Analogies between Quantum Waves and Classical Waves :

deceiving, surprising, and complementary……

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abstract

- Mesoscopic physics
- Some mesoscopic concepts with light
- Phase of ultrasound
- Optical vortices
- Mesoscopic seismology

CLASSICAL	QUANTUM
$c^2(\mathbf{r})\partial_t^2\Psi-\nabla^2\Psi=S(\mathbf{r},t)$	$i\partial_t\Psi-\nabla^2\Psi+V(\mathbf{r})\Psi=0$
Ψ measurable	$\lvert\Psi\rvert^2$ measurable
Energy ω^2	Energy E
Potential $[1-\varepsilon(\mathbf{r})]\,\omega^2$	Potential $V(\mathbf{r})$
Energy $\frac{1}{2}\varepsilon(\partial_t\Psi)^2+\frac{1}{2}(\nabla\Psi)^2$	Probability $\lvert\Psi\rvert^2$
Noise + absorption	decoherence
Product space $\mathrm{H}_1\otimes\mathrm{H}_2$	Sum Space $H_1\oplus H_2$
Source S, no mass, no charge	No source, mass, charge

Mesoscopic criterion

$$\bar{\tau} \quad < \quad \Delta T(L) \quad < \quad \tau^{\max} = \begin{cases} \tau^{\text{abs}} \quad \log \dfrac{\text{source}}{\text{noise}} \\ \tau_{\phi} \end{cases}$$

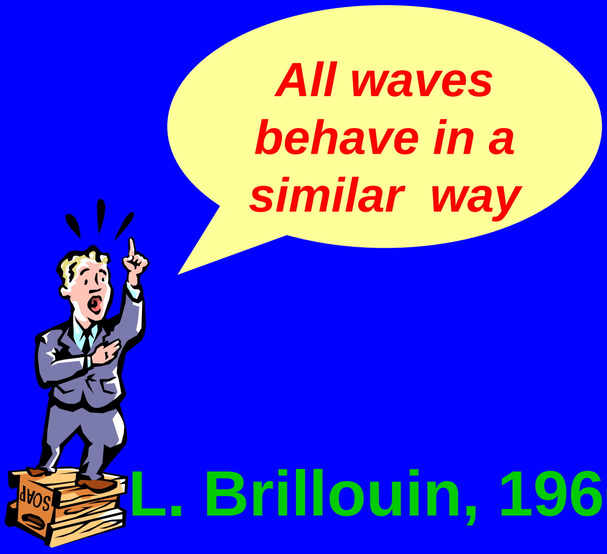
$\Delta T(L) \approx \dfrac{L^2}{2dD}$: Diffusion time $= \dfrac{\hbar}{E_{\text{Thouless}}}$

$D \propto \dfrac{1}{d} \mathrm{v} \ell$ Diffusion constant

$\ell \equiv \bar{\tau}\, \mathrm{v}$ Mean free path

$\sqrt{D\tau^{\max}} \equiv L_{\max}$ Absorption length /decoherence time

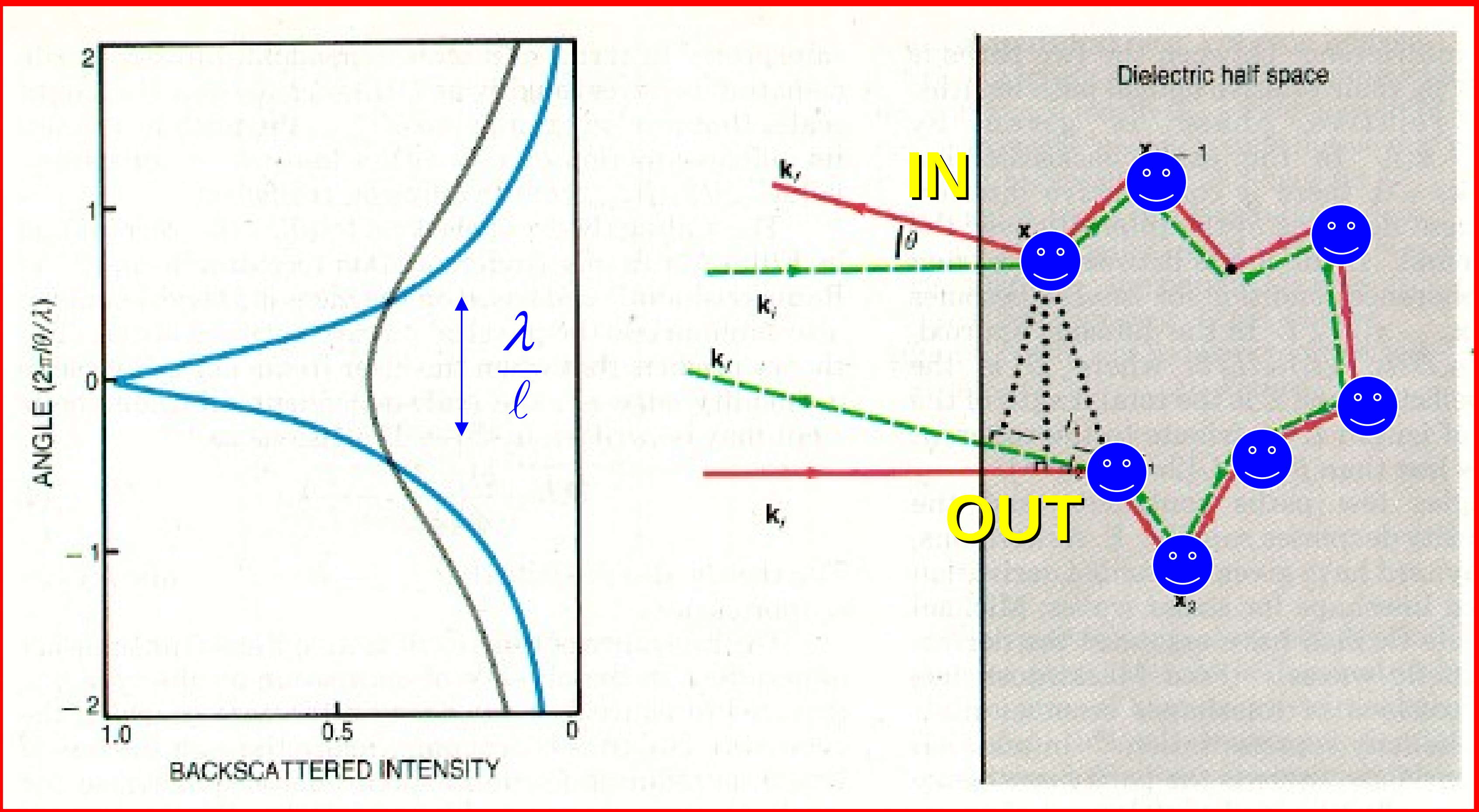
$$\ell \quad < \quad L \quad < \quad L_{\max}$$



L. Brillouin, 1960

Coherent Backscattering in Optics

Maret, Maynard, Akkermans & Wolf, Grenoble , PRL, 1985
Van Albada & Lagendijk, Amsterdam, PRL, 1985



$$\mathrm{CBS} \propto \int_{\mathrm{in,out}} \mathrm{G}(\mathrm{IN} \rightarrow \mathrm{OUT}) \exp\left[i\mathbf{k}\cdot\left(\mathbf{r}_{\mathrm{in}} - \mathbf{r}_{\mathrm{out}}\right)\right]$$

Transport Velocity in Random Media

$$D = \tfrac{1}{3} v_E \ell$$

Electron conduction

Standard literature

$$v_E = \frac{\hbar k_F}{m}$$

Fermi velocity

$$G = \frac{A}{L} \times \frac{4e^2}{3m} k_F^2 \ell$$

Electrical conductance

Transport Velocity in Random Media

$$D = \tfrac{1}{3} v_E \ell$$

Classical waves

Van Albada, Van Tiggelen, Lagendijk , Tip, 1990

$$v_E = \frac{\omega}{k} \frac{1}{1 + \phi'(\omega) / \bar{\tau}}$$

$$T = \frac{4}{3} A k^2 \frac{\ell}{L}$$

Not group velocity not phase velocity

Diffuse transmission

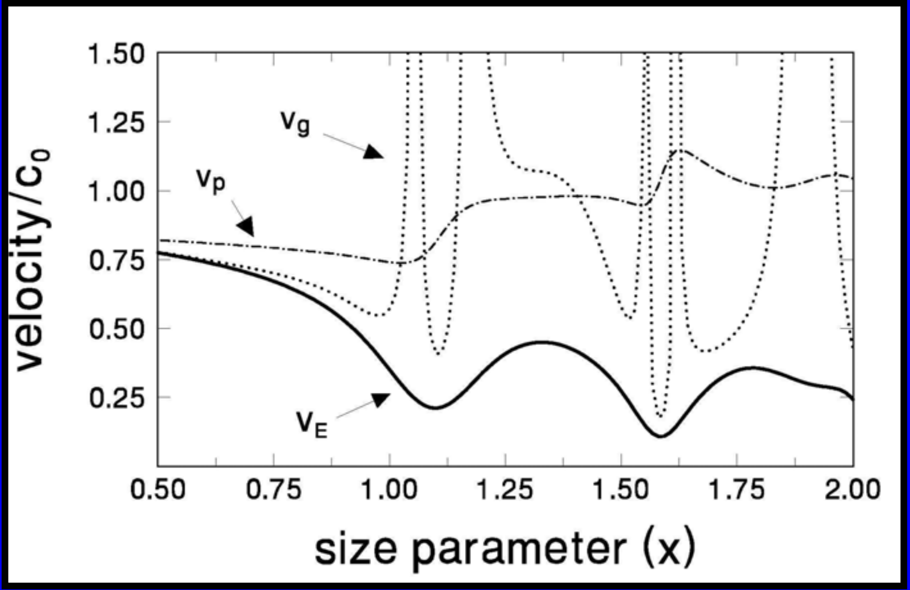
Transport Velocity in Random Media

$$D = \tfrac{1}{3} v_E \ell$$

Classical waves

Van Albada, Van Tiggelen, Lagendijk , Tip, 1990

$$v_E = \frac{\omega}{k} \frac{1}{1 + \phi'(\omega) / \bar{\tau}}$$



Not group velocity not phase velocity

Random Mie spheres: f= 25 %

Magneto-Optics in Diffuse Media

$$E_{\pm}(\mathbf{r},t) = \exp(-i\omega t + i\mathrm{k}\cdot\mathbf{r})\exp(\pm iV\mathbf{B}\cdot\mathbf{r})$$

Faraday effect

Radiatif transfer is well described by a diffusion equation

$$\mathbf{J} = -\mathbf{D}(\mathbf{B})\cdot\nabla\rho$$

Gradient imposed by parity !

