esa ACT Advanced Concepts Team European Space Agency

The Feigel Process

The Momentum of Quantum Vacuum

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What is claimed by A. Feigel ? Phys. Rev. Lett. **92**, 020404 (2004)

- Dielectric homogeneous media must obey Lorentz –invariance
- This solves Abraham-Minkowski Controversy
- This predicts a finite momentum density for zero-point fluctuations in Electro-Magnetic Matter

Vacuum Langrangian $L = \int \mathrm{d}t\,\mathrm{d}^3\mathbf{r}\;\left[\tfrac{1}{2}\mathbf{E}^2 - \tfrac{1}{2}\mathbf{B}^2\right]$

is Galilean – invariant (even Lorentz -invariant):

$$\begin{cases} \mathbf{E} \to \mathbf{E} + \mathbf{v}\times\mathbf{B} \\ \mathbf{B} \to \mathbf{B} - \mathbf{v}\times\mathbf{E} \\ \mathrm{d}t\,\mathrm{d}^3\mathbf{r} \to \mathrm{d}t\,\mathrm{d}^3\mathbf{r} \end{cases}$$

(for v <<1)

Vacuum stress tensor $T_{ij} = \frac{1}{4\pi}\left[E_i E_j + B_i B_j\right] - \frac{1}{8\pi}\left[\mathbf{E}^2 + \mathbf{B}^2\right]\delta_{ij}$

is symmetric

conservation of momentum → $\mathbf{P} = \int \mathrm{d}^3\mathbf{r}\;(\mathbf{E}\times\mathbf{B})$ $= \mathbf{S}/c^2$

conservation of angular momentum → $\mathbf{G} = \int \mathrm{d}^3\mathbf{r}\;\mathbf{r}\times(\mathbf{E}\times\mathbf{B})$

Homogeneous dielectric Langrangian

$$L = \int \mathrm{d}t\, \mathrm{d}^3\mathbf{r} \left[\tfrac{1}{2}\, \mathbf{E} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{E} - \tfrac{1}{2} \mathbf{B}^2 \right]$$

is NOT Galilean – invariant:

$$\begin{cases} \mathbf{E} \to \mathbf{E} + \mathbf{v} \times \mathbf{B} & \mathbf{D} \to \mathbf{D} + \mathbf{v} \times \mathbf{H} \\ \mathbf{B} \to \mathbf{B} - \mathbf{v} \times \mathbf{E} & \mathbf{H} \to \mathbf{H} - \mathbf{v} \times \mathbf{D} \\ \mathrm{d}t\, \mathrm{d}^3\mathbf{r} \to \mathrm{d}t\, \mathrm{d}^3\mathbf{r} & \end{cases}$$

dielectric stress tensor

$$T_{ij} = \frac{1}{4\pi} \left[E_i D_j + B_i B_j \right] - \frac{1}{8\pi} \left[\mathbf{E} \cdot \mathbf{D} + \mathbf{B}^2 \right] \delta_{ij}$$

is NOT NECESSARILY symmetric

Momentum is conserved

$$\mathbf{P} = \int \mathrm{d}^3\mathbf{r}\, (\mathbf{D} \times \mathbf{B})$$

Momentum of what?

$$\mathbf{P} \neq \mathbf{S} / c^2$$

$$\mathbf{G} = \int \mathrm{d}^3\mathbf{r}\, \mathbf{r} \times (\mathbf{D} \times \mathbf{B})$$

is angular momentum?

Feigel's approach: the two problems must be related, and must be solvable simultaneously

$$\left.\begin{aligned} \mathbf{E} &\to \mathbf{E} + \mathbf{v}\times\mathbf{B} \\ \mathbf{B} &\to \mathbf{B} - \mathbf{v}\times\mathbf{E} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \mathbf{D} &= \boldsymbol{\varepsilon}\cdot\mathbf{E} + \boldsymbol{\chi}\cdot\mathbf{H} \\ \mathbf{B} &= \mathbf{H} - \boldsymbol{\chi}\cdot\mathbf{E} \end{aligned}\right.$$

