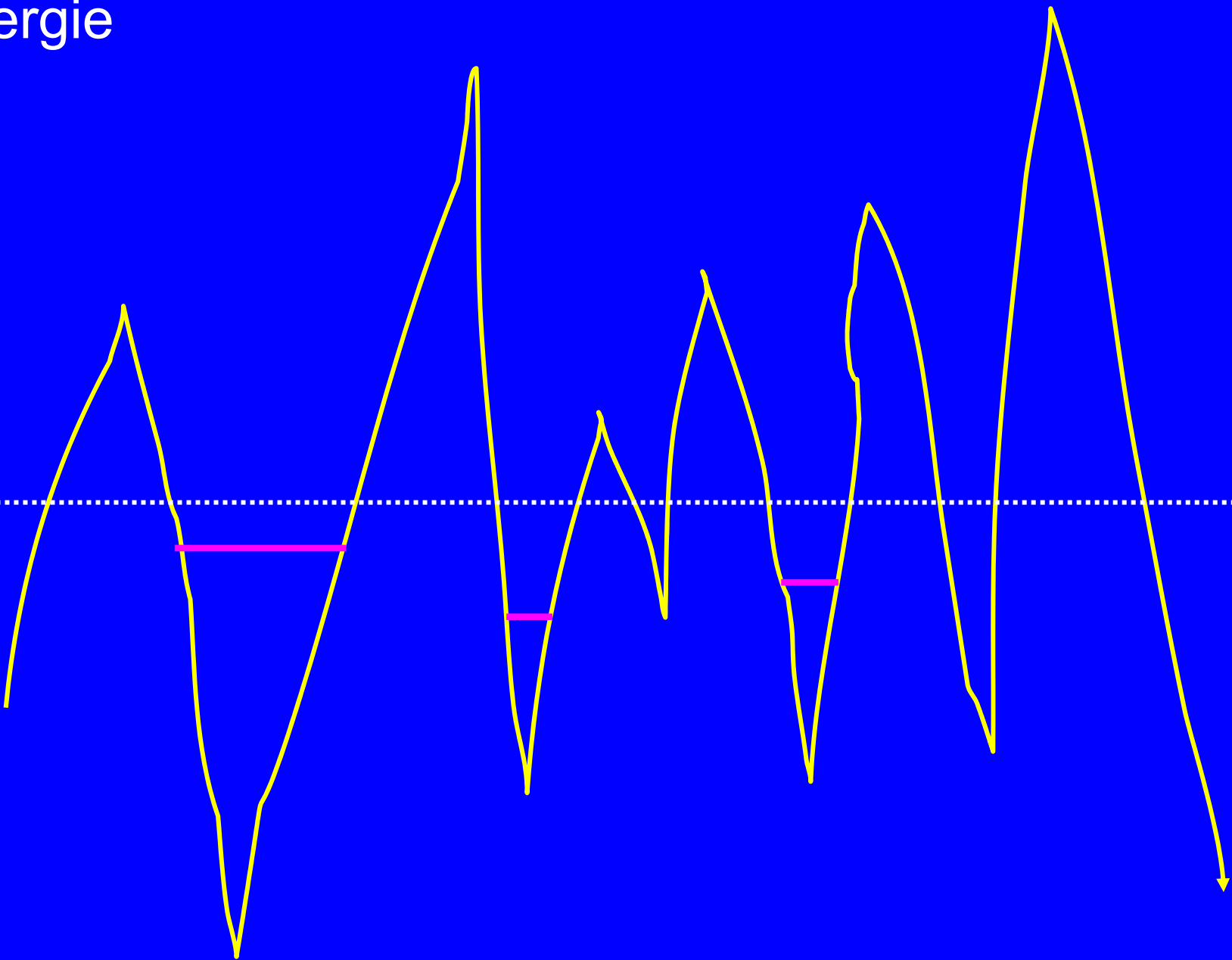
$\langle T \rangle = \exp(-L/\zeta)$ Transmission non-Ohmique

Etats liés avec un spectre dense en énergie ou en fréquence (absence de répulsion de niveau)