Do photons exhibit a Hall effect ?

$$\mathbf{J} = -D_H\mathbf{B}\times\nabla\rho$$

Yes!

Rikken & Van Tiggelen, Nature (1996)

Can photons diffuse without a gradient in density ?

$$\mathbf{J} = D_{MC}\,\mathbf{B}\rho$$

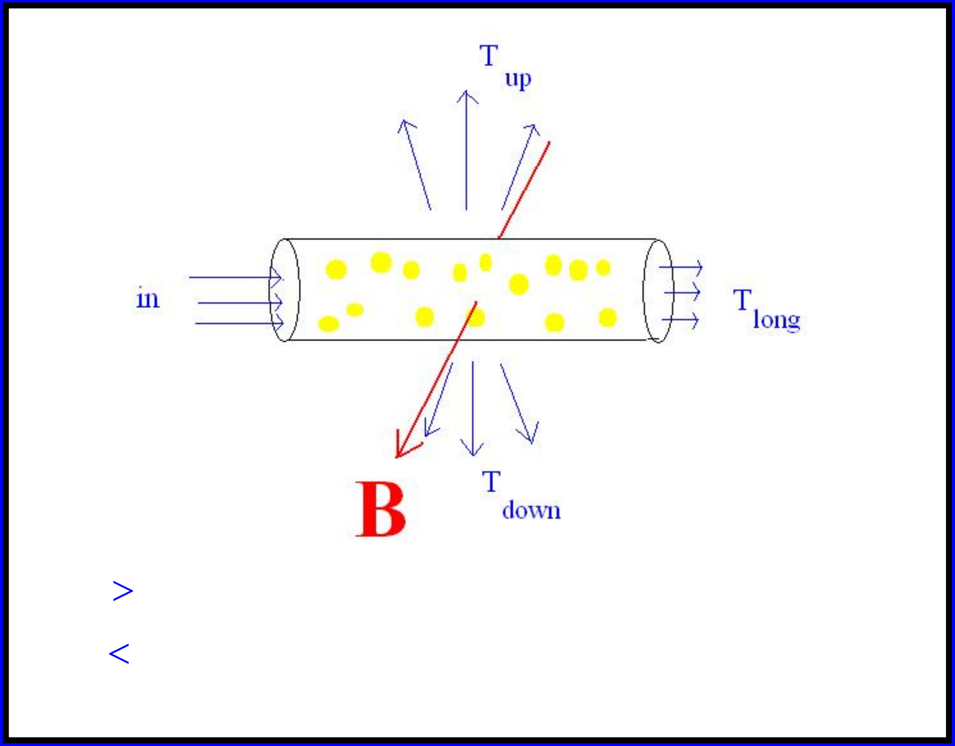
No!

Pinheiro & Van Tiggelen, JOSA A (2002)

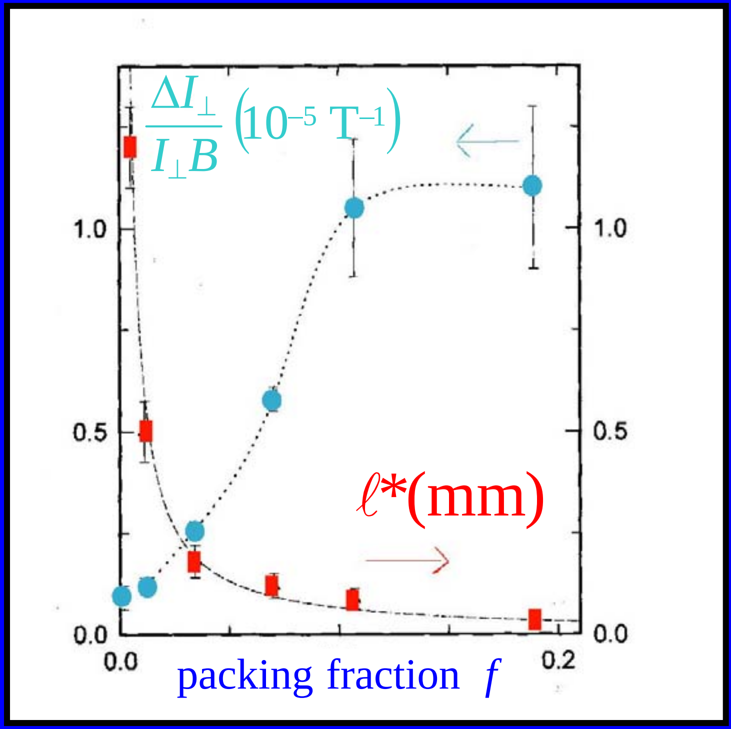
Photonic Hall Effect

Rikken & Van Tiggelen, Nature 381, 54 (1996)

$$\mathbf{J} = -D_H \mathbf{B} \times \nabla \rho$$



PHE of CeF_3 at 77 K



Photonic Spintronics

$$<\delta T_{a\to b}\,\delta\, T_{a'\to b'}> \;=\left|<t_{a\to b}\,t^{*}_{a'\to b'}>\right|^{2} + \frac{1}{g}\,C^{2}_{aa'\to bb'} + \frac{1}{g^{2}}\,C^{3}_{aa'\to bb'}$$

$$g = \frac{4}{3}\frac{N\ell}{L} = \frac{4}{3}\times 2\times\frac{Ak^{2}}{4\pi}\times\frac{\ell}{L}$$

Conductance de Thouless

$$\left|\left\langle t_{a\to b}\,t^{*}_{a'\to b'}\right\rangle\right|^{2} \equiv C^{1}_{aa'\to bb'} = \left\langle T_{a\to b}\right\rangle^{2} \times \delta_{aa'}\delta_{bb'}$$

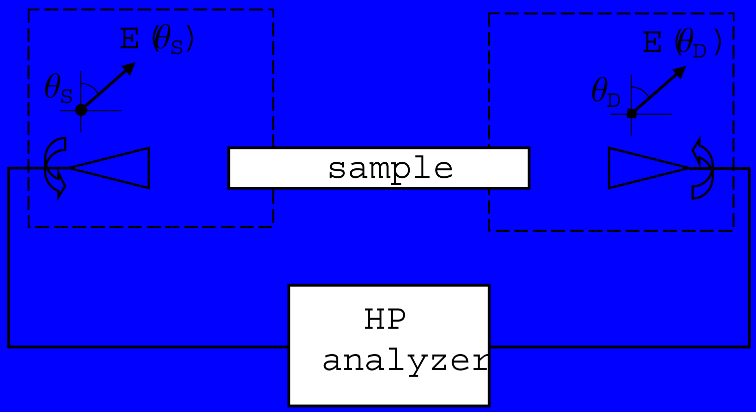
$$C^{2}_{aa'\to bb'} = \left\langle T_{a\to b}\right\rangle^{2} \times \frac{2}{3}\left(\delta_{aa'} + \delta_{bb'}\right)$$

$$C^{3}_{aa'\to bb'} = \left\langle T_{a\to b}\right\rangle^{2} \times \frac{2}{15}$$

Conductance de Landauer : $G = \sum_{ab} T_{a\to b}$

$$\begin{cases} \langle G\rangle = \dfrac{e^{2}}{h} N \left\langle T_{a\to b}\right\rangle = \dfrac{e^{2}}{h}\, g \\ (\Delta G)^{2} = \left(\dfrac{e^{2}}{h}\right)^{2} N^{2} \left\langle T_{a\to b}\right\rangle^{2} \times \dfrac{1}{g^{2}} \times \dfrac{2}{15} \\ \quad = \left(\dfrac{e^{2}}{h}\right)^{2} \dfrac{2}{15} \end{cases}$$

Photonic Spintronics



(Malus' law)

$\delta_{aa'} \rightarrow \cos^2\theta_S$

$\delta_{bb''} \rightarrow \cos^2\theta_D$

$\lambda = 5\,\text{mm}, \ \ell = 5\,\text{cm}, \ L = 1\,\text{m}, \ L_a = 50\ \text{cm}$

