3-Dimensional Anderson Localization of Elastic Waves

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50 years of Anderson localization

Localization [..] very few believed it at the time, and even fewer saw its importance, among those who failed was certainly its author.
It has yet to receive adequate mathematical treatment,
and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it.

P.W. Anderson, Nobel lecture, 1977

…..and now we have experiments !

My precious collaborators

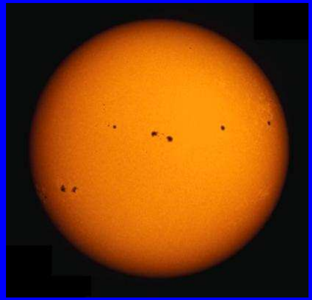
- John Page, Anatoli Strybulevych, Kurt Hildebrand
  (Winnipeg-Canada)

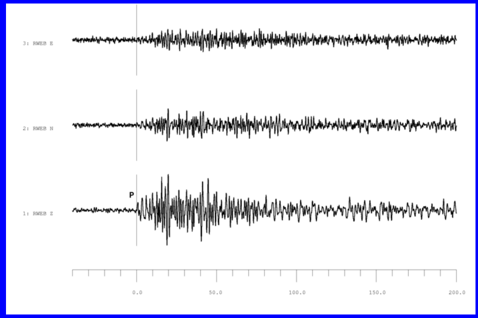
- Sergey Skipetrov and Nicolas Cherroret
  (Grenoble-France)

- Ad Lagendijk and Sanli Faez
  (Amsterdam-the Netherlands)

- Diederik Wiersma
  (Florence-Italy)

Diffusion of Waves





Diffusion = random walk of waves

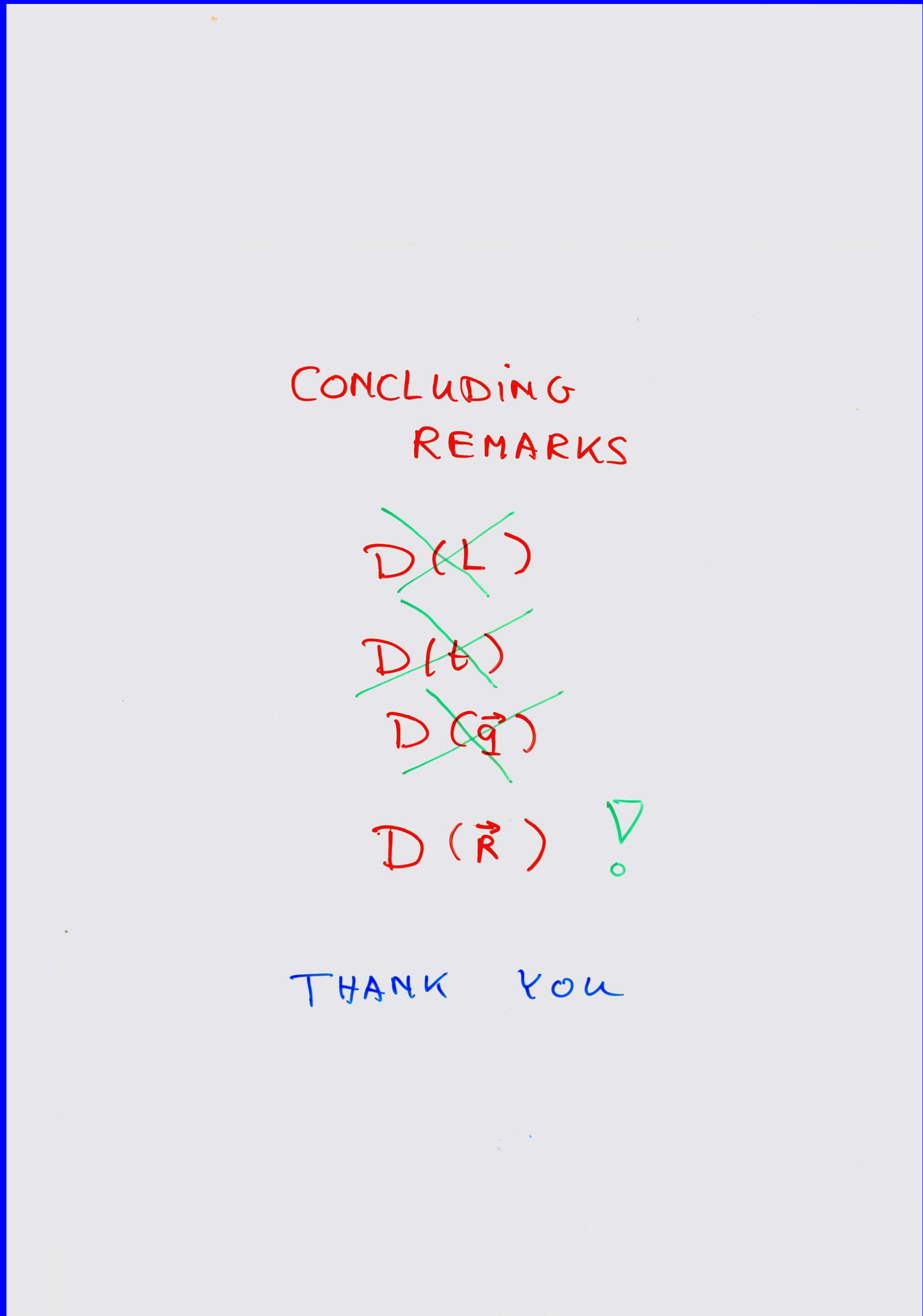
$$\partial_t \rho(\mathbf{r},t) \; - \; D \; \nabla^2 \rho(\mathbf{r},t) \; = S \;\; \delta(t) \;\; \delta(\mathbf{r}-\mathbf{r}_S)$$

$$\left\langle \mathbf{r}^2(t) \right\rangle = \frac{\left\langle \rho(\mathbf{r},t)\mathbf{r}^2 \right\rangle}{\left\langle \rho(\mathbf{r},t) \right\rangle} = \; 6D\,t$$