$$\chi_{ij} = (1-\varepsilon)\varepsilon_{ijk} v_k$$

$$\begin{aligned} L &= \int \mathrm{d}t\, \mathrm{d}^3\mathbf{r} \left[\tfrac{1}{2}\rho\, \mathbf{v}^2 + \tfrac{1}{2}\mathbf{E}\cdot\mathbf{D} - \tfrac{1}{2}\mathbf{B}\cdot\mathbf{H} \right] \\ &= \int \mathrm{d}t\, \mathrm{d}^3\mathbf{r} \left[\tfrac{1}{2}\rho\, \mathbf{v}^2 + \tfrac{1}{2}\mathbf{E}^2 - \tfrac{1}{2}\mathbf{B}^2 + \mathbf{E}\cdot\boldsymbol{\chi}\cdot\mathbf{B} \right] \end{aligned}$$

is Galilean–invariant :

$$L(\not{x}_\alpha, v_\alpha, A_\alpha, \partial_\beta A_\alpha)$$

Conservation of Noether momentum:

$$\mathbf{G} = \int \mathrm{d}^3\mathbf{r} \left[\rho\, \mathbf{v} + \frac{1}{4\pi}\mathbf{E}\times\mathbf{H} \right]$$

Conservation of Pseudo-momentum:

$$\frac{\partial L}{\partial \mathbf{v}} = \int \mathrm{d}^3\mathbf{r} \left[\rho\, \mathbf{v} + \frac{(\varepsilon-1)}{4\pi}\mathbf{E}\times\mathbf{B} \right]$$

Feigel concludes:

$$\rho\,\mathbf{v} = \frac{(\varepsilon - 1)}{4\pi}\mathbf{E}\times\mathbf{B} = \mathbf{P}\times\mathbf{B}$$

?

$$\mathbf{G} = \int d^3\mathbf{r}\left[\rho\,\mathbf{v} + \frac{1}{4\pi}\mathbf{E}\times\mathbf{H}\right] = \int d^3\mathbf{r}\,\frac{1}{4\pi}\mathbf{D}\times\mathbf{B}$$

(Minkowski Momentum)

$$\rho\,\mathbf{v} = \langle 0|\mathbf{P}\times\mathbf{B}|0\rangle$$

$$= \int d\omega \int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\delta[\omega - \omega(\mathbf{k})]\,\langle 0|\mathbf{P}(\mathbf{k})\times\mathbf{B}^*(\mathbf{k})|0\rangle$$

$$\frac{1}{\rho}\int d\omega \int d^2\hat{\mathbf{k}}\;\frac{\omega^2}{(2\pi)^3}\times \tfrac{1}{2}\hbar\omega \times \frac{\mathbf{k}}{\omega} = 0$$

Empty vacuum

$$v_n = \frac{2}{3}\,\frac{\hbar\,\omega_c^4}{\pi^3\rho\,c^4}(1+\varepsilon)\,\varepsilon_{nkl}\,\chi^{kl}$$

Bi-anisotropic medium in vacuum

Classical Electrodynamics of bi-anisotropic media:

$$\partial_t \mathbf{D} = \nabla \times \mathbf{H} \qquad \nabla \cdot \mathbf{D} = 0$$

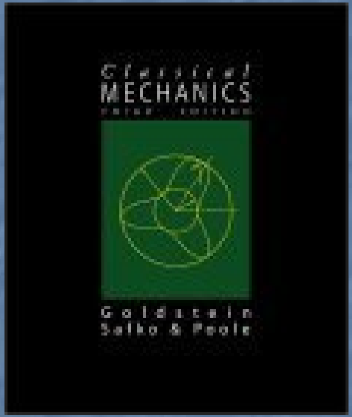
$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = 0$$

Macroscopic Maxwell equations

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} + \boldsymbol{\chi} \cdot \mathbf{H}$$

$$\mathbf{B} = \mathbf{H} - \boldsymbol{\chi} \cdot \mathbf{E}$$

Constitutive equations



Newton -Lorentz equation

$$\rho(\mathbf{r}) = \sum_a m_a \delta(\mathbf{r} - \mathbf{r}_a) \qquad \mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \approx \mathbf{v}_a = \frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t}$$

$$m_a \frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t} = q_a \left(\mathbf{E} + \mathbf{v}_a \times \mathbf{B} \right)$$