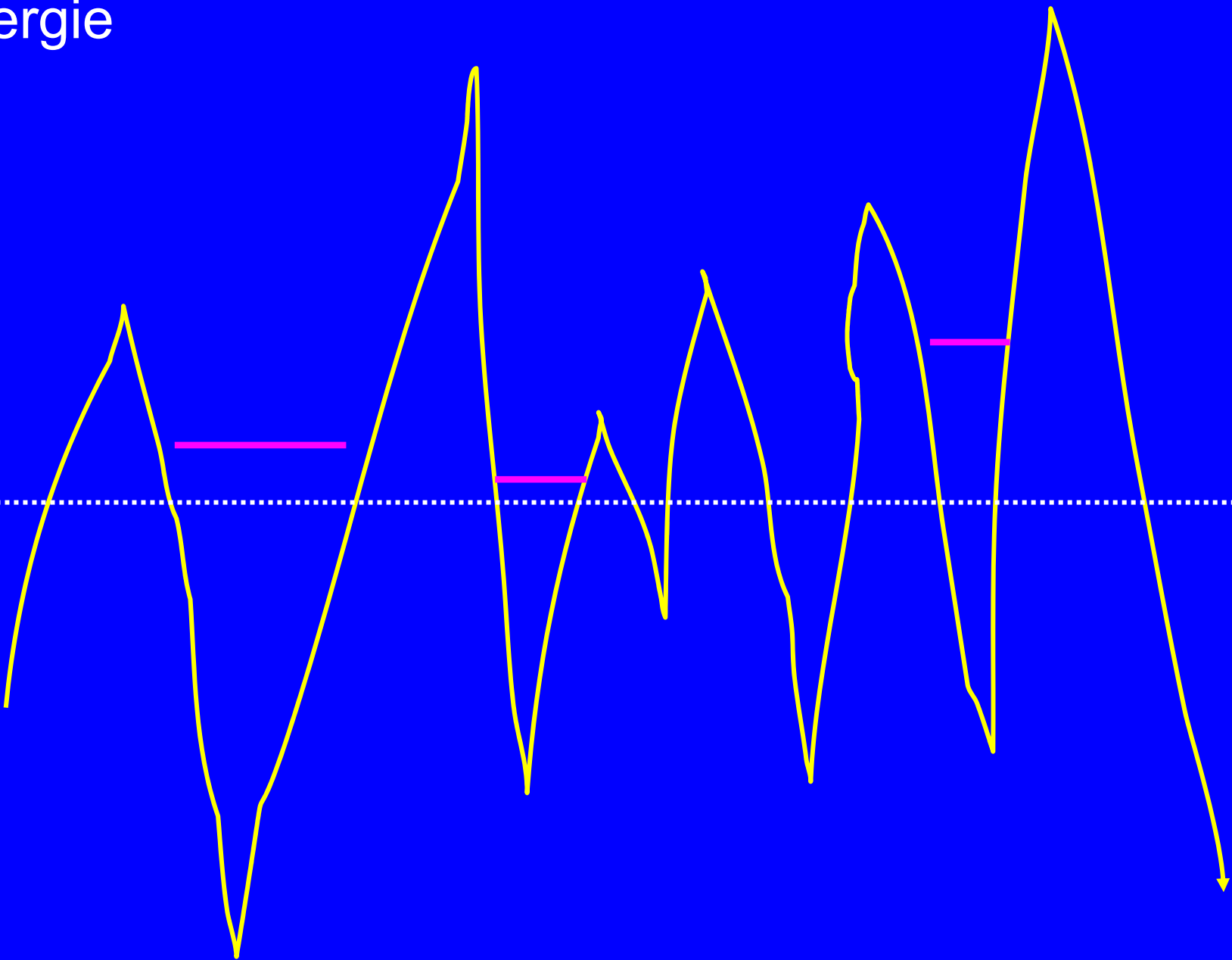
Fluctuations géantes de transmission

Les définitions ne sont pas équivalentes

Energie

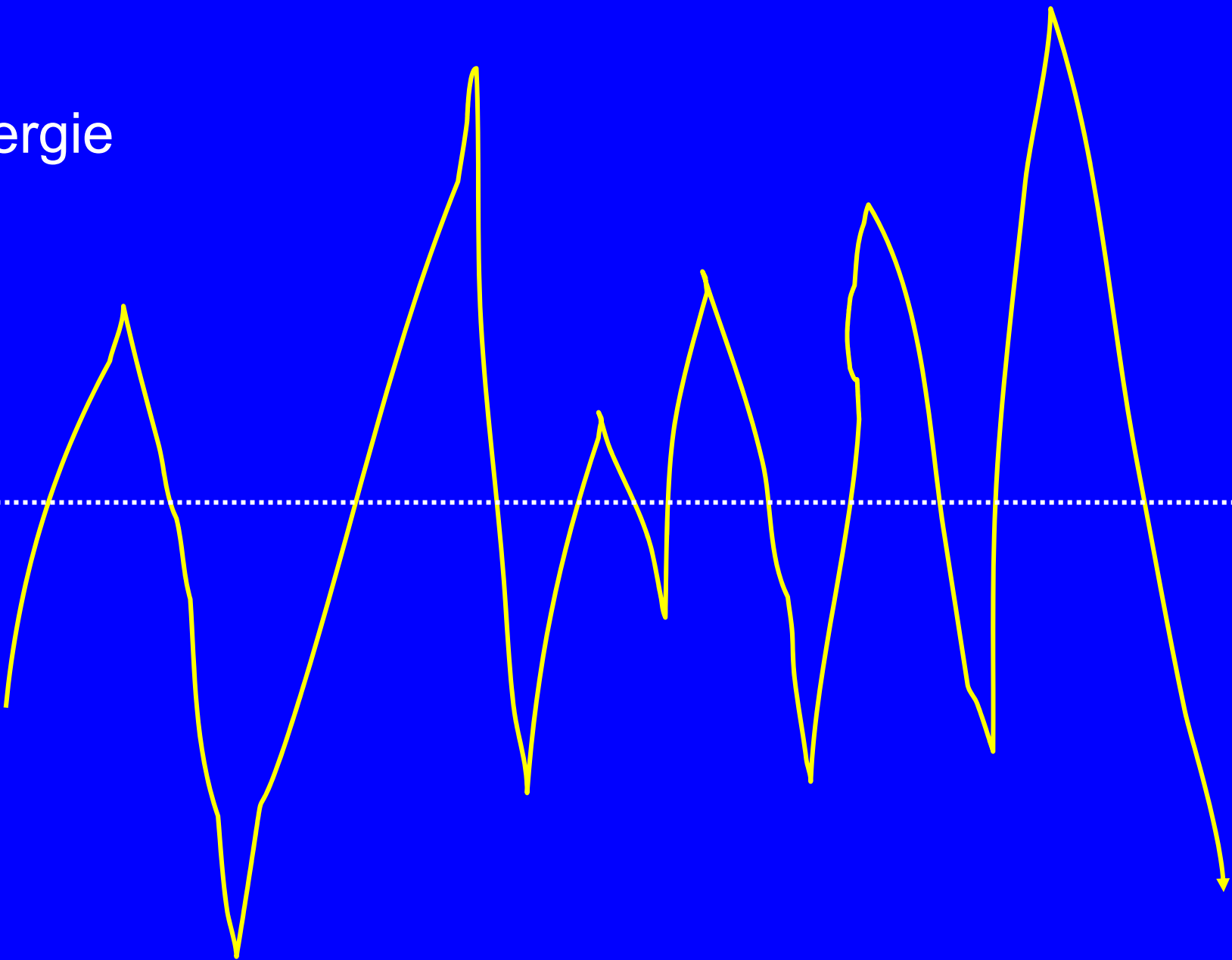


Energie

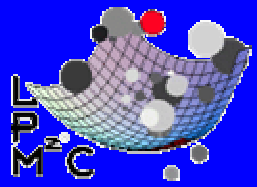




Energie



Description théorique



Théorie des matrices aléatoires

Elégante, exacte et « complète » mais technique, restriction à Q1D
(Beenakker, RMP 1997)

Ab initio

Anderson Tight Binding Model (80-90), Dipôles aléatoires 2000),
grands systèmes difficiles

Théorie des champs