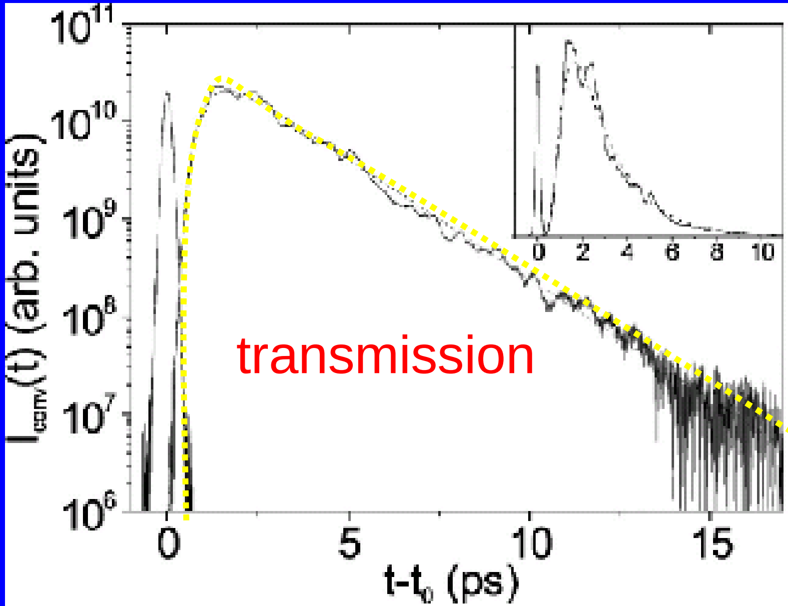
$$D = \tfrac{1}{3} \;\, \mathrm{v} \;\; \ell^*$$

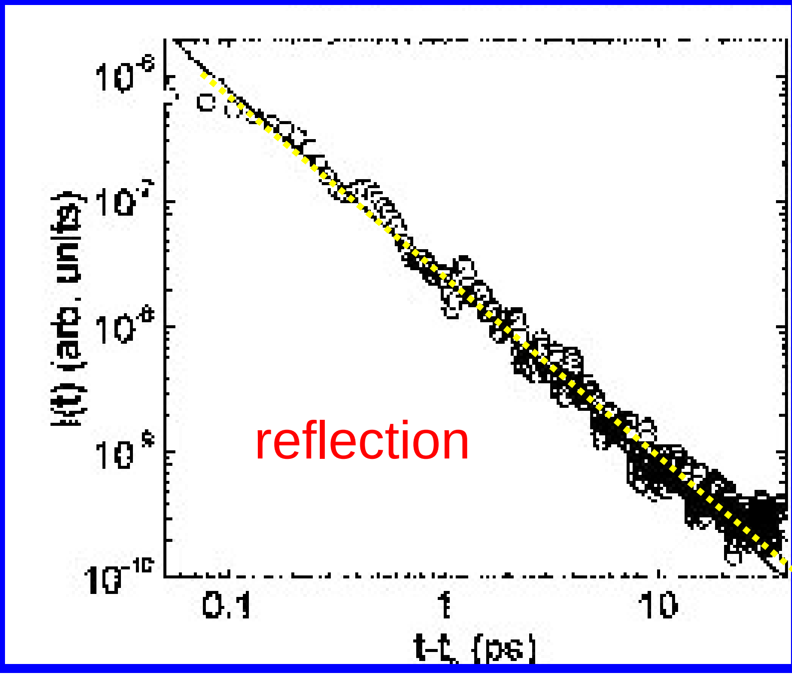
diffusion constant

Crete, 2000



Diffusion, works even better than expected!





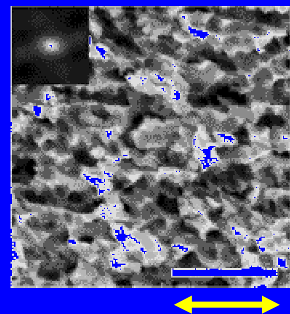
porous GaP

$L = 20\ \mu m$

$\lambda = 739\ nm$

Lagendijk etal, PRE 2003

Diffusion equation



1 µm

$D = 23\ m^2 / s$

$\ell^* = 250\ nm \quad (k\ell^* = 2.1)$

$1/t^{3/2}$

Anderson localisation?

« Unrecognizable monster »

Measurable?

- *Vanishing of diffusion : $r^2(t) \rightarrow$ constant « $D(t) \sim 1/t$ »*

- *Scale-dependence of conductance $T(L)$, $G(L) \sim \exp(-L/\xi)$*
  « $D(L)$ »

- *Anomalous, dynamic leaking $T(t) \sim \exp(-D(t,L)\, t / L^2$*
  $R(t) \sim 1/t^2$

- *Dense « pure point » spectrum / no level repulsion*

Measurable?

- *Giant nongaussian fluctuations*

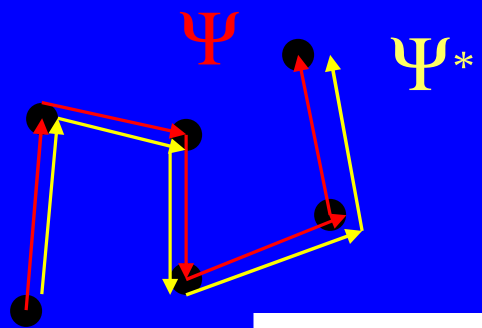
- *Multifractality of wave function*

Theoretical Description

$$k\ell >> 1$$

Diffusion equation

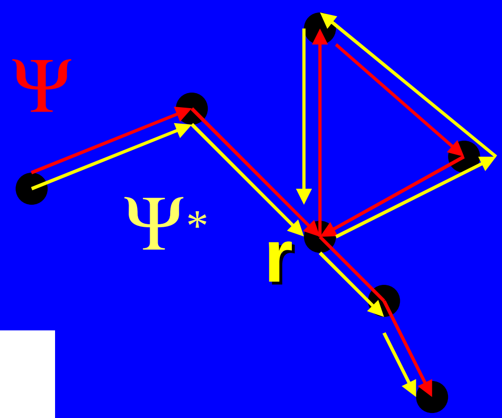
Schwarzshild/Milne , 1900
Chandrasekhar, 1950
Van de Hulst, 1950



$$D_B = \tfrac{1}{3}\, v_E\, \ell^*$$

$$k\ell \approx 1$$

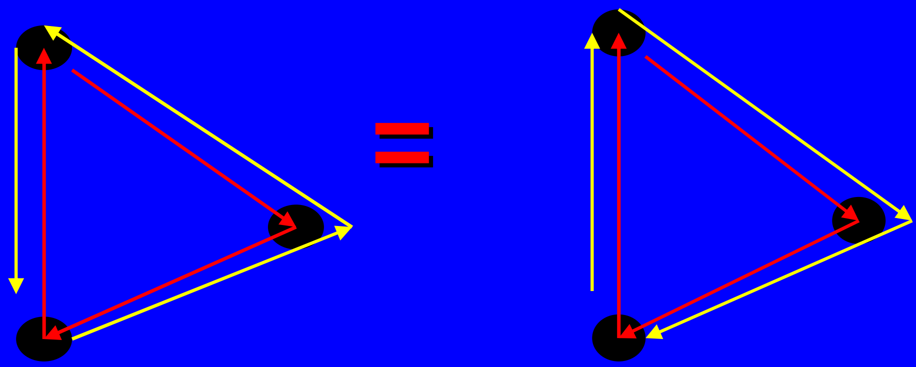
Self consistent theory



$$\frac{1}{D} = \frac{1}{D_B} + \frac{C(\mathbf{r},\mathbf{r})}{\pi v_E N(\omega)}$$