Microscopic Matter

Macro-Maxwell

Newton-Lorentz

$$\partial_t \mathbf{G} + \nabla\cdot\mathbf{T} = \mathbf{f} \qquad \partial_t E + \nabla\cdot\mathbf{S} = 0$$

$$\mathbf{G} = \frac{1}{4\pi}\mathbf{D}\times\mathbf{B} \qquad \mathbf{S} = \frac{1}{4\pi}\mathbf{E}\times\mathbf{H}$$

$$E = \frac{1}{8\pi}(\mathbf{H}\cdot\mathbf{B} + \mathbf{E}\cdot\mathbf{D}) \qquad T_{ij} = E\delta_{ij} - \frac{1}{4\pi}\left(H_i B_j + E_i D_j\right)$$

Minkowski version of Energy/momentum conservation

$$\partial_t(\rho\,\mathbf{v} - \mathbf{P}\times\mathbf{B}) + \nabla\cdot\mathbf{U} = -\mathbf{f}$$

$$U_{ij} = \rho v_i v_{\mathbf{i}} + E_i P_j - M_i B_j - \frac{1}{2}(\mathbf{E}\cdot\mathbf{P} - \mathbf{M}\cdot\mathbf{B})\delta_{ij}$$

« pseudo momentum » conservation (Feigel

EM force density

$$f_j = \frac{1}{8\pi}\left[\mathbf{E}\cdot\left(\partial_j\boldsymbol{\varepsilon}\right)\cdot\mathbf{E} + \mathbf{H}\cdot\left(\partial_j\boldsymbol{\mu}\right)\cdot\mathbf{H} + 2\mathbf{E}\cdot\left(\partial_j\boldsymbol{\chi}\right)\cdot\mathbf{H}\right]$$

$$\partial_t\left(\rho\,\mathbf{v} + \frac{1}{4\pi}\mathbf{E}\times\mathbf{B}\right) + \nabla\cdot\mathbf{T}^0 = 0$$

$$T_{ij}^0 = \frac{1}{8\pi}\left(\mathbf{E}^2 + \mathbf{B}^2\right)\delta_{ij} - \frac{1}{4\pi}\left(E_i E_j + B_i B_j\right)$$

Rigorous conservation of total momentum (matter + field)

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$$\partial_t \mathbf{G} + \nabla \cdot \mathbf{T} = \mathbf{f} \qquad \partial_t E + \nabla \cdot \mathbf{S} = 0$$

$$\mathbf{G} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{H} \qquad \mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{H}$$

$$E = \frac{1}{8\pi}\left(\mathbf{H} \cdot \mathbf{B} + \mathbf{E} \cdot \mathbf{D}\right) \qquad T_{ij} = E\delta_{ij} - \frac{1}{8\pi}\left(H_i B_j + H_j B_i + E_i D_j + E_j D_i\right)$$

$$\mathbf{f} = \mathbf{f}(\text{Minkowski}) + \frac{1}{4\pi} \partial_t \left(\mathbf{D} \times \mathbf{B} - \mathbf{E} \times \mathbf{H}\right) + \partial_i \left(\cdots\right)$$

Abrahams version of energy/momentum conservation

Problems with the Feigel paper

1. Conservation of momentum and pseudo momentum density follow from ordinary classical electrodynamics applied to bi-anisotropic media, with actually a sloppy flaw corrected

   Classical electrodynamics obeys Galilean Invariance.

2. The Abraham-Minkowski is not solved because it is not even addressed

   AB controversy involves a discussion of the EM force density (thus inhomogeneous media or time-dependent fields) and the (symmetry of the) stress tensor. There is no unique solution (on macroscopic level).

3. From $\partial_t \int d^3\mathbf{r} \; (\rho\,\mathbf{v} - \mathbf{P}\times\mathbf{B}) = 0$ it is concluded that $\boxed{\rho\,\mathbf{v} \equiv \mathbf{P}\times\mathbf{B}}$

   This assumes no motion and no field at some distant time, whereas already no sources (free charges) have been adopted. The final result is also not Galilean invariant

   The solution $\rho\,\mathbf{v} \equiv 0$ and $\mathbf{P}\times\mathbf{B} =$ constant leads to no contradiction.