Supersymétrie et Goldstone modes, valable pour toutes les dimensions,
description de la transition, très loin des manip....

Théorie self-consistante

Equation de diffusion généralisée, « simple »,
très proche des manip, théorie « champ moyen »
(Vollhardt & Wölfle, 1980)

Anderson Tight Binding Model 3D

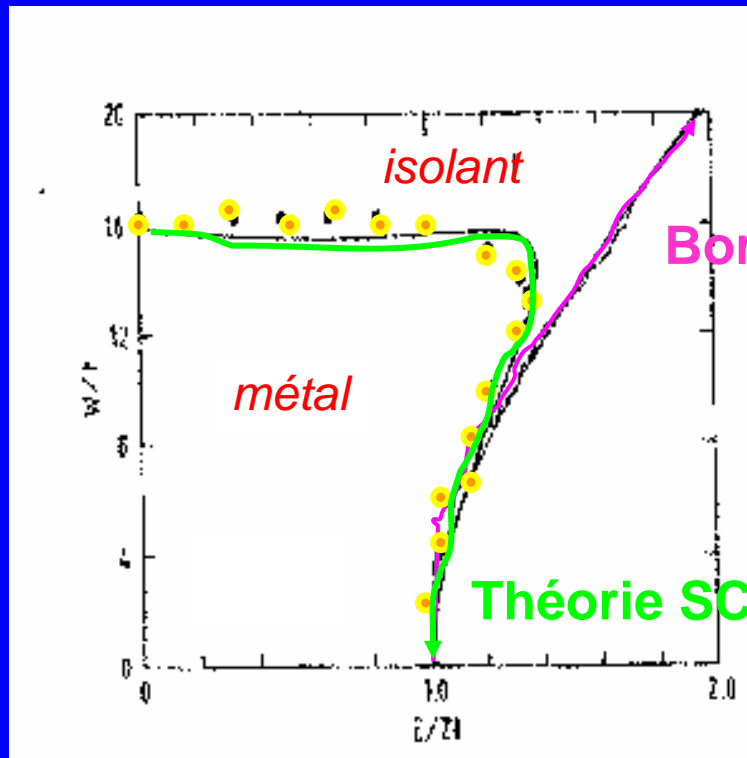
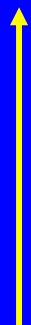
$$H = \sum_{nn'} t |n\rangle\langle n'| + \sum_n V_n |n\rangle\langle n|$$

Hopping ($t=1$)

Désordre:

$$-\frac{W}{2} < V_n < \frac{W}{2} \text{ aléatoire}$$

Désordre
W

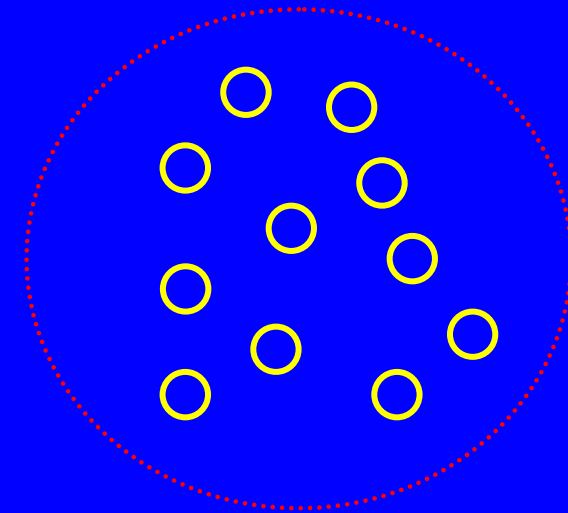
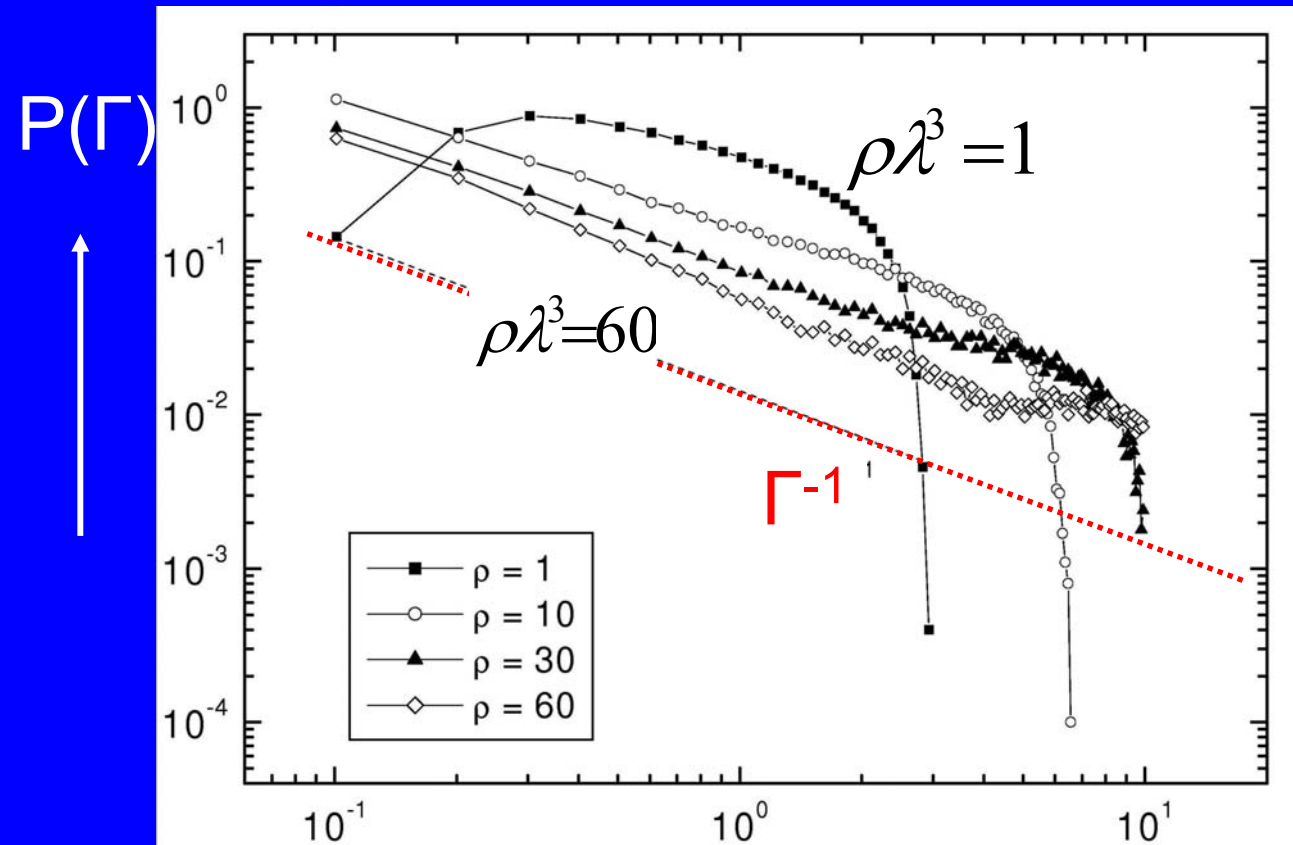


Bord du spectre

→ Energie E

Distribution de taux de fuite! ::

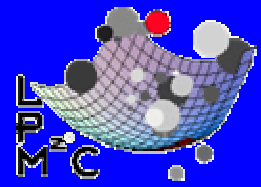
N dipôles résonnantes *ab initio* dans une sphère
Pinheiro/van Tiggelen PRE 2004