Vollhardt & Wölfle, 1980

reciprocity $\Rightarrow$ $C(\mathbf{r},\mathbf{r}) = G(\mathbf{r},\mathbf{r})$



$$-\nabla \cdot D(\mathbf{r})\nabla\, G(\mathbf{r},\mathbf{r}') = \frac{4\pi}{\ell}\delta(\mathbf{r}-\mathbf{r}')$$

$$\frac{1}{D(\mathbf{r})} = \frac{1}{D_B} + \frac{G(\mathbf{r},\mathbf{r})}{\pi v_E N(\omega)}$$

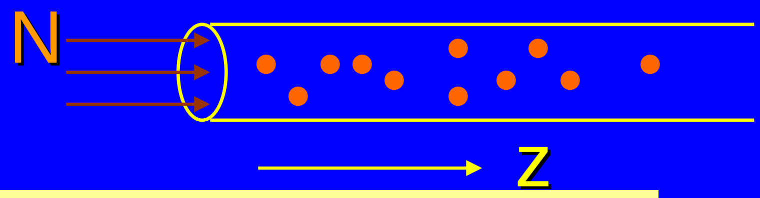
3D unbounded medium

$$G(\mathbf{r},\mathbf{r}') = G(\mathbf{r}-\mathbf{r}');\; G(\mathbf{r},\mathbf{r}) = G(0) = \frac{4\pi}{\ell}\int_{q<1/\ell} d^3\mathbf{q}\,\frac{1}{Dq^2}$$

$$\Rightarrow \boxed{D = D_B\left(1-\frac{1}{(k\ell)^2}\right)}$$

OK Mott

Quasi-1D wave guide



$$d\tau \equiv \frac{dz}{D(z)} \Rightarrow -\partial_\tau^2 G(\tau,\tau') = \frac{4\pi}{\ell}\delta(\tau-\tau') \Rightarrow G(\tau,\tau) = \frac{4\pi}{\ell}\tau$$

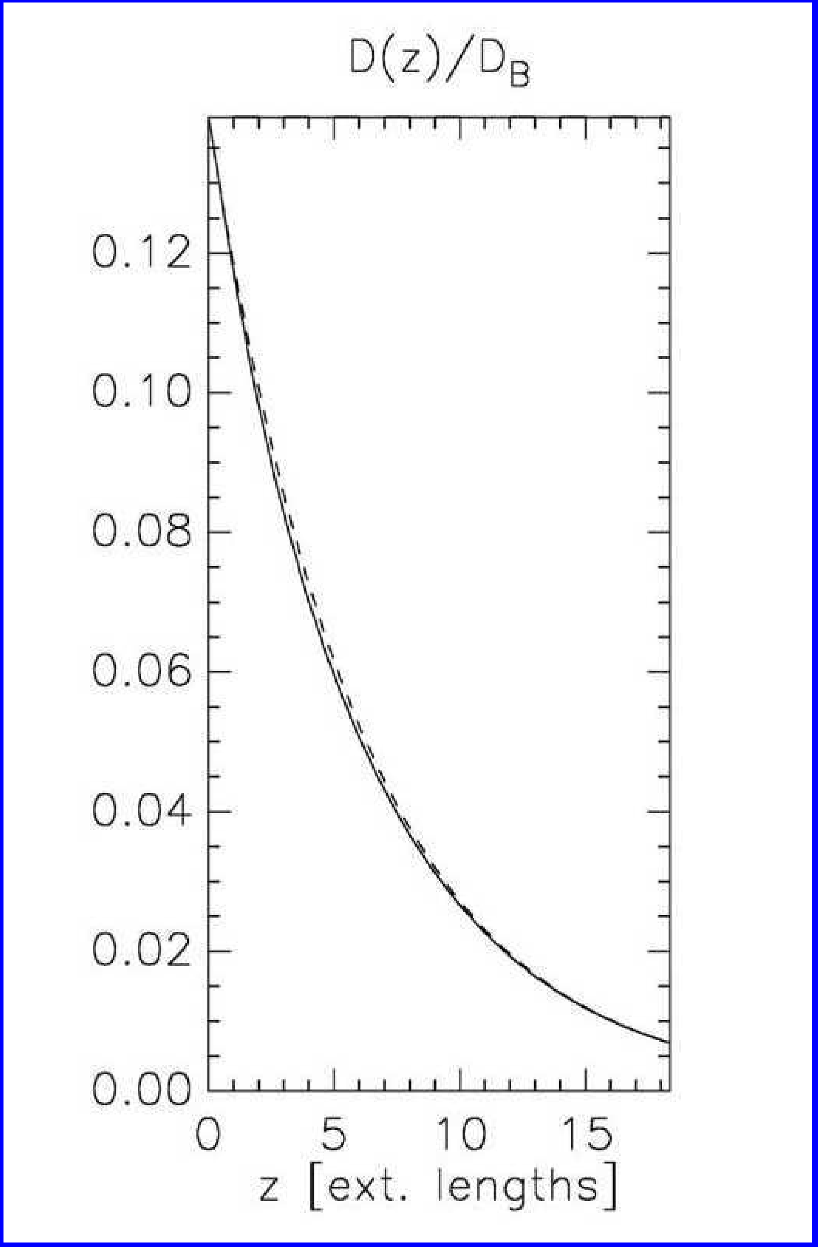
$$\Rightarrow \frac{d\tau}{dz} := \frac{1}{D} = \frac{1}{D_B} + \frac{2}{\xi}\tau \Rightarrow \boxed{D(z) = D_B \exp\left(-\frac{2z}{\xi}\right)}$$