Need to address the dynamic problem !

$$\mathbf{D} = \varepsilon\mathbf{E} + \boldsymbol{\chi}\cdot\mathbf{H}$$

$$\mathbf{B} = \mathbf{H} + \boldsymbol{\chi}^T\cdot\mathbf{E}$$

1. The tensor χ changes under time reversal
2. The tensor χ changes under parity

Fresnel Dispersion law

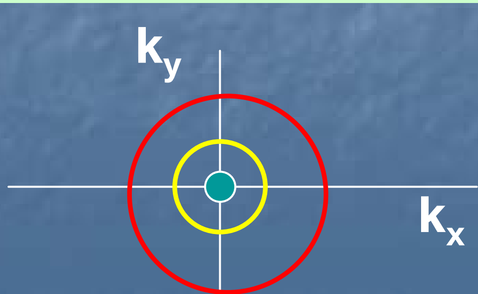
$$\det\left(\varepsilon\omega^2 - p^2 + \mathbf{pp} + \omega\,\boldsymbol{\chi}\cdot\boldsymbol{\Phi}_{\mathbf{p}} + \omega\,\boldsymbol{\Phi}_{\mathbf{p}}\cdot\boldsymbol{\chi}\right) = 0$$

$$\left(\Phi_{\mathbf{p}}\right)_{inm} = i\varepsilon_{nml}\,p_l$$

Rotatory power;

$$\chi_{ij}(\omega) = g\,\omega\,\delta_{ij}$$

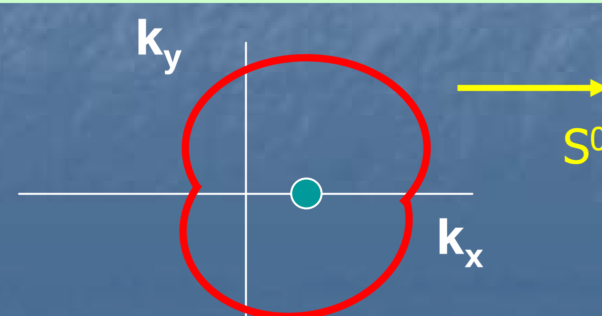
$$\varepsilon_{ij}(\omega,\mathbf{p}) = \varepsilon\delta_{ij} + 2g\,i\varepsilon_{ijk}\,p_k$$



Magneto-electrical effect

$$\chi_{ij}(\omega) = \chi\left(E_i^0 B_j^0 - B_i^0 E_j^0\right)$$

$$\varepsilon_{ij}(\omega,\mathbf{p}) = \varepsilon\delta_{ij} + \frac{2\chi}{\omega}\,i\left(S_i^0 p_j - p_i S_j^0\right)$$



Subtle theorems

$$\langle 0|\mathbf{S}|0\rangle = \langle 0|\frac{1}{4\pi}\mathbf{E}\times\mathbf{H}|0\rangle = 0$$

So in Abraham's version:

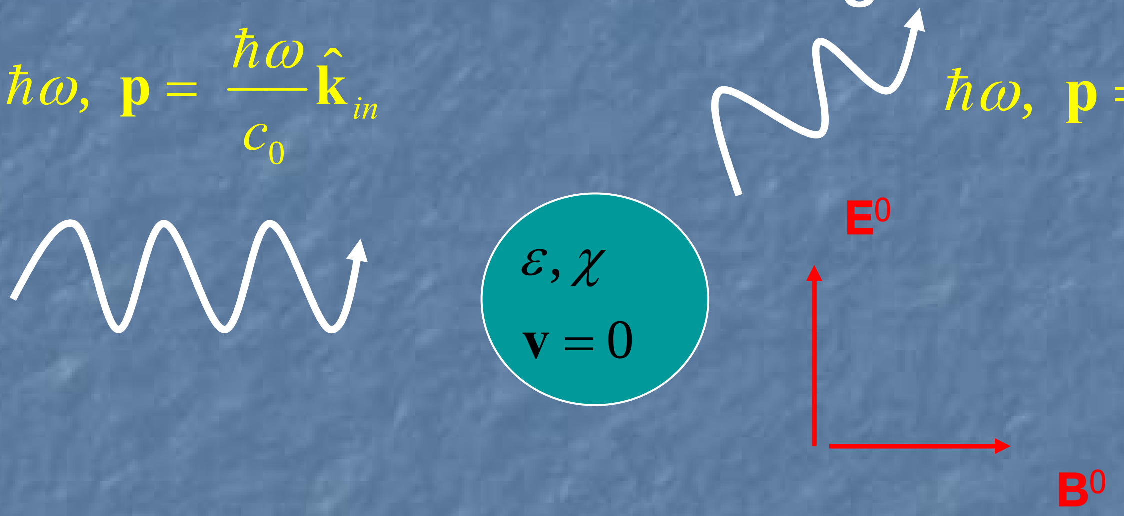
$$\langle 0|\mathbf{G}|0\rangle = \langle 0|\mathbf{S}|0\rangle = 0$$