$$P(\Gamma) \propto \frac{1}{\Gamma}$$

—————> Fuite Γ / fuite dans le vide

Description théorique



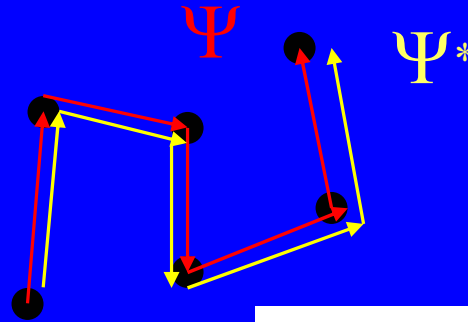
$$kl \gg 1$$

Transfert radiatif

Schwarzschild/Milne, 1900

Chandrasekhar, 1950

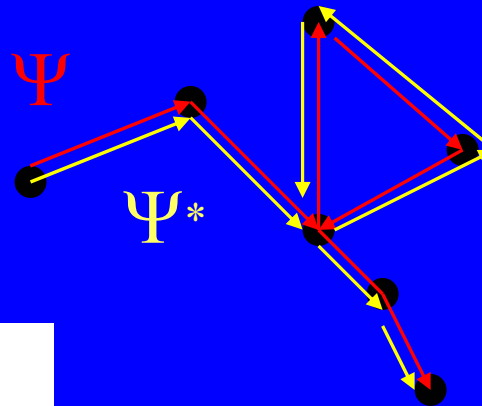
Van de Hulst, 1950



$$D_B = \frac{1}{3} v_E l^*$$

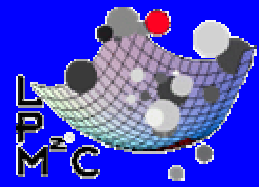
$$kl \approx 1$$

Théorie self-consistante

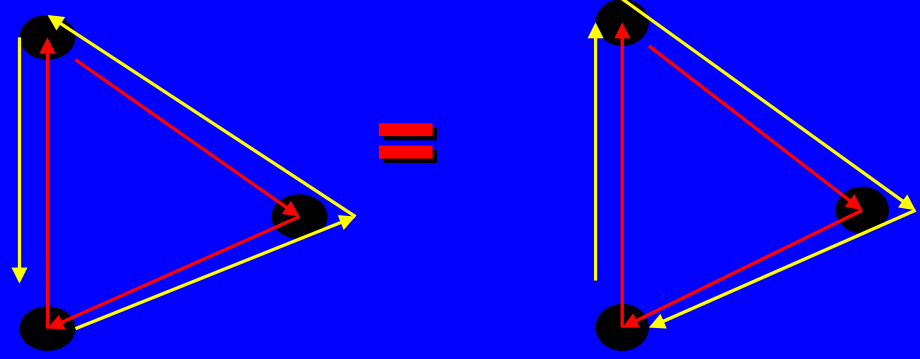


$$\frac{1}{D} = \frac{1}{D_B} + \frac{C(\mathbf{r}, \mathbf{r})}{\pi v_E N(\omega)}$$

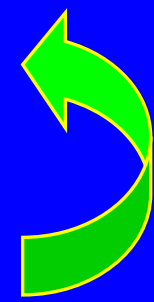
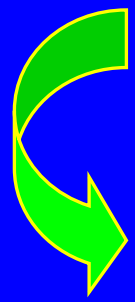
Vollhardt & Wölfle, 1980

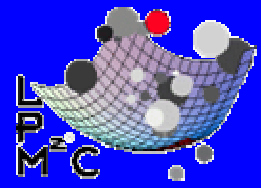


réciprocité $\Rightarrow C(\mathbf{r},\mathbf{r}) = G(\mathbf{r},\mathbf{r})$



$$-\nabla \cdot D(\mathbf{r}) \nabla G(\mathbf{r}, \mathbf{r}') = \frac{4\pi}{\ell} \delta(\mathbf{r} - \mathbf{r}')$$
$$\frac{1}{D(\mathbf{r})} = \frac{1}{D_B} + \frac{G(\mathbf{r}, \mathbf{r})}{\pi v_E N(\omega)}$$





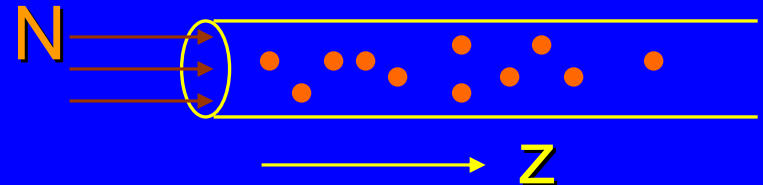
Milieu 3D infini

$$G(\mathbf{r}, \mathbf{r}') = G(\mathbf{r} - \mathbf{r}'); \quad G(\mathbf{r}, \mathbf{r}) = G(0) = \frac{4\pi}{\ell} \int_{q < 1/\ell} d^3\mathbf{q} \frac{1}{Dq^2}$$

$$\Rightarrow D = D_B \left(1 - \frac{1}{(k\ell)^2} \right)$$

OK Mott

Demi espace Quasi-1D



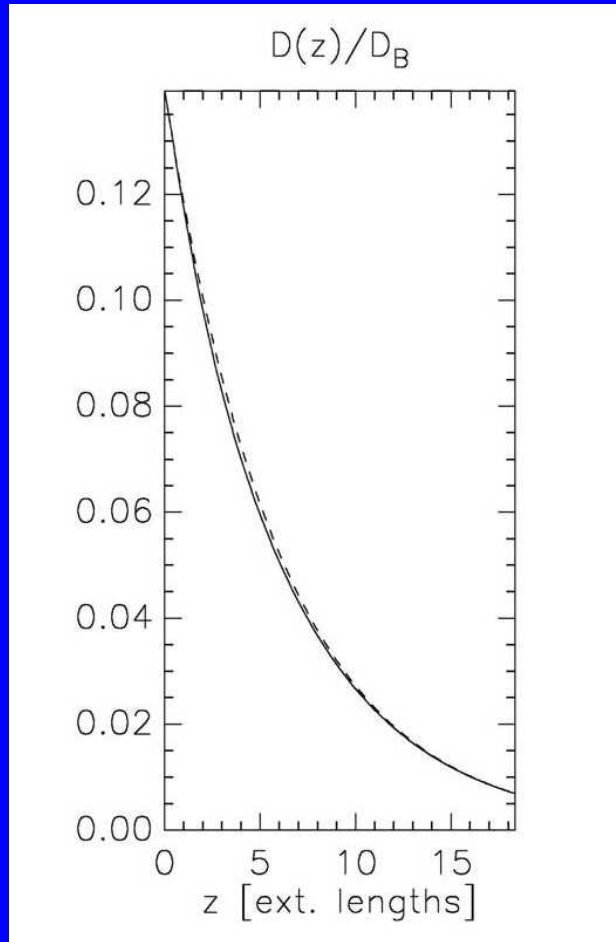
$$d\tau \equiv \frac{dz}{D(z)} \Rightarrow -\partial_z^2 G(\tau, \tau') = \frac{4\pi}{\ell} \delta(\tau - \tau') \Rightarrow G(\tau, \tau) = \frac{4\pi}{\ell} \tau$$