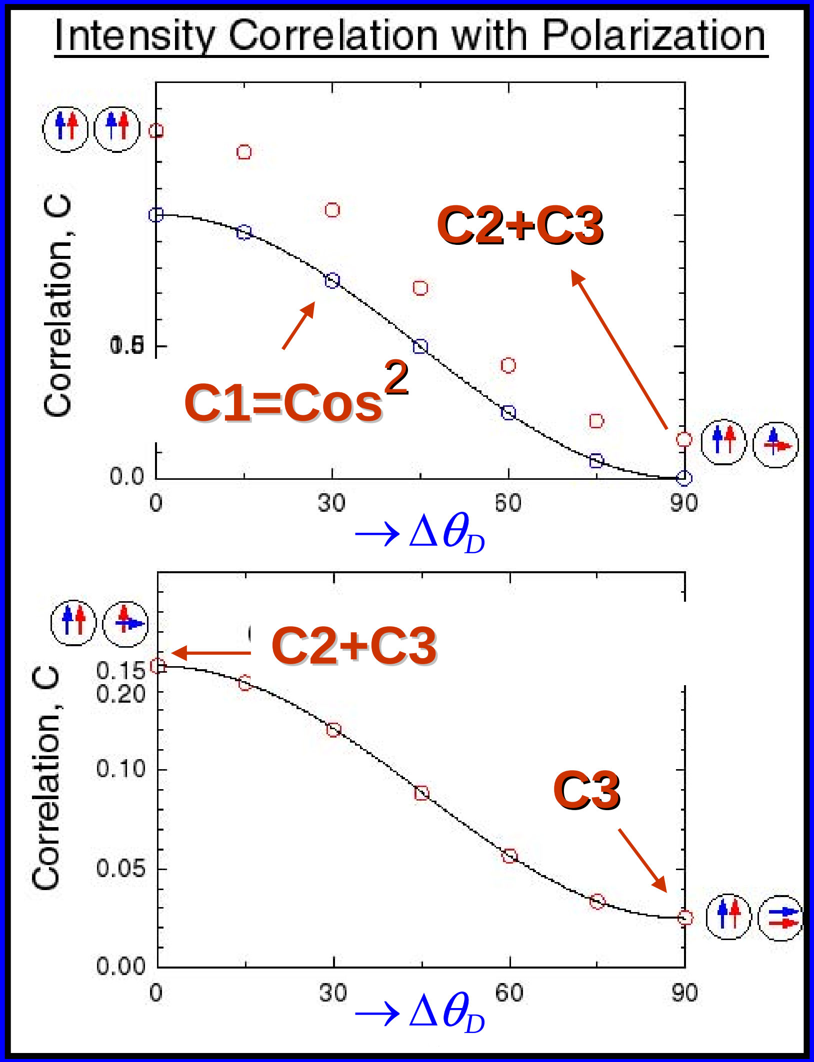
$$g = \frac{N\,\ell}{L} \approx \frac{Ak^2}{2\pi}\,\frac{\ell}{L} = 2.3 \pm 0.05$$

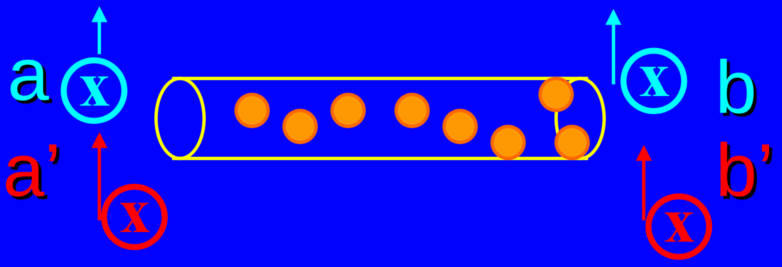
$$\left\langle \delta T(0,0)\ \delta T(\theta_a,\theta_b)\right\rangle = \cos^2\theta_a\ \cos^2\theta_b + \frac{1}{g}\frac{2}{3}\left[\cos^2\theta_a + \cos^2\theta_b\right] + \frac{1}{g^2}\frac{2}{15}$$

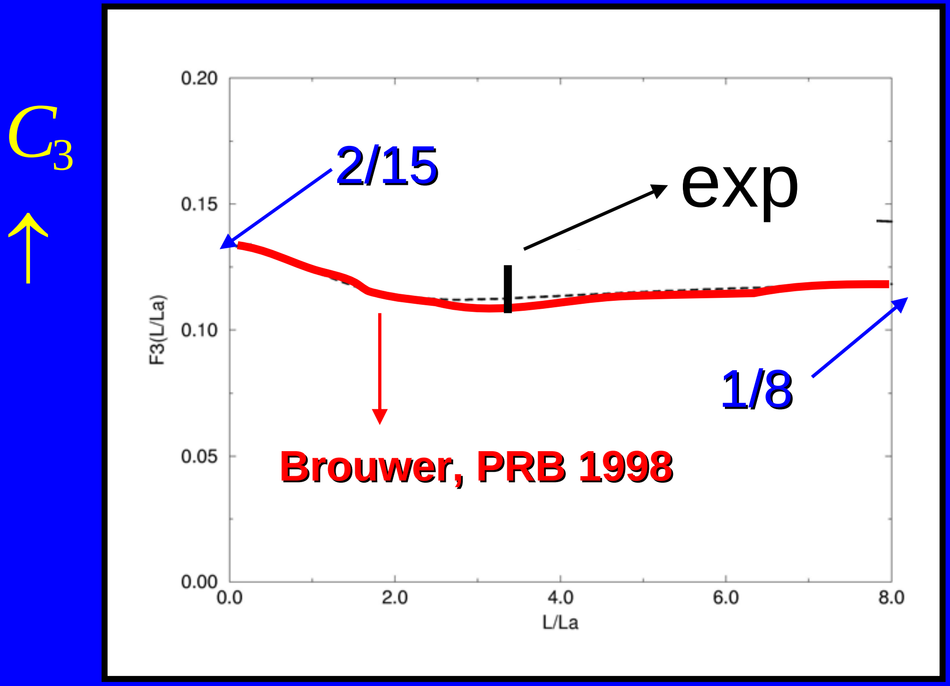
Genack, Chabanov, Trégourès, Van Tiggelen, 2003

Photonic Spintronics

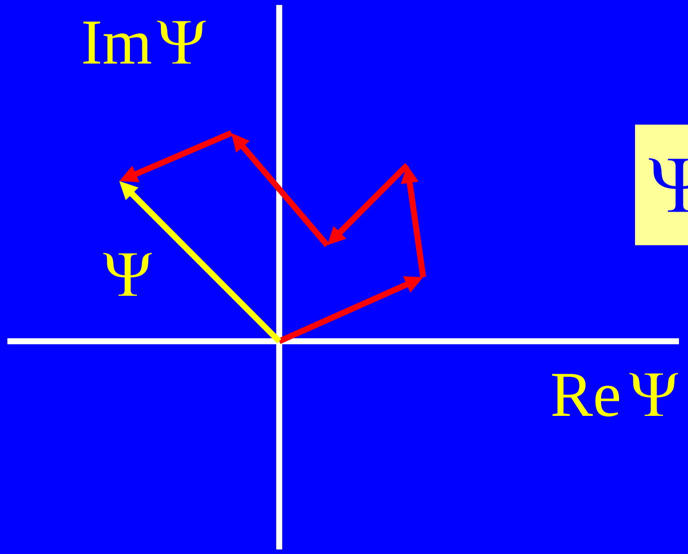
Genack, Chabanov, Trégourès, Van Tiggelen, 2003







$\rightarrow$ absorption L/L_a



$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 + \ldots$$

probability distribution

$$P(\Psi_1, \Psi_2, \ldots \Psi_N) = \frac{1}{\pi^N \det \mathbf{C}} \exp\left(-\boldsymbol{\Psi}^* \cdot \mathbf{C}^{-1} \cdot \boldsymbol{\Psi}\right) \qquad C_{ij} \equiv <\Psi_i \Psi_j^*>$$

diffusion equation

Gaussian Speckles

$$\Psi = \sqrt{I}\ \mathrm{e}^{\,i\phi}$$

→ intensity

→ phase

1. **Stationary**: Distribution of speckle intensity

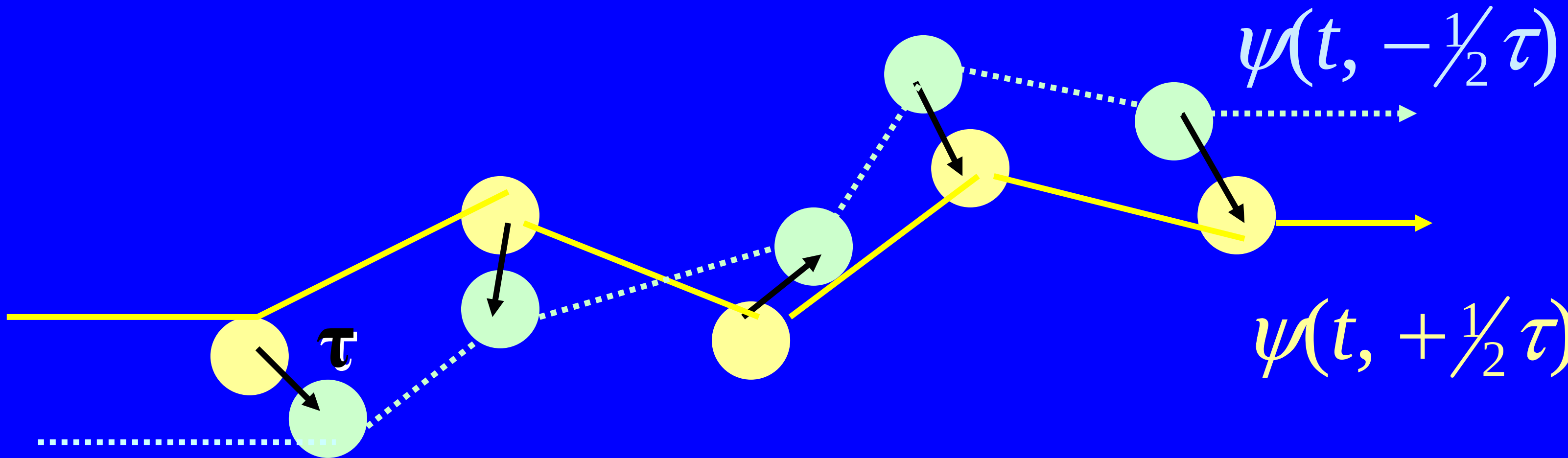
$$P(I,\phi) = \frac{1}{<I>}\exp\left(-I/<I>\right)$$