But in Minkowski's version:

$$\langle 0|\mathbf{G}|0\rangle \propto \frac{\hbar\omega_c^4}{\rho\, c^4}(1+\varepsilon)\,\chi\,\mathbf{S}^0$$

Finite classical macroscopic medium: elastic scattering

$$\hbar\omega,\ \mathbf{p} = \frac{\hbar\omega}{c_0}\hat{\mathbf{k}}_{in} \qquad\qquad \hbar\omega,\ \mathbf{p} = \frac{\hbar\omega}{c_0}\hat{\mathbf{k}}_{out}$$



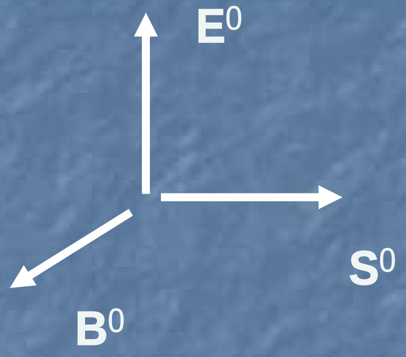
$$\begin{aligned}
\Delta\mathbf{p}(\omega) &= \hbar\omega^2\Delta t \int d^2\hat{\mathbf{k}}_{in} \int d^2\hat{\mathbf{k}}_{out} \frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{in},\hat{\mathbf{k}}_{out},\mathbf{S}_0)\left(\mathbf{k}_{in}-\mathbf{k}_{out}\right) \\
&\underbrace{=}_{PT} \hbar\omega^2\Delta t \int d^2\hat{\mathbf{k}}_{in} \int d^2\hat{\mathbf{k}}_{out} \frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{out},\hat{\mathbf{k}}_{in},\mathbf{S}_0)\left(\mathbf{k}_{in}-\mathbf{k}_{out}\right) \\
&= \hbar\omega^2\Delta t \int d^2\hat{\mathbf{k}}_{in} \int d^2\hat{\mathbf{k}}_{out} \frac{d\sigma}{d\Omega}(\hat{\mathbf{k}}_{\text{int}},\hat{\mathbf{k}}_{out},\mathbf{S}_0)\left(\mathbf{k}_{out}-\mathbf{k}_{in}\right) \\
&= 0
\end{aligned}$$