$(\xi \propto N\ell)$ OK DMPK

Half space 3D
$k\ell=1$

Van Tiggelen, Lagendijk, Wiersma, PRL 2000

$$D(z) = \frac{D(0)}{1+z/\zeta} \Rightarrow T \propto \frac{1}{L^2}$$



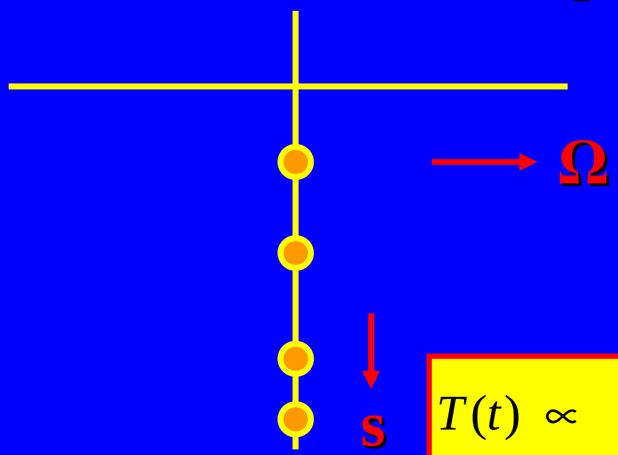
Complex Dynamics in finite open media

Skipetrov & Van Tiggelen, PRL 2004,2006

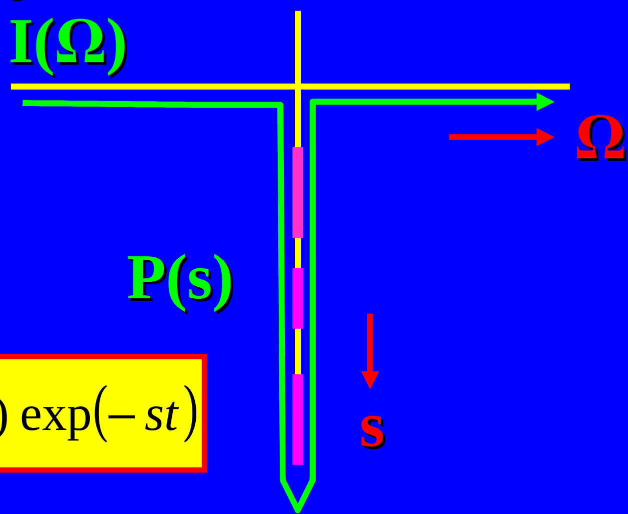
$$-s\, G(z,z',q,s) + \partial_z D(z,s)\, G(z,z',q,s) + q^2 G(z,z',q,s) = \delta(z-z')$$

$$\frac{1}{D(z,s)} = \frac{1}{D_B} + \frac{2}{k^2\ell}\int_{q<1/3\ell} d^2\mathbf{q}\; G(z,z,q,s)$$

Complex Frequency Ω+is



$$T(t) \propto \int_0^\infty ds\, G(z_s, z, s)\exp(-st)$$

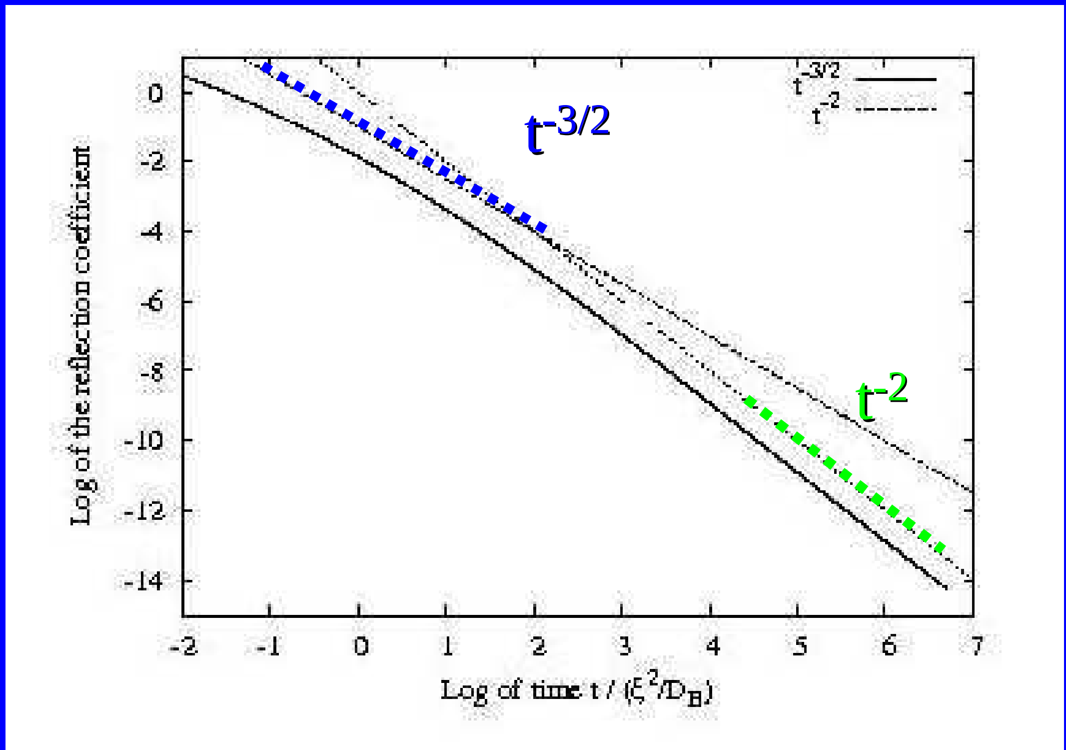


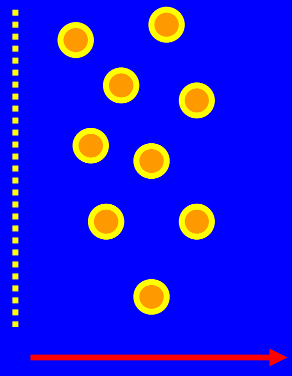
Diffuse regime: simple poles

Localized regime: Riemann sheets

3D, localized half space : kℓ=0.7

R(t)





time/(ζ² /D_B)