$$\Rightarrow \frac{d\tau}{dz} := \frac{1}{D} = \frac{1}{D_B} + \frac{2}{\xi} \tau \Rightarrow D(z) = D_B \exp\left(-\frac{2z}{\xi}\right)$$

$(\xi \propto N\ell)$

OK RMT

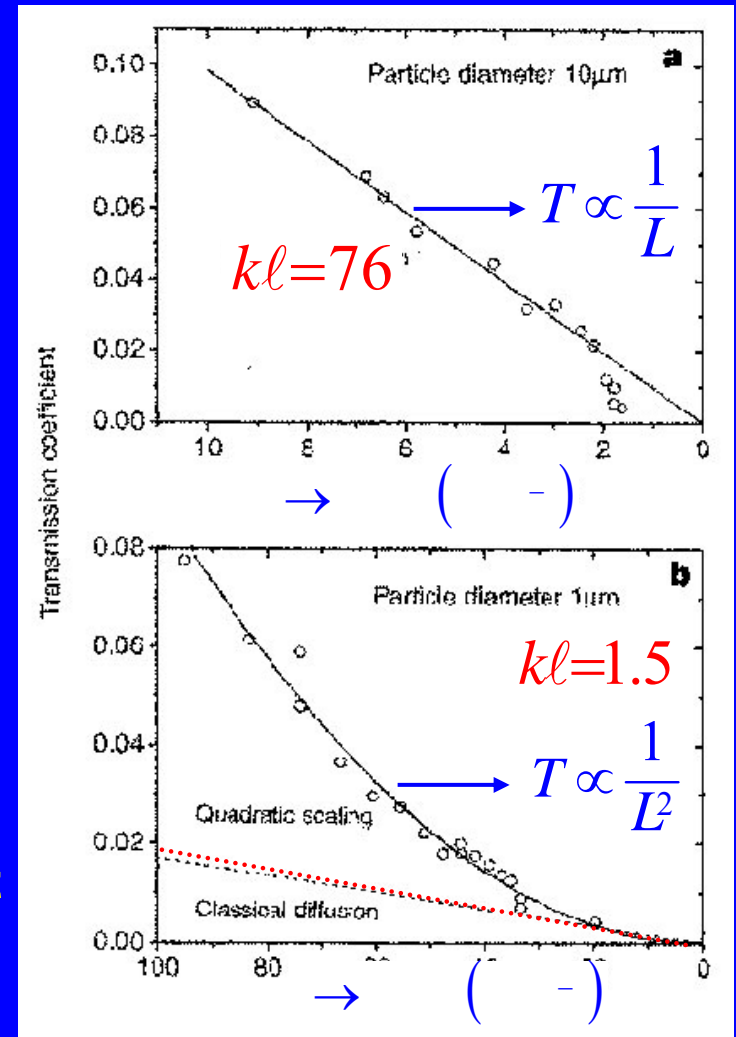
Demie espace 3D prédiction théorique $k\ell=1$