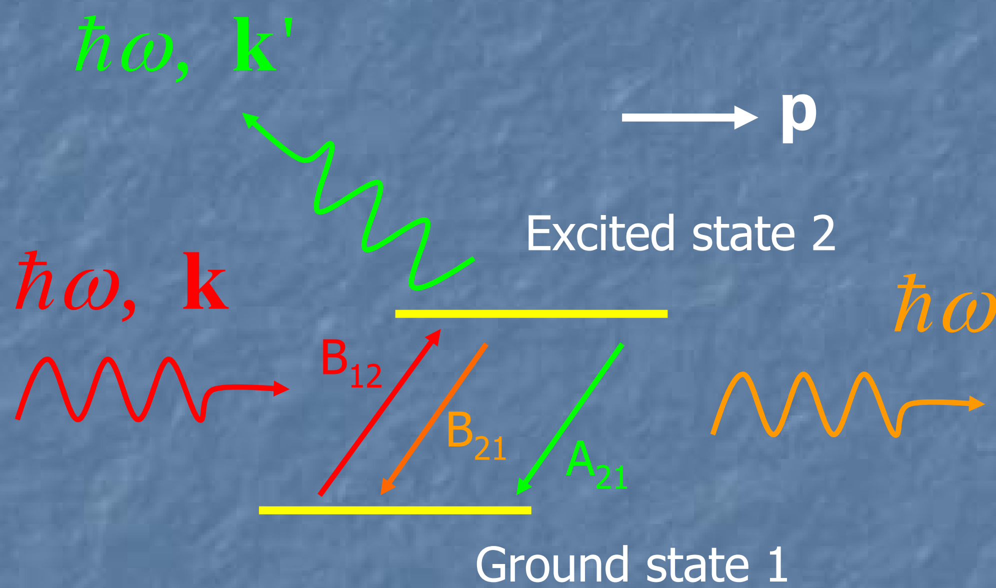
Object stays at rest

Van Tiggelen and Rikken, Comment Phys. Rev. Lett. (2004)

Quantum electrodynamics:

Inelastic scattering from molecules in a radiation field and subject to E^0 and B^0





$$B_{12} = B\left(1 + \underline{\chi\, \mathbf{k} \cdot \mathbf{S}^0}\right)$$

$$B_{21} = B_{12}$$

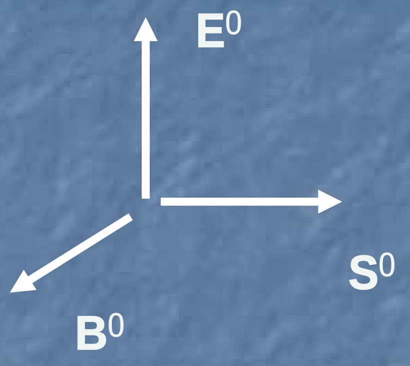
$$A_{21} = B_{12} \times \frac{\hbar\omega^3}{\pi^2 c_0^3}$$

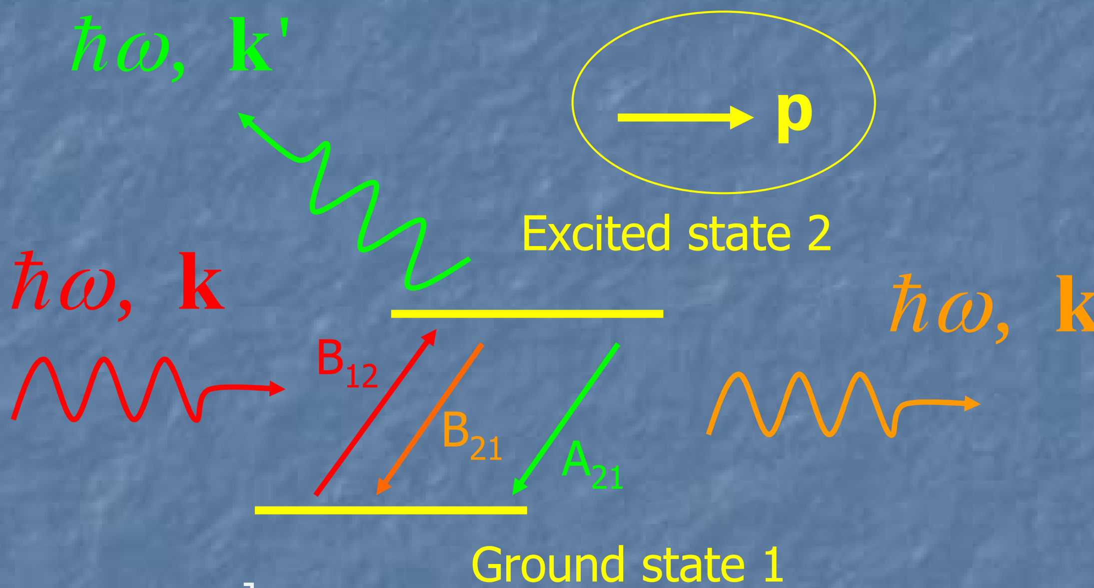
Magneto-electric QED

Parity : OK
Time-reversal: OK
Lorentz-invariance?