2. **Dynamics** : Distribution of « Wigner delay » time $\dfrac{\mathrm{d}\phi}{\mathrm{d}\omega}$

$$P\left[\Psi\left(\omega-\frac{\Omega}{2}\right),\ \Psi\left(\omega+\frac{\Omega}{2}\right)\right] = \frac{1}{\pi^2 \det \mathbf{C}}\exp\left(-\mathbf{\Psi}^*\cdot\mathbf{C}(\Omega)^{-1}\cdot\mathbf{\Psi}\right)$$

$$\Rightarrow\quad P\left(\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \phi'\right) = \frac{Q}{2}\ \frac{1}{\left[Q + \left(\hat{\phi}' - 1\right)^2\right]^{3/2}}$$

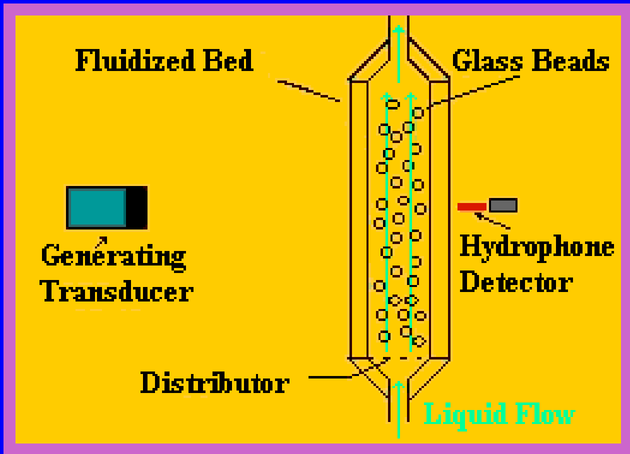
Diffuse Acoustic Wave Spectroscopy



$$n = \frac{ct}{\ell^*}$$

$$\frac{\left\langle \psi\left(t, -\tfrac{1}{2}\tau\right)\psi\left(t, +\tfrac{1}{2}\tau\right)\right\rangle}{\left\langle \psi(t)^2 \right\rangle} = g(\tau) = \exp\left(-\tfrac{1}{6} k^2 n \left\langle \Delta\mathbf{r}^2(\tau) \right\rangle\right)$$

$$g(\tau) \approx \exp\left(-\frac{1}{6}\frac{\tau^2}{t_{\mathrm{DAWS}}^2}\right)$$



Diffuse Acoustic wave Spectroscopy

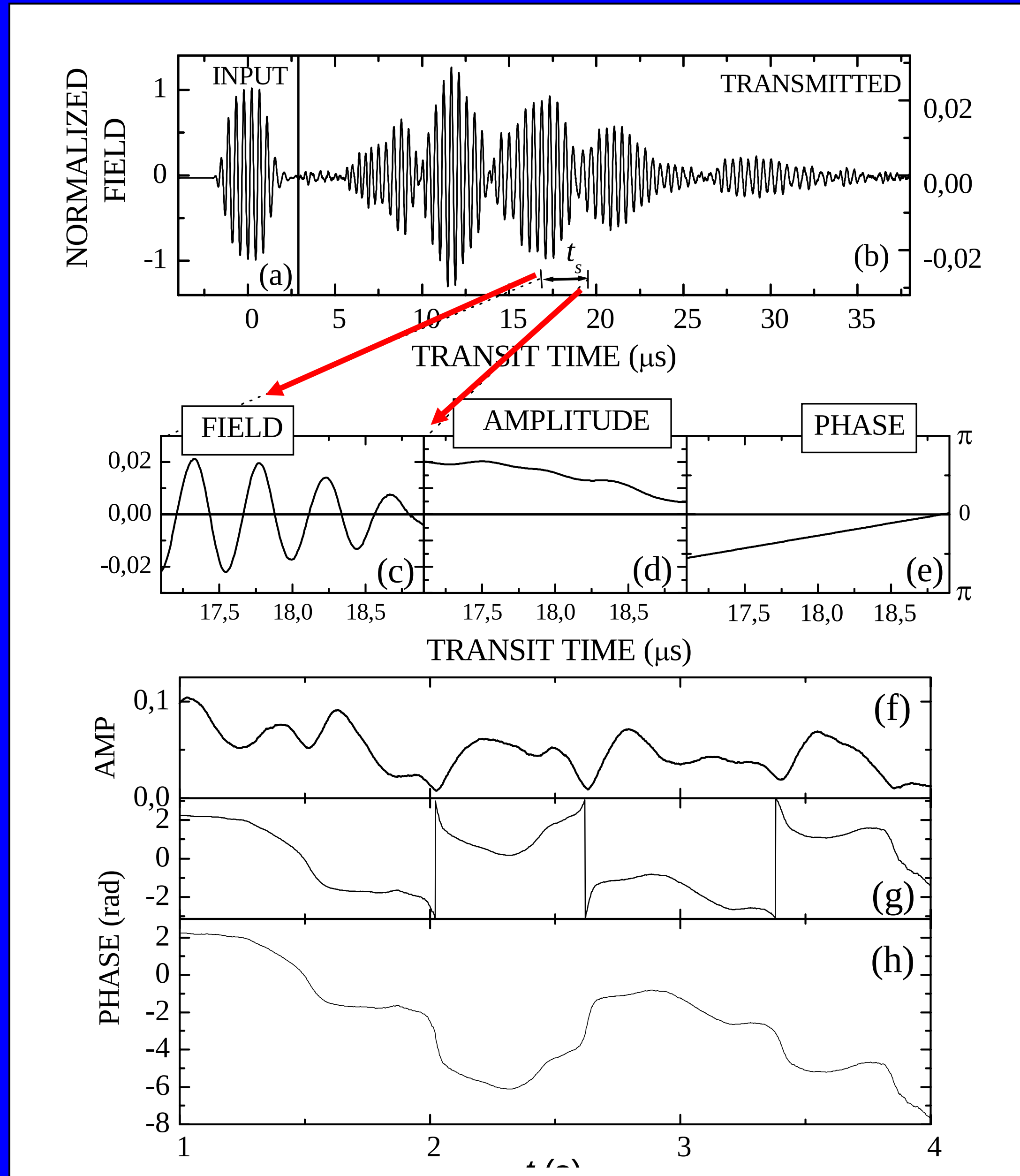
John Page, Dave Weitz,
Michael Cowan

amplitude →

Wrapped phase →

unwrapped phase →

$\ell^* = 1.5\,mm; \quad \tau^* = 1\,\mu s$



Time (seconds!)

Probability distribution $P(\Delta\Phi)$

for phase shift $\Delta\Phi(\tau)$

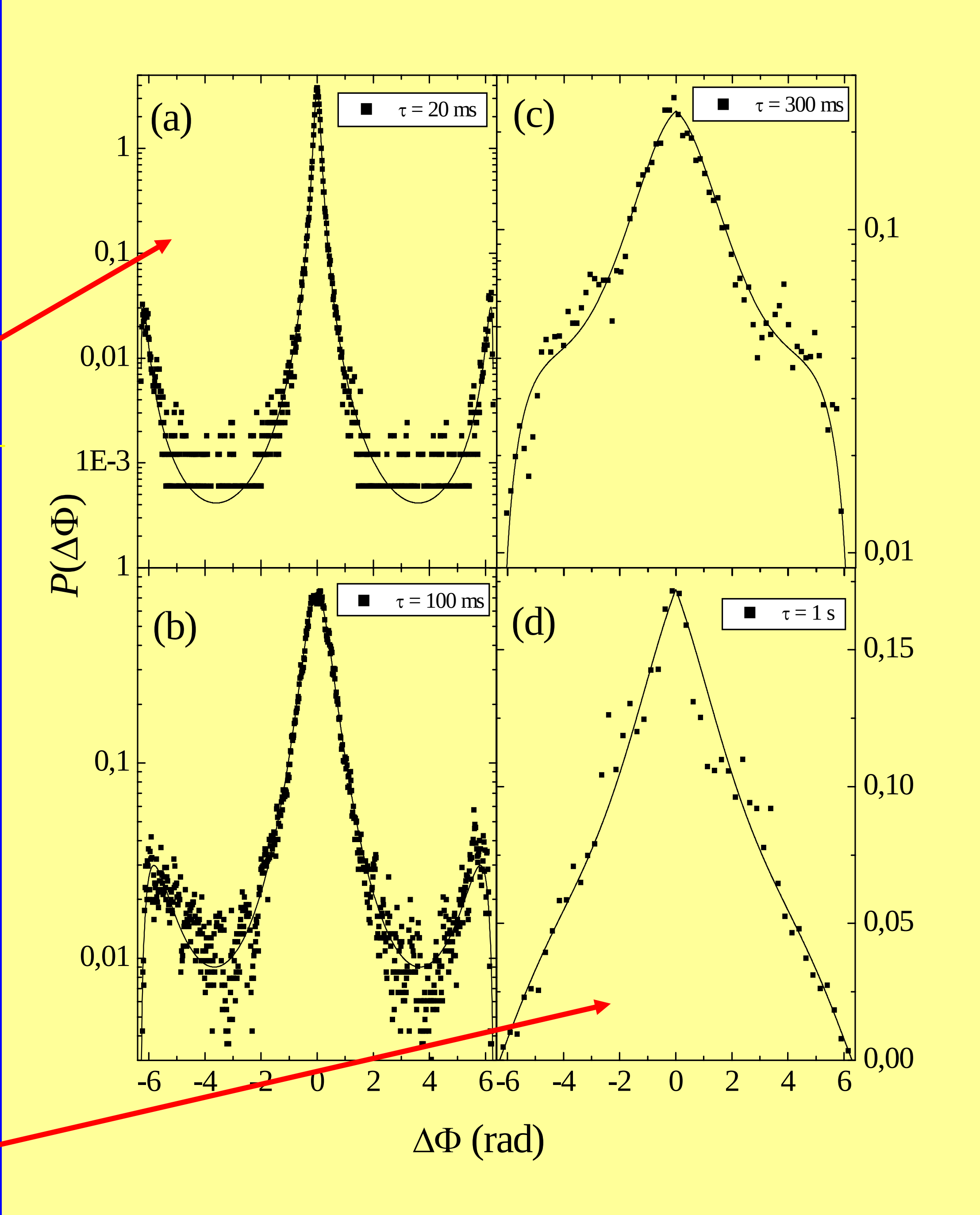
after time τ

$$P\left(\frac{d\phi}{d\tau}\right) = \frac{Q}{\left[2Q + \left(\frac{d\phi}{d\tau}\right)^2\right]^{3/2}}$$