**Loi de Ohm
Conducteur
photonique**

**Isolant
photonique?
ou absorbant
photonique**

Transmission expérimentale Wiersma, Lagendijk, Nature 1997



$$D(z) = \frac{D(0)}{1+z\zeta} \Rightarrow T \propto \frac{1}{L^2}$$

Van Tiggelen, Lagendijk, Wiersma,
PRL 2000

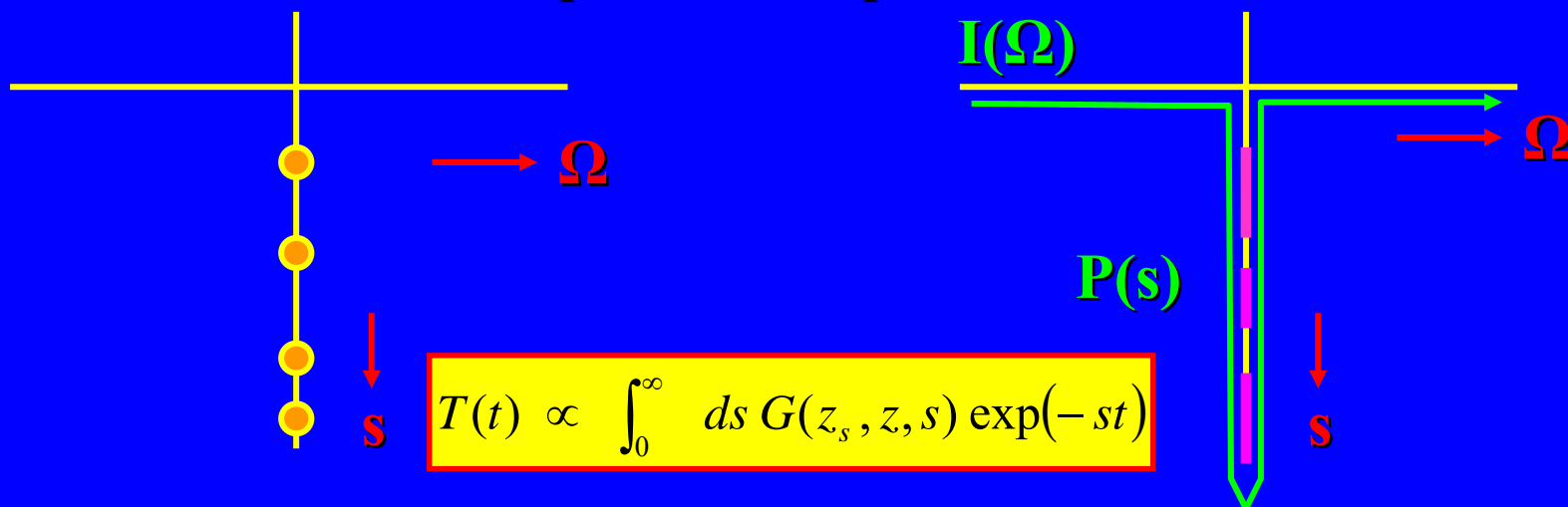
Dynamique de la Localisation

Skipetrov & Van Tiggelen, PRL 2004,2005

$$-\mathbf{s} G(z, z', q, \mathbf{s}) + \partial_z D(z, \mathbf{s}) G(z, z', q, \mathbf{s}) + q^2 G(z, z', q, \mathbf{s}) = \delta(z - z')$$

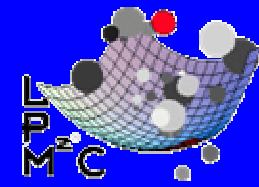
$$\frac{1}{D(z, \mathbf{s})} = \frac{1}{D_B} + \frac{2}{k^2 l} \int_{q < 1/3l} d^2 \mathbf{q} G(z, z, q, \mathbf{s})$$

Fréquence complexe $\Omega + is$



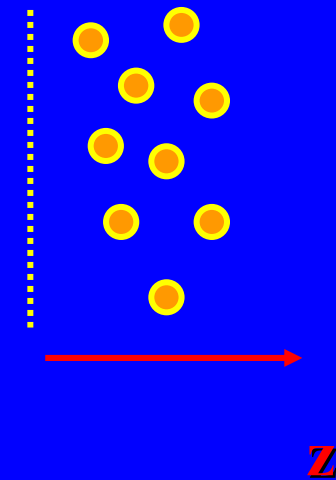
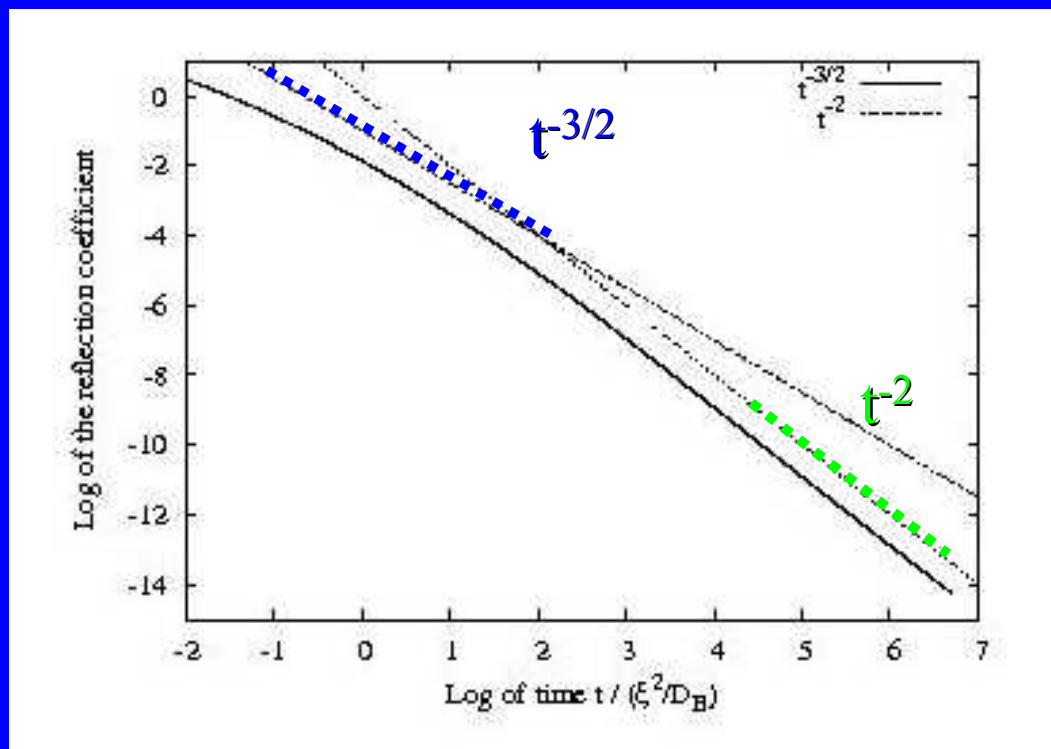
Régime diffus: pôles simples