Value of χ ?

Quantum electrodynamics:





$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\int \mathrm{d}\hat{\mathbf{k}}'\,\hbar\mathbf{k}'\,N_2(t)A_{21}(\mathbf{k}')$$

$$+\int \mathrm{d}\hat{\mathbf{k}}\,\hbar\mathbf{k}\,\rho(\omega)\left[N_1(t)-N_2(t)\right]B_{21}(\mathbf{k}')$$

Einstein Doppler friction $\longrightarrow$ $-\dfrac{\hbar\omega}{mc_0^2}B_0\left[N_1(t)-N_2(t)\right]\left(\rho-\dfrac{\omega}{3}\dfrac{d\rho}{d\omega}\right)\mathbf{p}$

$$\mathbf{p}(t\to\infty)\ \propto\ \chi\ \mathbf{S}^0\ ?$$

Lorentz/Galilean invariance ?

Vacuum
Thermal equilibrium

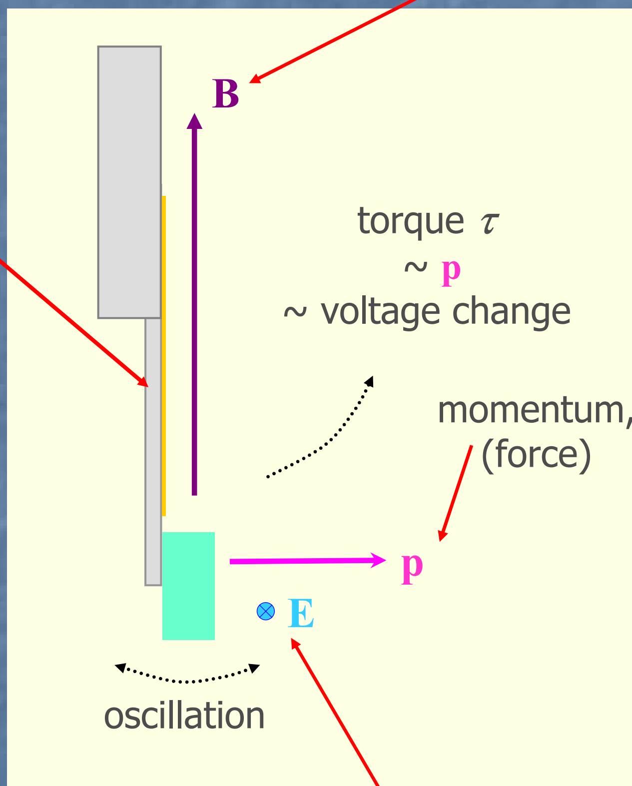
Experimental verification of Feigel theory

phase-sensitive torque-measurements on micro-sized samples

side-view:

static magnetic field

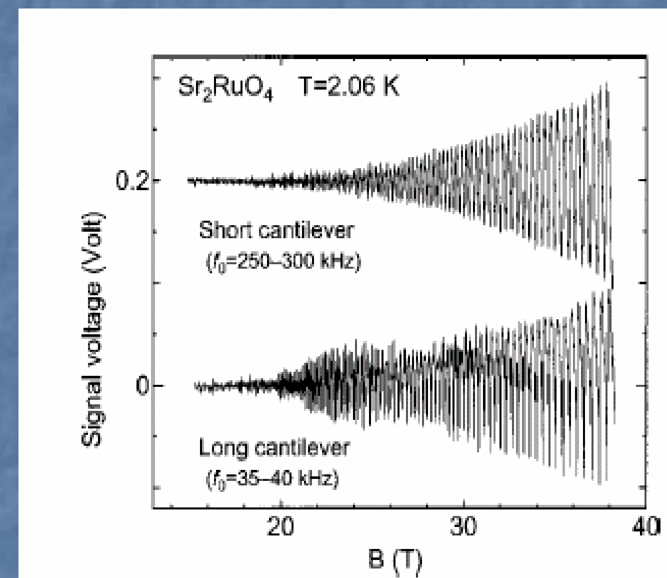
cantilever



AC electric field,
perpendicular to B,
applied by electrode-plates
(sample holder/stick)

example: E. Ohmichi, Yt Osada, Rev. Sci. Instr. **73**, 3022 (2002)

de Haas - van Alphen oscillations of Sr_2RuO_4



resolution $\delta V \sim \delta\tau \approx \pm 10^{-11}$ Nm

some numbers on the Feigel process...