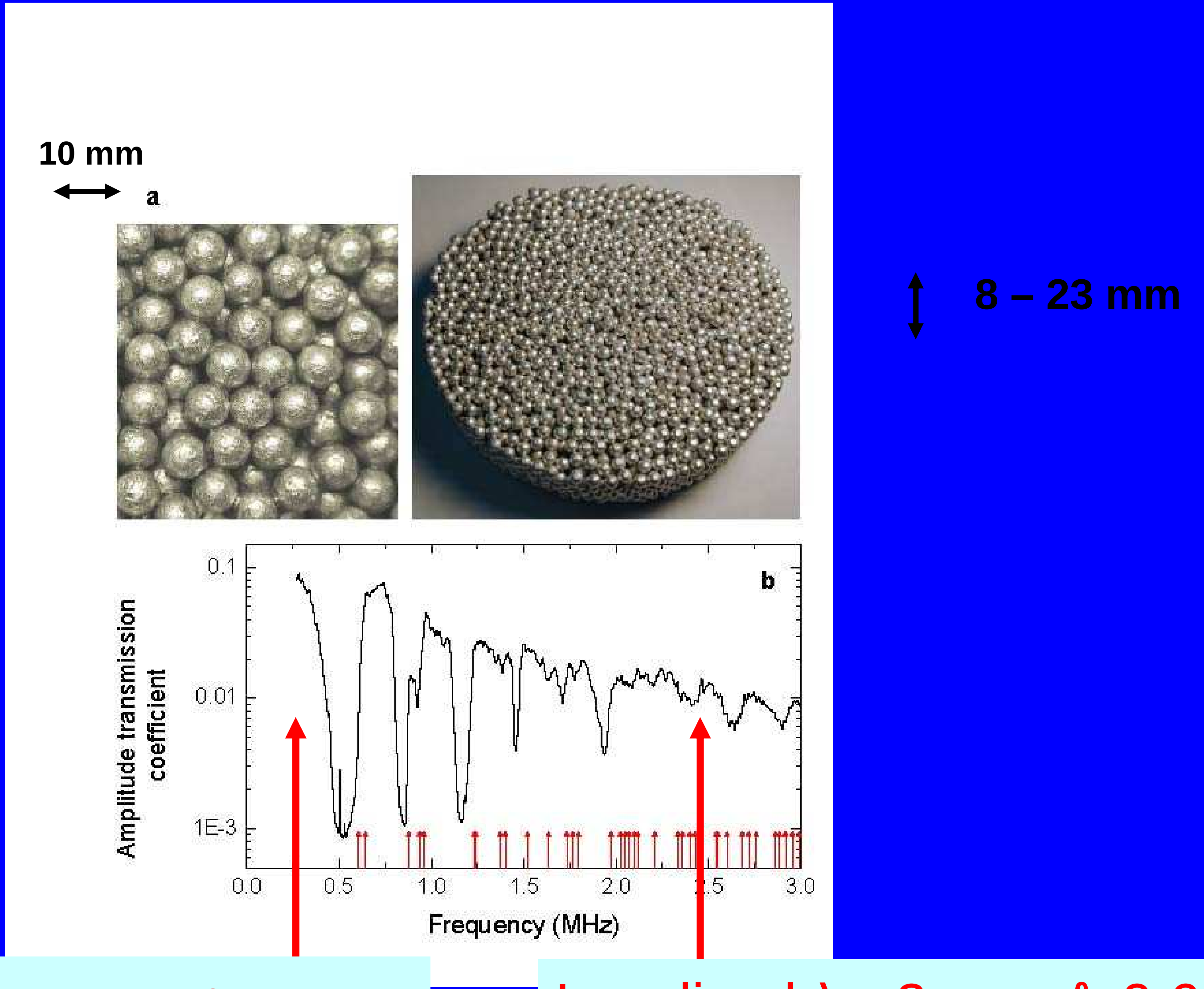
$$R(t) \propto \frac{1}{t^2}$$

1D sismologie : Sheng Papanicolaou, 1987
Q1D (DMKP) Titov, Beenakker, 2000

3D localisation of ultrasound

Hu, Page and Strybulevych, Skipetrov, Van Tiggelen

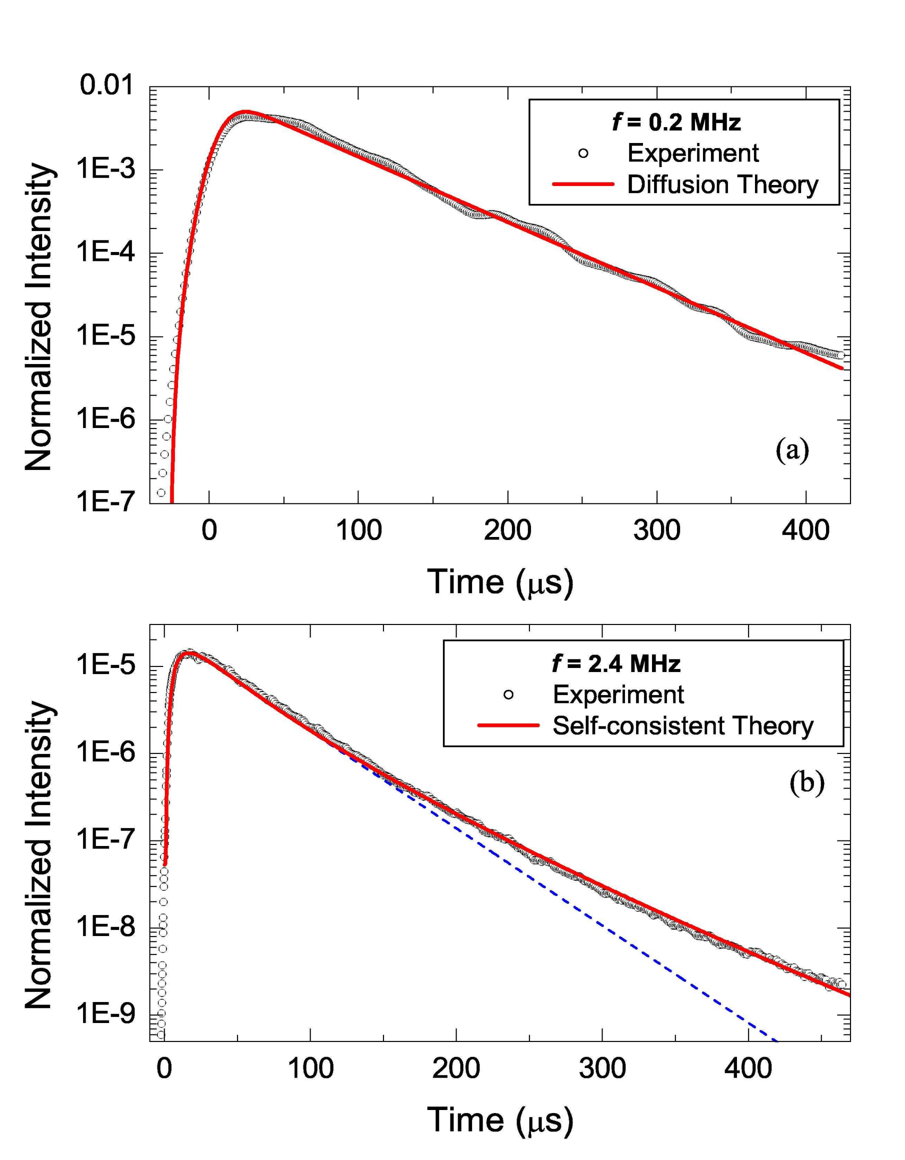
Published online: 19 October 2008; corrected online: 28 October 2008; doi:10.1038/nphys1101