Régime localisé: coupures de Riemann



3D Demie espace localisée : $k\ell=0.7$

$R(t)$



time/ (ζ^2 / D_B)

$$R(t) \propto \frac{1}{t^2}$$

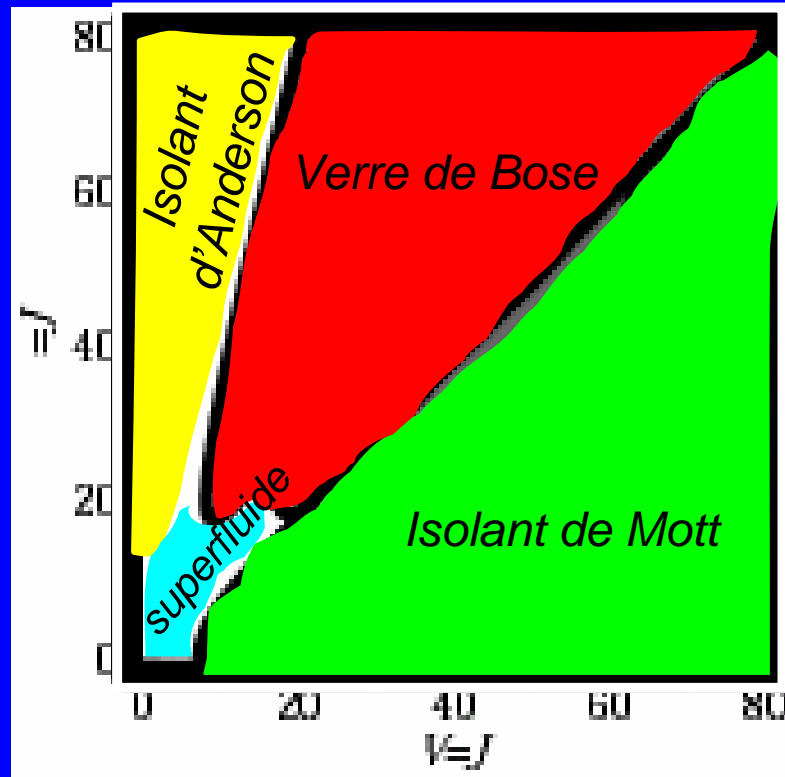
1D sismologie : Sheng Papanicolaou, 1987
Q1D (RMT) Titov, Beenakker, 2000

Les perspectives

Localisation of Cold Bosons in Disordered Optical Lattices

Roth & Burnett, 2002

désordre



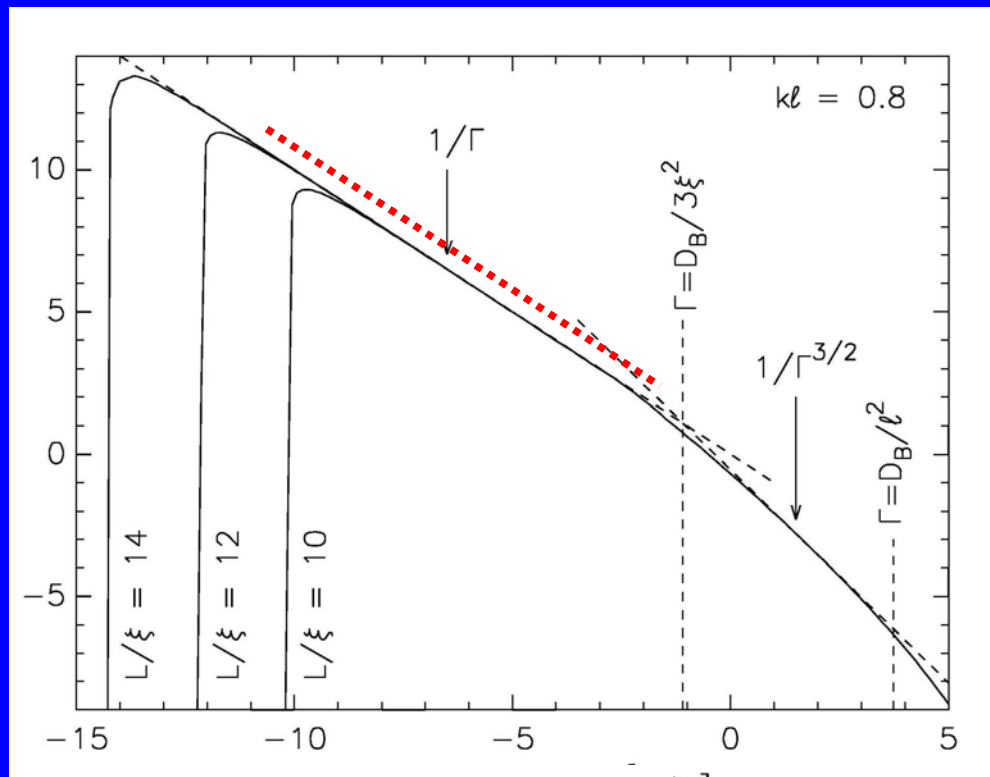
interaction

Prédiction distribution des taux de fuites

$$\frac{dW}{dt} = -R(t, \mathbf{r}_s) - T(t, \mathbf{r}_s)$$

$$\langle W(t, \mathbf{r}_s) \rangle_{\mathbf{r}_s} = \int d\Gamma P(\Gamma) \exp(-\Gamma t)$$

$P(\Gamma)$



Fuite Γ