$$Q = \frac{1}{6t_{\mathrm{DAWS}}^2}$$

$$t_{\mathrm{DAWS}} = 100\,\mathrm{ms}$$

$$P(\Delta\phi) = \frac{1}{2\pi^2}\left(2\pi - |\Delta\phi|\right)$$



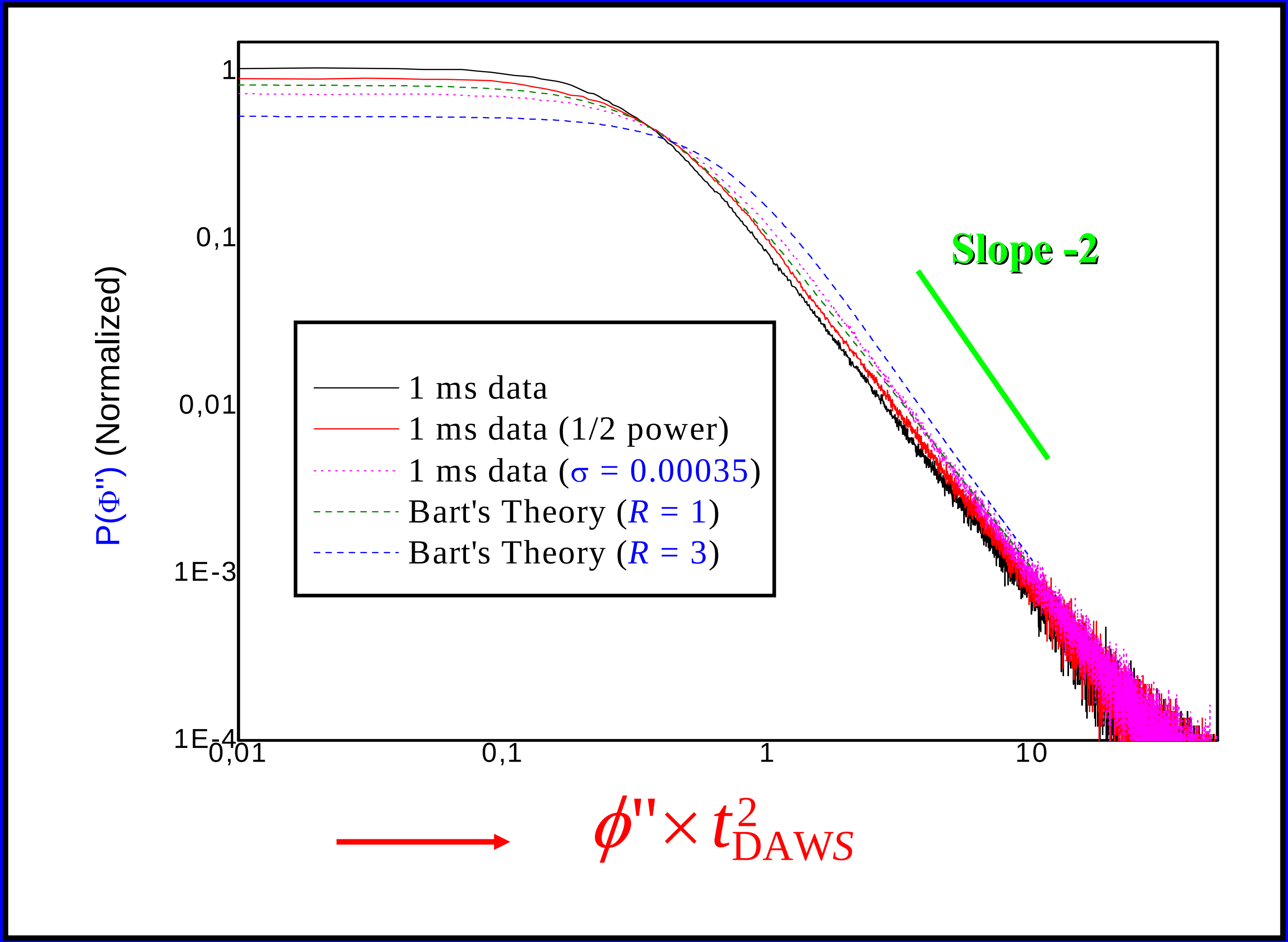
Probability distribution of SECOND derivative

$$P\left[\bar{\phi}''\right]=\frac{1}{\pi}\int_0^\infty dx\,\frac{\left(4x^2+R\right)^{3/2}}{\left[\left(\bar{\phi}''\right)^2+\left(x^2+\frac{1}{2}\right)\left(4x^2+R\right)\right]^2}$$

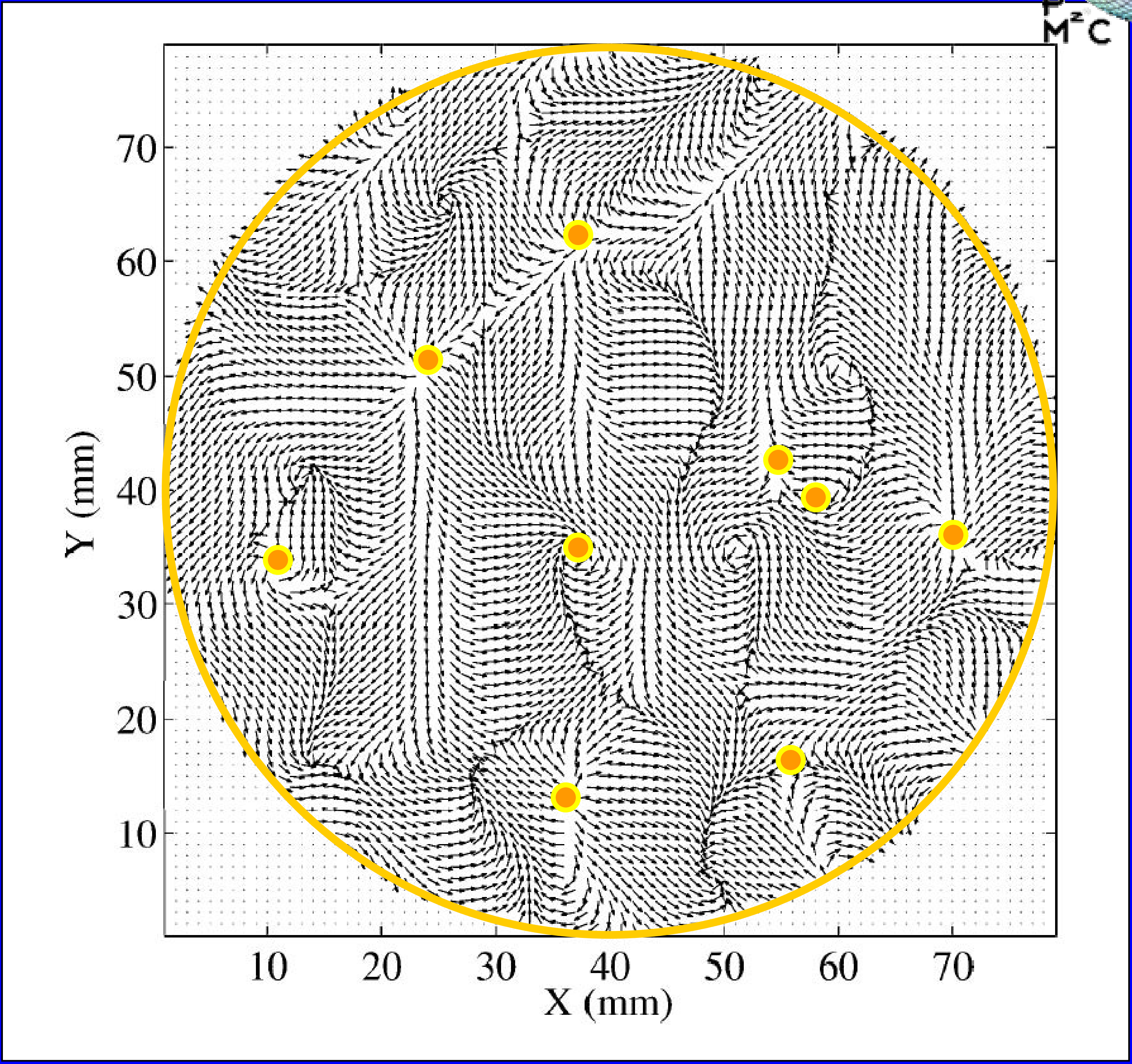
$$P\left[\Delta\phi''\equiv\phi_+''-\phi_-''\right]=\frac{1}{4T}\,\frac{1}{\left[\frac{\left(\Delta\phi''\right)^2}{T}+\frac{1}{2}\right]^{3/2}}$$

$$R=\frac{1}{2}\left[\frac{g^{(4)}(0)}{\left(g''(0)\right)^2}-1\right] \qquad T=\frac{4}{3}\,\frac{g^{(4)}(0)}{\left(g''(0)\right)^2}$$