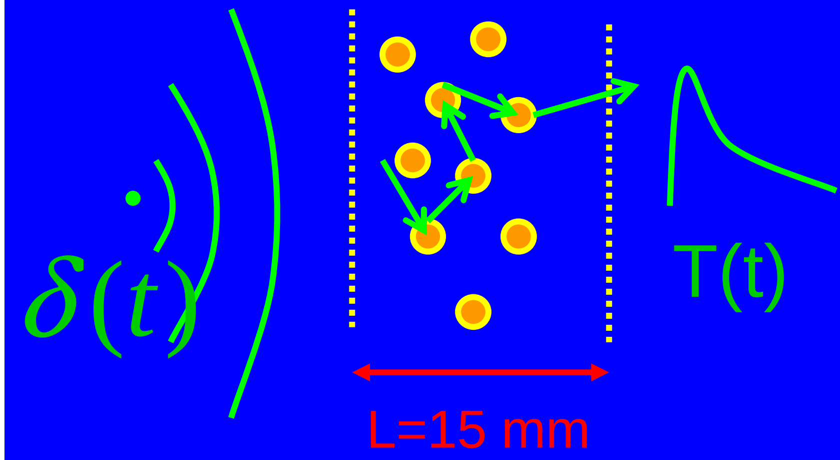


Diffuse λ_p=9 mm, ℓ=2 mm

Localized λ_p=2 mm, ℓ=0.6 mm

Time-dependent transmission



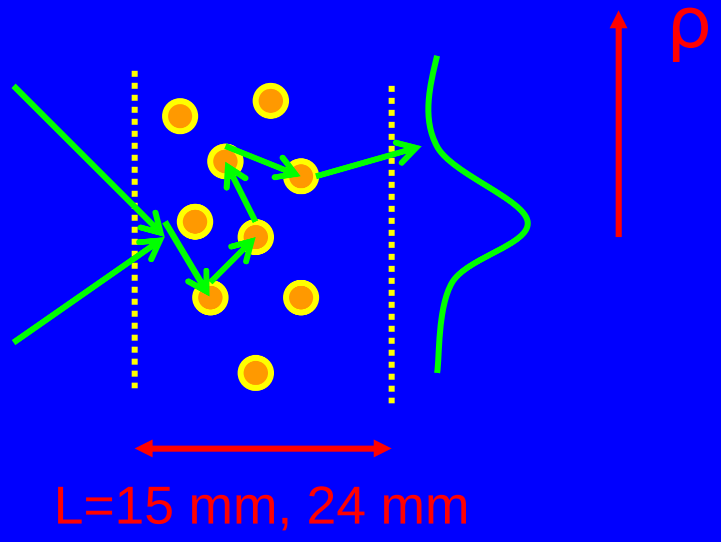


D(t) ?

BvT1

3D transverse loQUAlisation of ultrasound

Transverse loCalisation: De Vries, Lagendijk and De Raedt, PRL 1989



Diffuse: $\langle \rho^2 \rangle = 4Dt$

Near transition: $\langle \rho^2 \rangle \sim L^2$, not $t^{2/3}$

Localized: $\langle \rho^2 \rangle \sim L\xi$, not ξ^2

Skipetrov and Cherroret

Slide 16

BvT1 Bart van Tiggelen; 07/06/2009

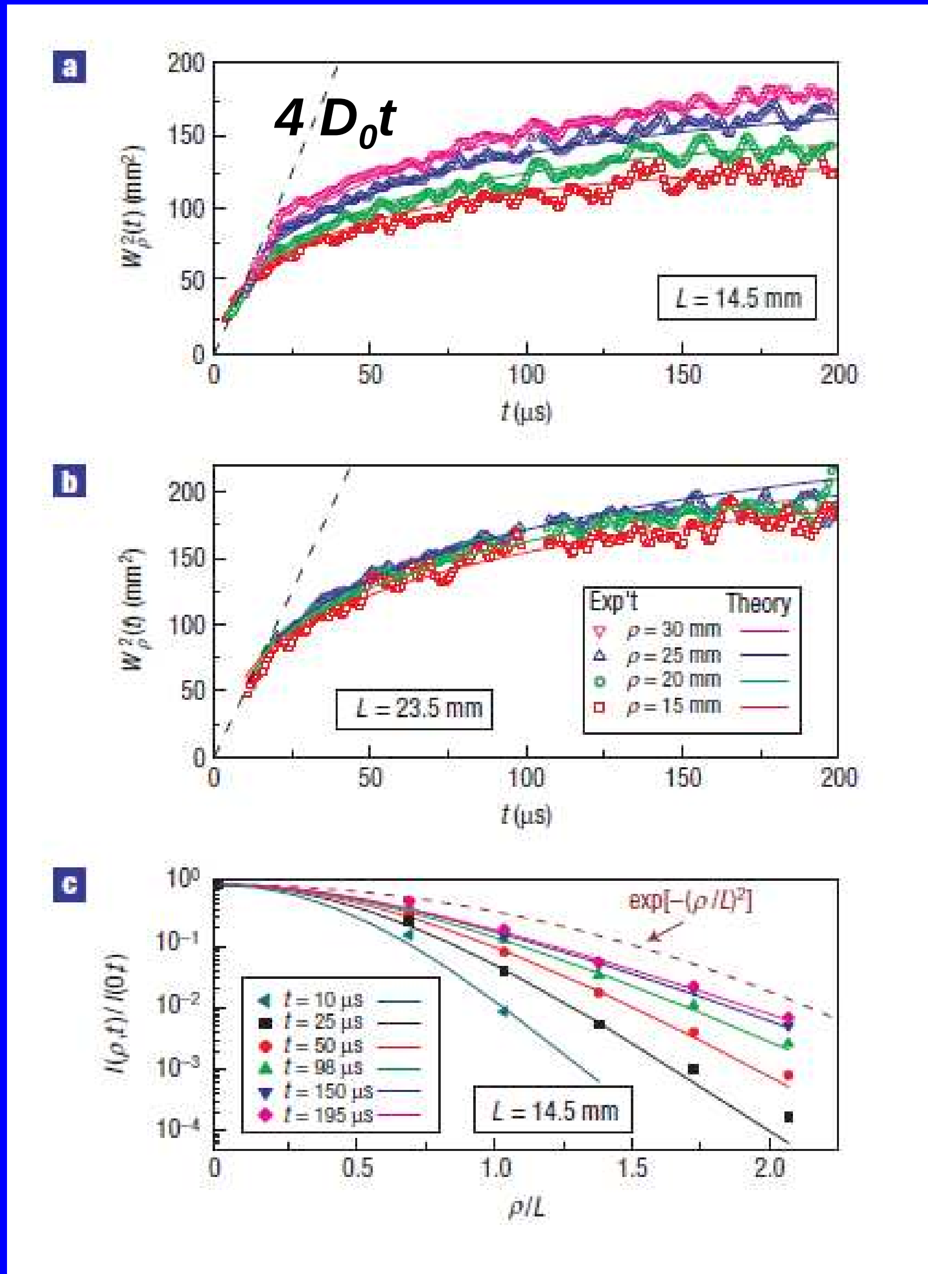
3D Transverse loQUAlisation of ultrasound