material: diamond

$\varepsilon \approx 6$, $\mu \approx 1$, $\rho \approx 3.52$ g/cm^3, m ≈ 10 μg

B : 16 T

AC electric Field: 10^7 V/m at cantilever eigen-frequency (reduced to 8.72 kHz due to sample mass)

following Feigel:

vacuum fluctuations

$\lambda_{max} = 0.1$ nm

↓

$\mathbf{v} \approx 3.7$ μm/s

$\mathbf{p} \approx 37 \cdot 10^{-15}$ kgm/s

yields

$\tau \approx 1 \cdot 10^{-9}$ Nm

✓

exp. resolution

$\delta\tau \approx \pm 10^{-11}$ Nm

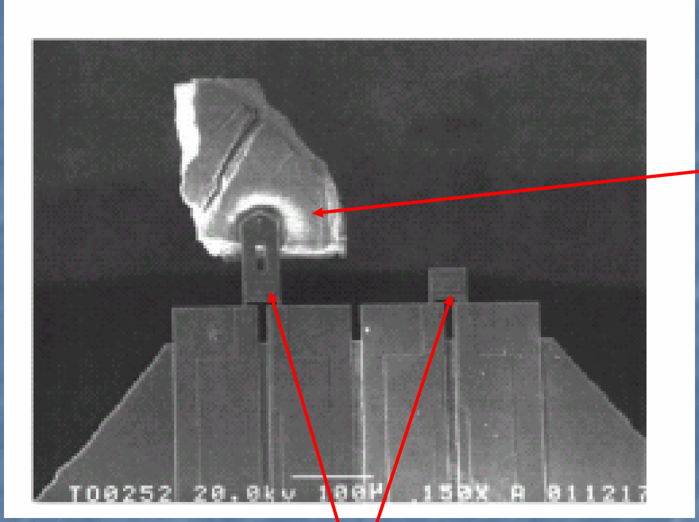
alternatives:

isotropic high intensity field
($\mathbf{p}$ ~ intensity)

→ *laser-field*

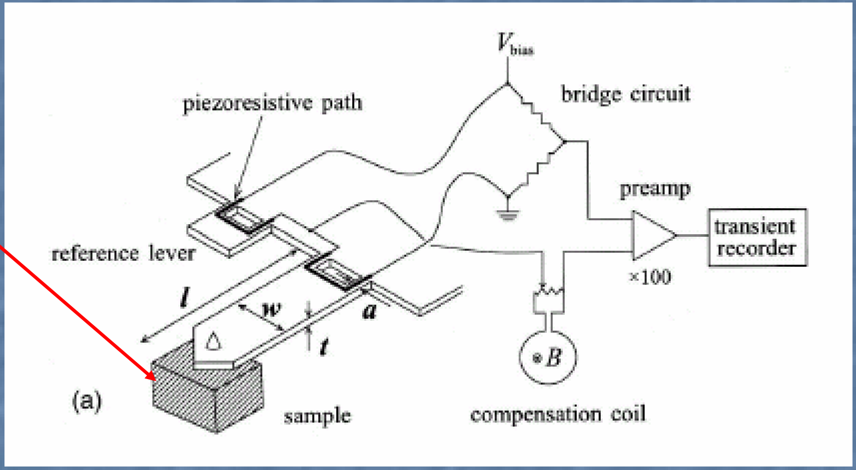
Possible experimental approach:

cantilever-magnetometry:



sample

piezoresistive path / element



l = 120 - 400 µm
w = 50 µm
t = 4 - 5 µm
f = 250 - 300 kHz

<u>small samples possible!</u>

force/torque applied
~ mechanical stress
~ voltage / resistance change on piezoresistive element