Probability distribution of SECOND derivative



Optical Vortices

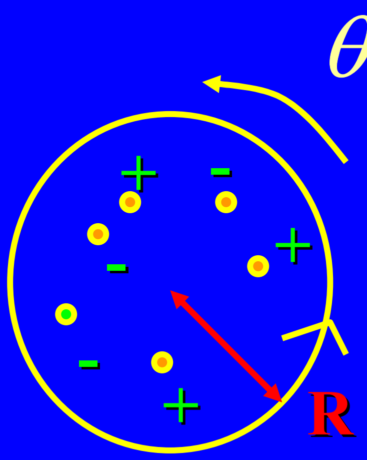


Patrick Sebbah
Azriel Genack

theorem

$$\oint d\mathbf{l}\cdot\nabla\phi(\mathbf{r}) = 2\pi Q$$

$$Q = \sum_{zero\ i} q_i$$



$$\langle Q\rangle = 0$$

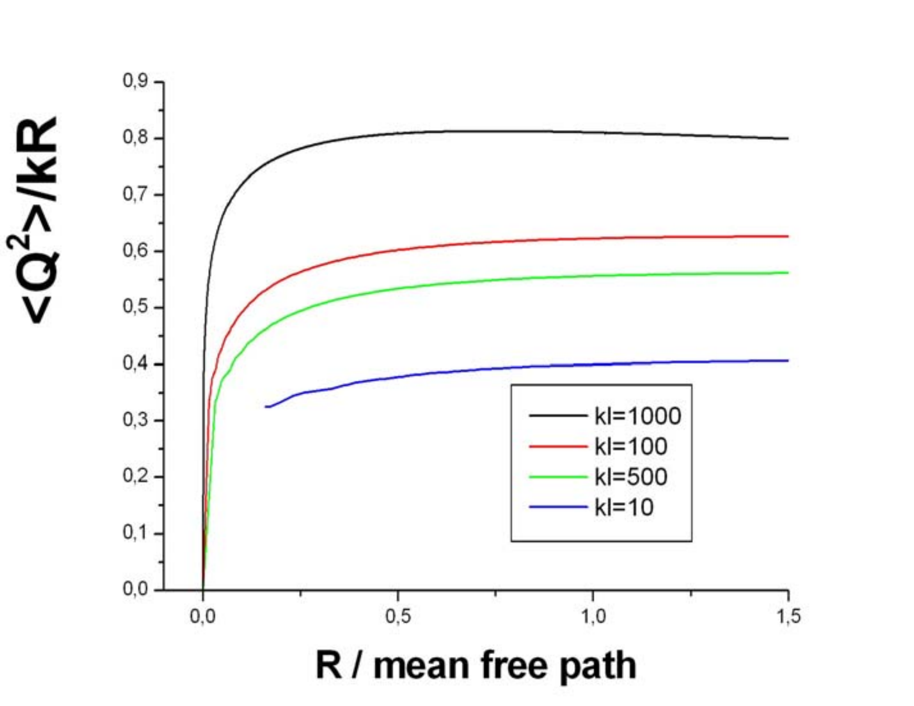
$$\langle Q^2(\text{circle})\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

Count the mean free path?

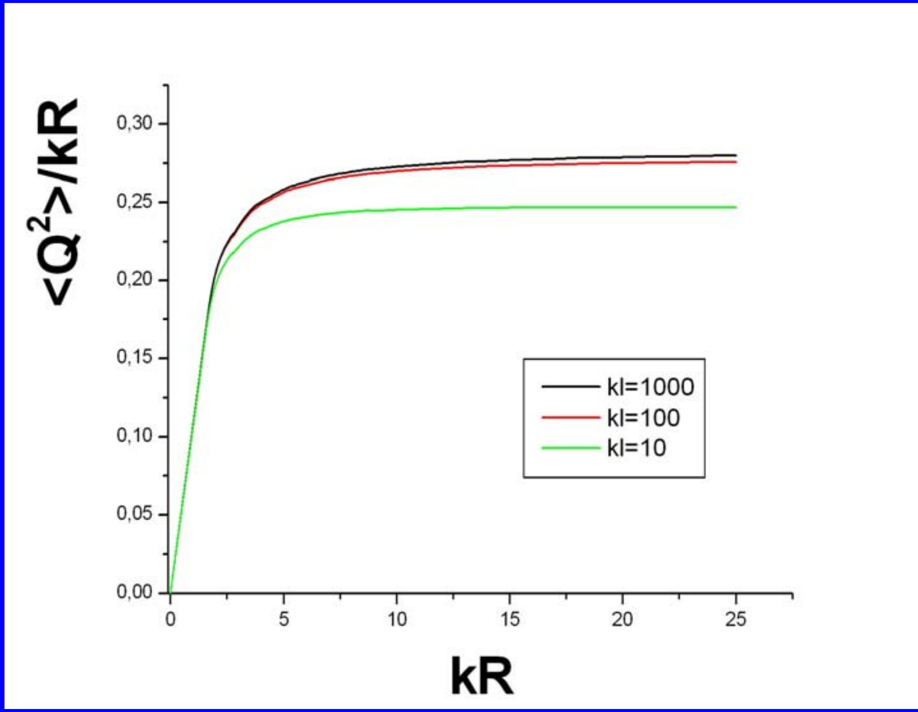
$$\left\langle Q^2(\text{circle})\right\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

$P\left[\psi(\mathbf{r}_1),\psi(\mathbf{r}_2),\psi(\mathbf{r}_3),\psi(\mathbf{r}_4)\right]$

$\left\langle \psi(\mathbf{r})\psi^*(\mathbf{r}')\right\rangle = J_0(k\Delta r)\exp(-\Delta r/2\ell)$



2 dimensions



3 dimensions

$$\left\langle Q^2 \right\rangle \propto R$$

implies screening of topological charge

(Halperin, 1981, Berry 2000, Wilkinson, 2004)

$$\rho(\mathbf{r}) = \sum_i q_i \ \delta^{(2)}(\mathbf{r}-\mathbf{r}_i)$$

Topological charge density

$$Q = \int_A d^2\mathbf{r} \ \rho(\mathbf{r})$$

Topological charge

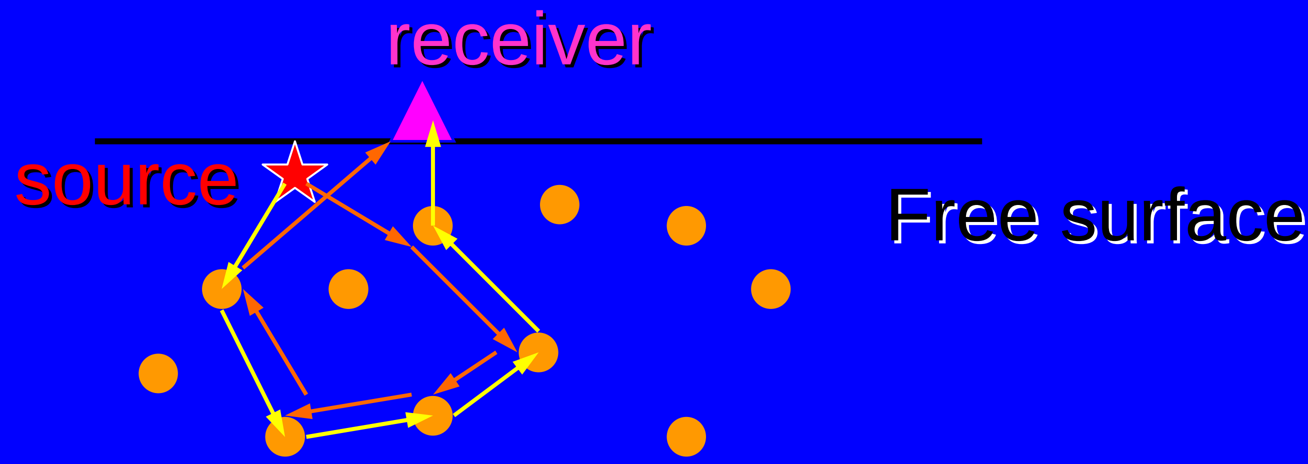
$$C(\mathbf{x}) = \left\langle \rho(\mathbf{r}+\mathbf{x})\rho(\mathbf{r}) \right\rangle$$

Topological pair correlation

$$\left\langle Q^2(R) \right\rangle = \pi R^2 \int_{\Re^2} d^n\mathbf{x} \ C(\mathbf{x}) + O(R) \Rightarrow \int_{\Re^2} d^2\mathbf{x} \ C(\mathbf{x}) = 0$$

$$\left\langle Q^2(R) \right\rangle \to \text{constant} \quad ???$$

Berry & Dennis, 2000



1. Distance source receiver < wavelength

$$CBS(r) \propto 1 + J_0^2\left(\frac{2\pi r}{\lambda}\right) \quad \times \quad 1 - e^{-t/\tau}$$

2. Symmetry source = symmetry receiver & magnitude

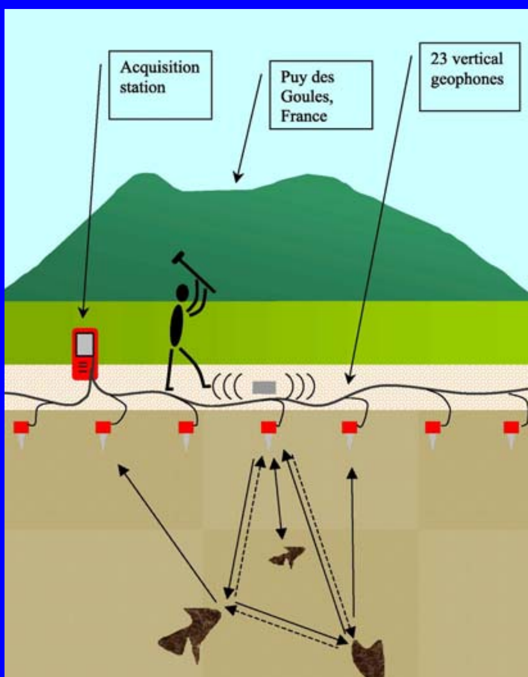
Earth quake	Explosion	Sledge hammer
measure $\left\| \partial_y u_x + \partial_x u_y \right\|^2$	measure $\left\| \mathrm{div}\, \mathbf{u} \right\|^2$	measure $\left\| u_z \right\|^2$