$$T(\rho,t) = T(0,t)\exp\left(-\frac{\rho^2}{w(\rho,t)^2}\right)$$

Transverse

Size $w(\rho,t)$

$T(\rho,t)/T(0,t)$



kℓ ≈ 1,82

v_E=17,4 km/s

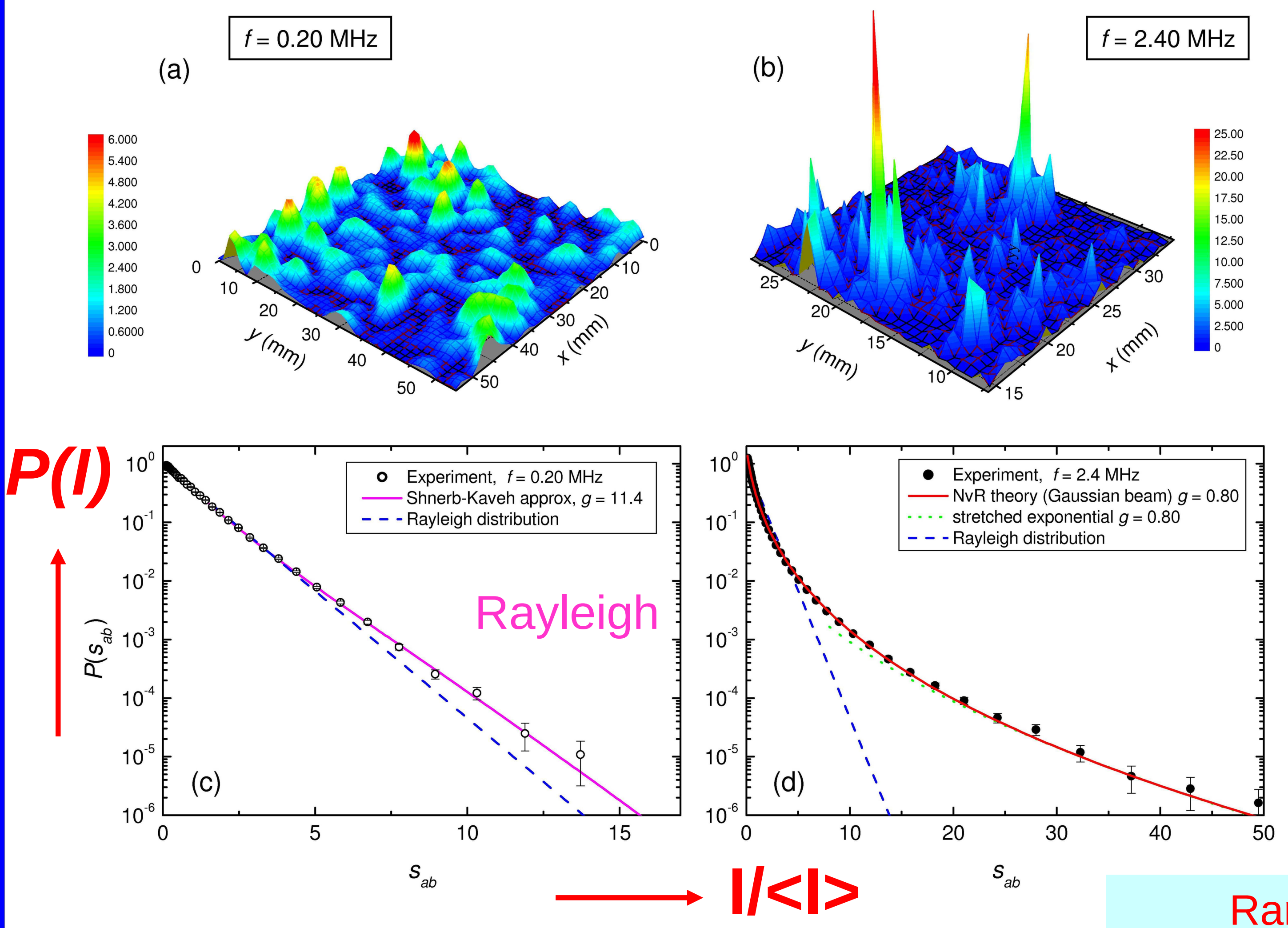
=3.5 v_p

ξ= 16,3 mm

~~D(t)~~

~~D(L)~~

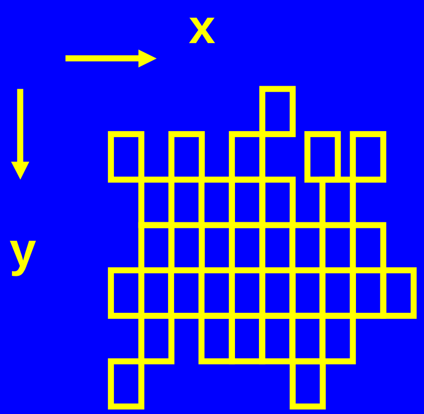
Speckle distribution of transmission



Random matrix
Rossum & Nieuwenhuizen
g=0.8 +/- 0.02

Multifractality of wave function

Faez, Page, Lagendijk and Van Tiggelen



Box counting

$$I_b = \frac{\int_{b^d} d^d\mathbf{r}\, I(\mathbf{r})}{\int_{L^d} d^d\mathbf{r}\, I(\mathbf{r})} \qquad \lambda << b << L$$

$$P_q = \sum_b (I_b)^q = \left(\frac{L}{b}\right)^{-d(q-1)+\Delta(q)}$$

Generalized Inverse Participation Ratio

$$P(\log I_b) \propto \left(\frac{L}{b}\right)^{f\left(-\frac{\log I_b}{\log L/b}\right)}$$

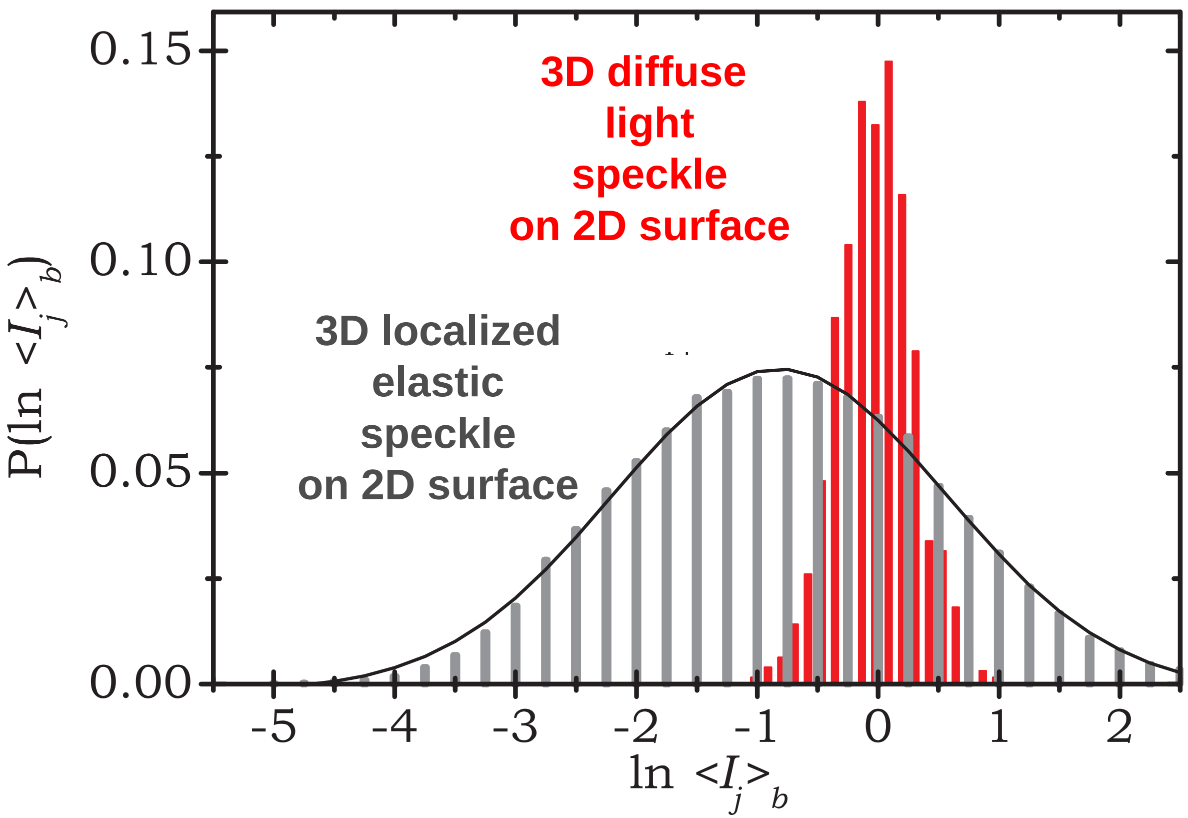
Probability Distribution Function

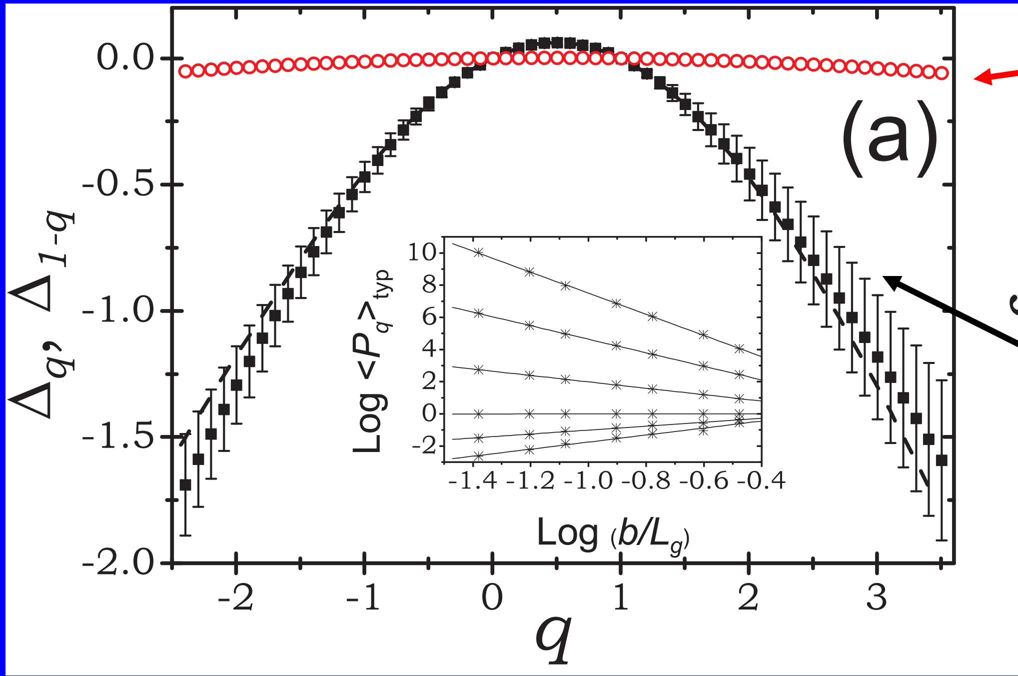
$$\Delta(q) = \gamma q(1-q) \Leftrightarrow f(\alpha) = -\frac{1}{4\gamma}(\alpha - d - \gamma)^2$$

Anomalous gIPR **Lognormal PDF**

Multifractality of wave function

Faez, Page, Lagendijk and Van Tiggelen

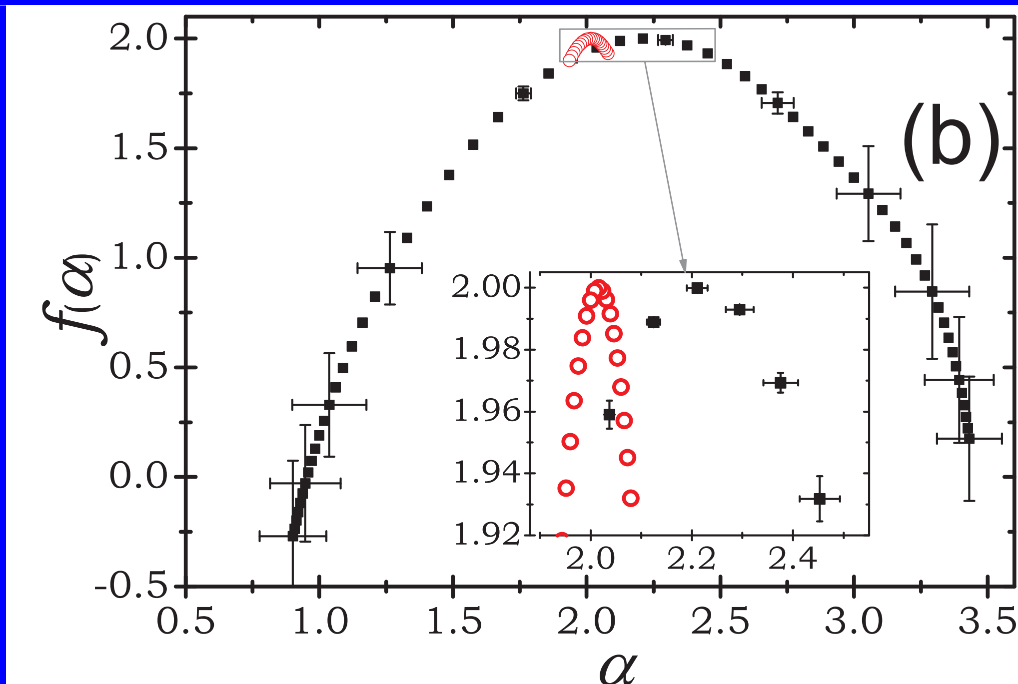




Diffuse light

Localized ultrasound

$$\Delta_q \approx 0.21\, q(1-q)$$



is

$$\gamma \propto \frac{1}{g}$$

?

Observation of 3D ultrasound Localization

- ✓ Transverse confinement: D(r)!
- ✓ Anomalous dynamic transmission
- ✓ Giant non-poissonian fluctuations $g < 1$
- ✓ Multifractal wave function
- ? $1/t^2$ reflection
- ? Critical exponent