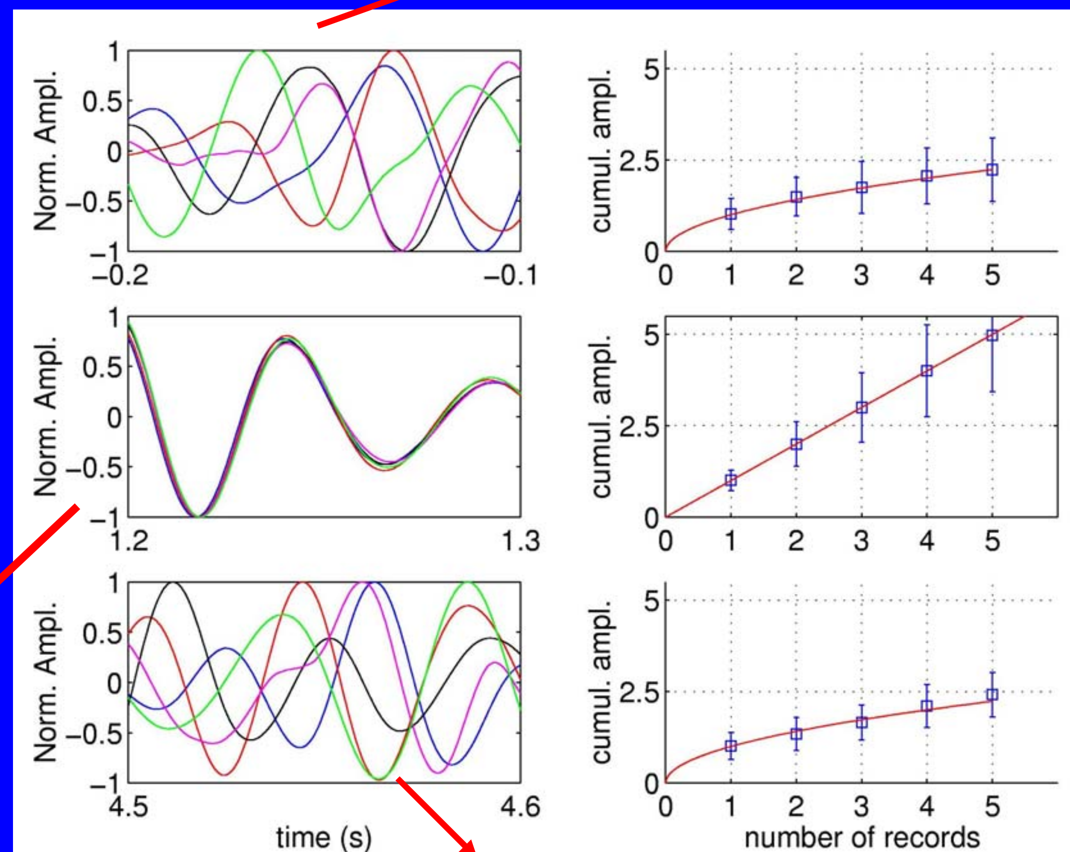
← magnitude

Seismic waves in the French Auvergne

Eric Larose, Ludovic Margerin, Michel Campillo et Bart van Tiggelen , PRL, July 2004

Operator noise

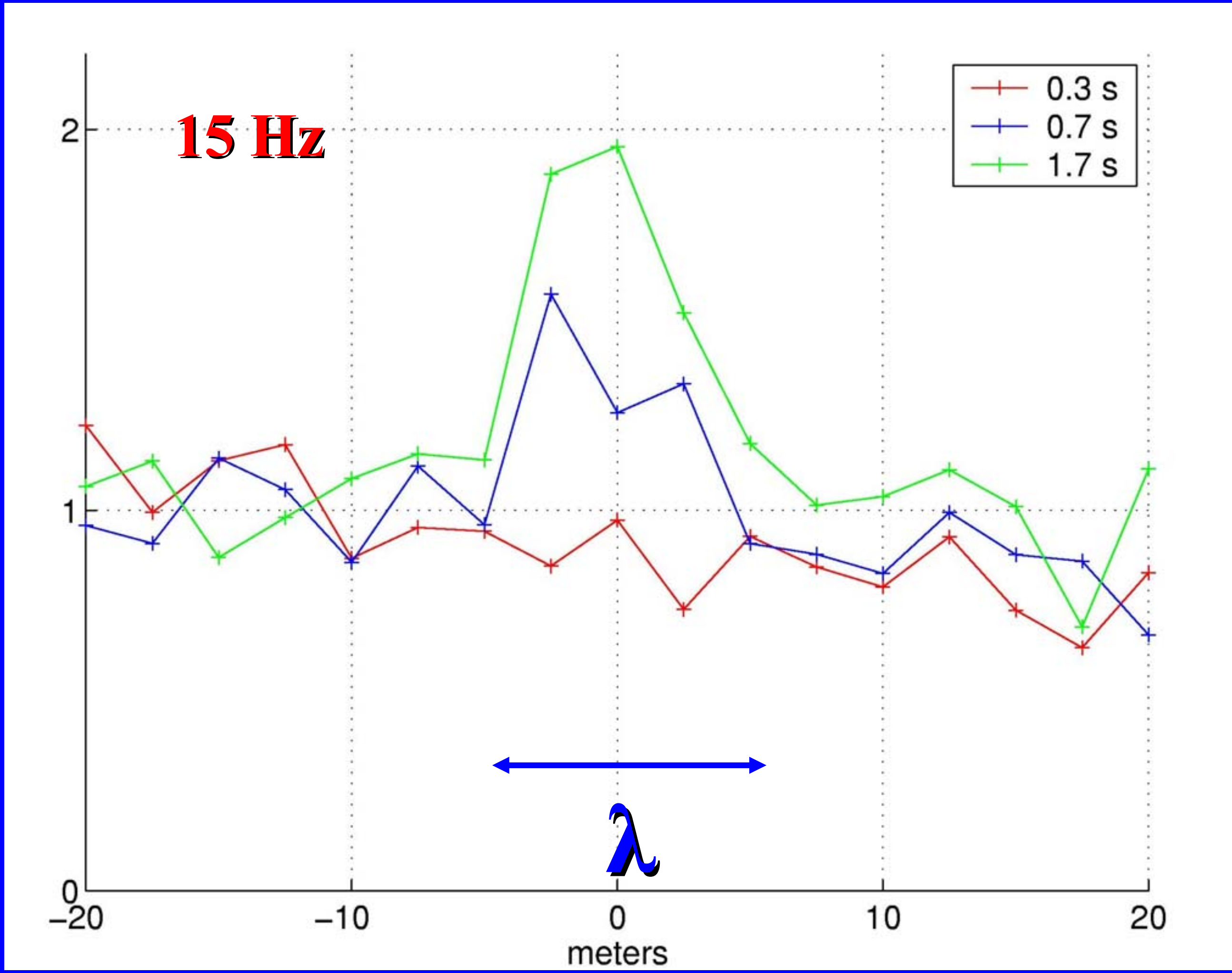




Mesoscopic signal

Background noise

Coherent Backscattering in the French Auvergne



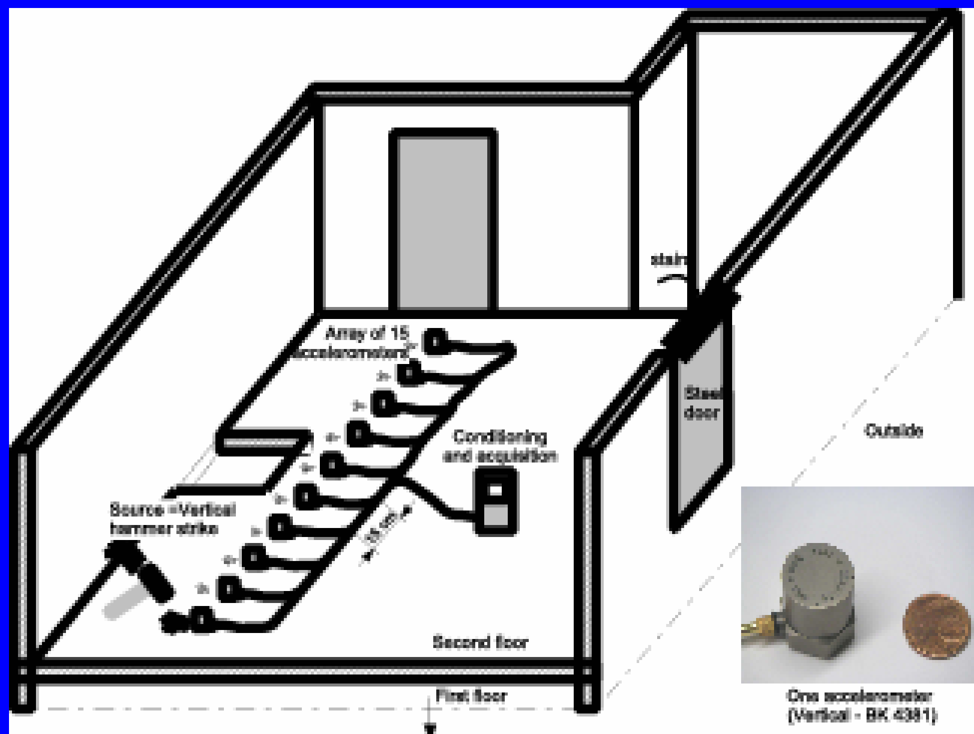
Mean free time=0.7 seconds

Wavelength= 20 meter $c_{Rayleigh}$ **= 300 m/s**

Mean free path = 210 m

Coherent Backscattering in Concrete Structure

Larose, De Rosny, Goudeard, Anache, Margerin, Campillo, Van Tiggelen, 2005

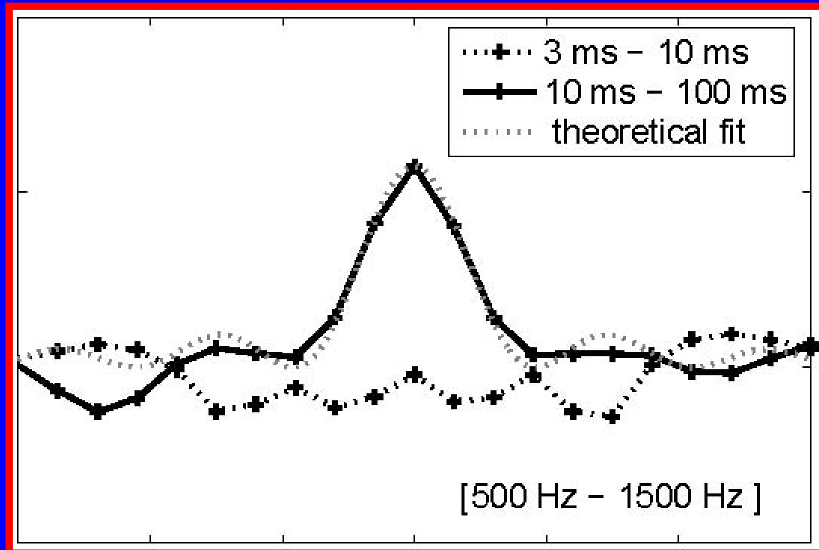


Coherent Backscattering in Concrete Structure

Larose, De Rosny, Goudeard, Anache, Margerin, Campillo, Van Tiggelen, 2005

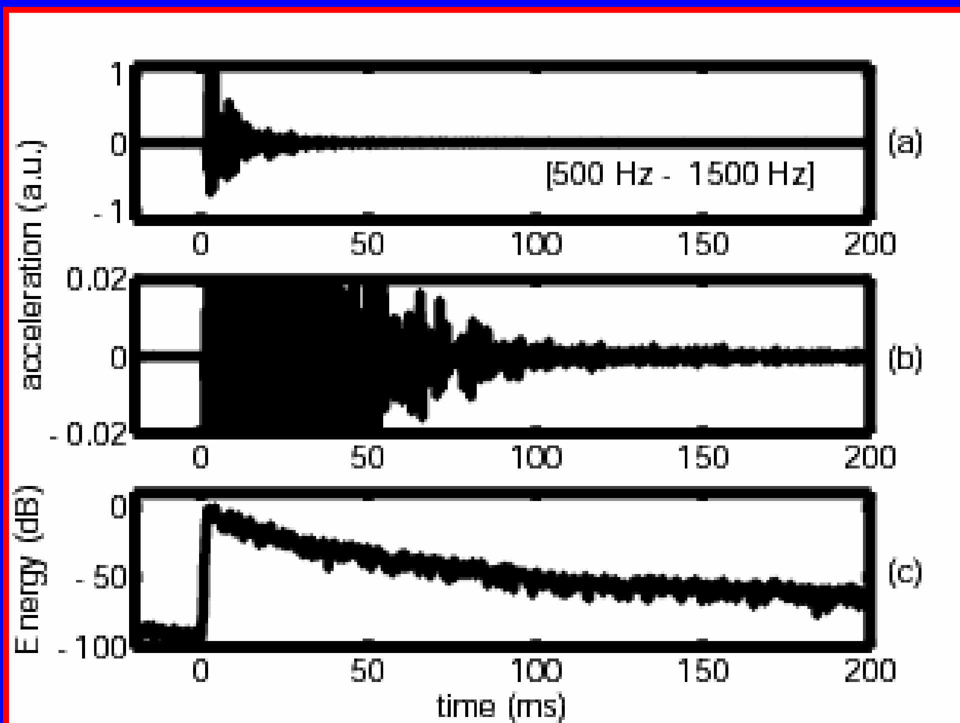
Energy enhancement

2



-1 **0** **+1**

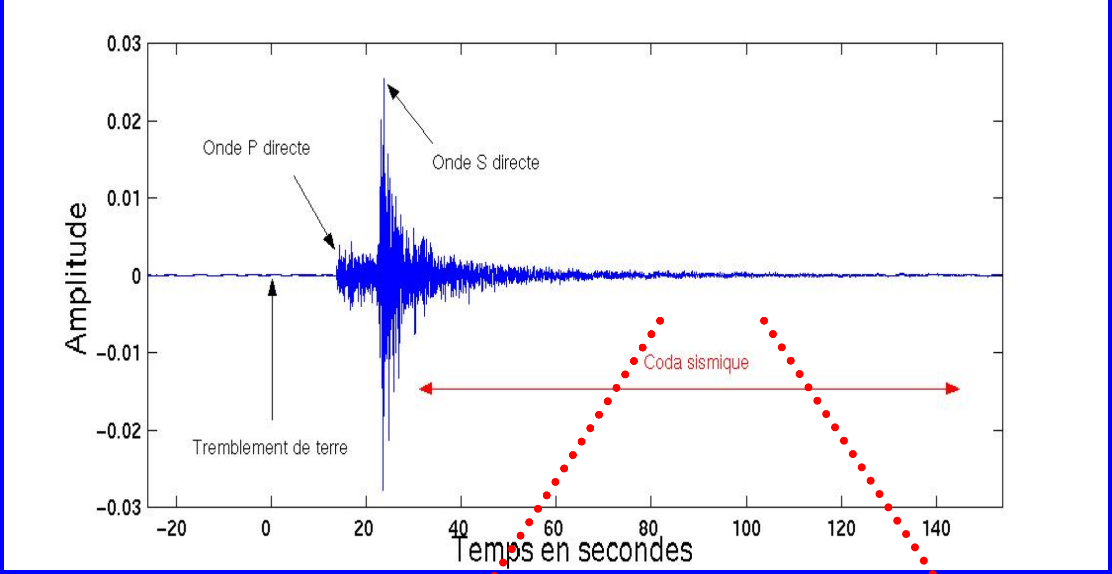
Δr (meters)



$$CBS(r) \propto 1 + J_0^2\left(\frac{2\pi r}{\lambda}\right) \times \; 1 - e^{-t/\tau}$$

Imaging without a source

Equipartition → **Correlation = Green function**



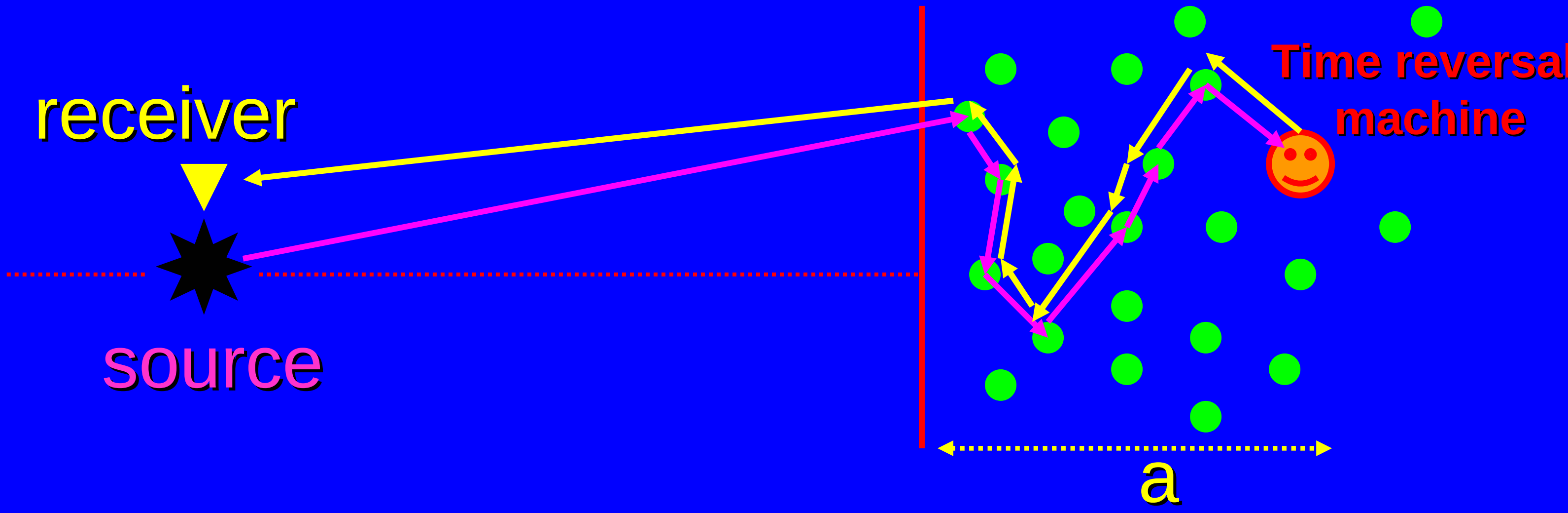
Helio-seismology
Duval, Nature 1993
Thermal phonons
Weaver & Lobkis, PRL 2001
Seismic coda/noise
Campillo etal Science 2003, 2005

$$\left\langle u\left(\mathbf{r}=A, t-\frac{1}{2}\tau\right) u\left(\mathbf{r}=B, t+\frac{1}{2}\tau\right)\right\rangle$$

$$\propto$$

$$G(A\rightarrow B,\tau)+G(A\rightarrow B,-\tau)$$

Relation with Time-Reversal and Coherent backscattering



$$[S \to TRM \to R](\tau) = \int dt\,[TRM \to S](t-\tau)\,[TRM \to R](t+\tau)$$

Time-reversal ⟷ **correlation method**

$$R(z,\tau) = S(\tau) \times \mathrm{CBS}\left(\theta \frac{\ell}{\lambda} \to \theta \frac{a}{\lambda}\right) + \text{speckle}$$

$$\approx \sqrt{\frac{D}{Wa^2}} << 1$$

Stable time-reversal at source……

….. with CBS cusp !!