Quasi-Two-Dimensional Trapped Bose Gases

Markus Holzmann

LPTMC, CNRS, Universite Pierre et Marie Curie, Paris

LPMMC, CNRS, Université Joseph Fourier, Grenoble, France

- quasi2D trapped Bosons: Kosterlitz-Thouless ↔ BEC
- density profile: universal correlations
- QMC ↔ experiment: direct comparisions

  density profile
  coherence properties
  superfluidity
- disorder ?

Physics in two dimensions (2D)

- Enhanced (quantum) fluctuations:

  → Absence of long-range order at finite temperture T
  (Mermin, Wagner, Hohenberg,...theorem)

  → Bose-Einstein phase transition (BEC) only at T=0

- Kosterlitz-Thouless phase transtion (KT) at $T_c=\pi\rho_s/2m^2$

  "topological phase transition" ρ_s: superfluid mass density

- Exception: Ideal Bose gas in 2D harmonic trap:
  BEC transition at finite T no KT

$$N=\sum_{\nu_x,\nu_y}\frac{1}{e^{\beta(\varepsilon_\nu-\mu)}-1}\simeq\int\frac{d^2\mathbf{r}\,d^2\mathbf{k}}{(2\pi)^2}\left(\exp\left[\beta(k^2/2m+v(r)-\mu)\right]-1\right)^{-1}$$

V. Bagnato, D. Kleppner, PRA 44, 7439 (1991)

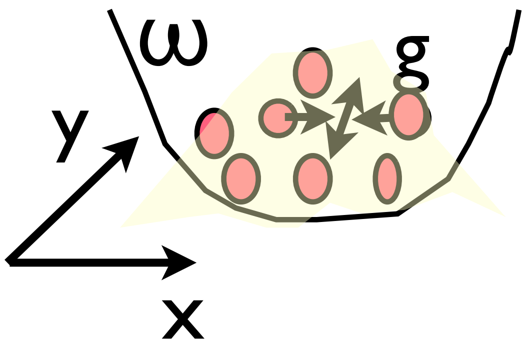
$$=-\int\frac{d^2\mathbf{r}}{\lambda^2}\log\left(1-\exp\left[\beta(\mu-v(r))\right]\right)\leq N_c(T)$$

critical number of Bosons, N_c, at finite T $\lambda^2=2\pi/mT$

Two-dimensional Ultracold Bosons in a harmonic trap: Bose-Einstein Condensation (BEC) or Kosterlitz-Thouless transition (KT)?

basic model: N Bosons

with short-ranged (hard-core) interaction in isotropic trap



ω: trap frequency
g: interaction constant
N: number of Bosons

$g \sim (\log na^2)^{-1}$ for hard disks of diameter a

thermodynamic limit in 2D trap: $N \to \infty$ and $\omega \to 0$ with $N\omega^2$=const

mean extension R_T of cloud: $R_T = (T/m\omega^2)^{1/2}$ for temperature T=const

➡ Certainly superfluid phase transition at finite T!

Bose-Einstein Condensation (BEC) or Kosterlitz-Thouless transition (KT)?

Superfluidity <=> non-classical moment of inertia:

$$I_{nc} < I_{cl} = \int d\mathbf{r}\, r^2 n(r) \qquad \text{jump in } I_{nc} \text{ at } T_c?$$

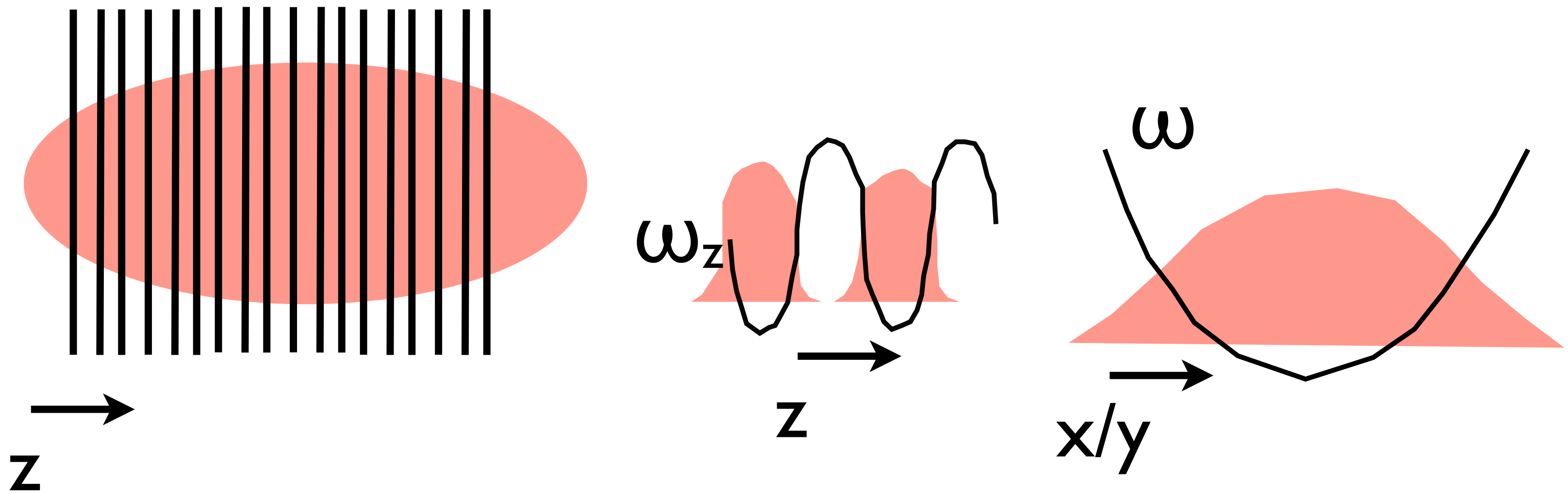
BEC => Reduced 1-body density:

$$\langle \psi^+(\mathbf{r})\psi(\mathbf{r}')\rangle = \sum_{nl} N_{nl}\, \varphi^*_n(r)\varphi_n(r') \cos(l\theta_{rr'})$$

BEC: long-range order	KT: algebraic order
$T > T_{BEC}$: $N_{nl} \sim 1$, $I = I_{cl}$	$T > T_{KT}$: $N_{nl} \sim 1$, $I = I_{cl}$
$T < T_{BEC}$: $N_{00} \sim N$, $I_{nc} < I_{cl}$	$T < T_{KT}$: $N_{00} \sim N^{1-\eta(T)/2}$, $I_{nc} < I_{cl}$

quasi 2D Bose gas: Experimental realisation

Z. Hadzibabic, P. Krüger, M. Cheneau, B. Battelier, J. Dalibard, Nature 441, 1118 (2006);
P. Krüger, Z. Hadzibabic, J. Dalibard, PRL 99, 040402 (2007).



3D BEC + 1D optical lattice=> quasi 2D slices with $\omega_z/\omega \approx 300$

relatively strong interaction $g \simeq 0.13$ Boson number $N \sim 10^4$/slice

quasi 2d situation:

- chemical potential $\mu \lesssim \omega_z$
- few excited states in z thermally occupied $T \lesssim \omega_z$

QMC simulations of Quasi-2D Bose gases

M. H., W. Krauth, PRL 100, 190402 (2008)

based on Feynman's path integral representation of the N-particle density matrix

$$\rho(\mathbf{R},\mathbf{R}') = \frac{1}{Z}\langle \mathbf{R}|e^{-\beta H}|\mathbf{R}'\rangle \qquad \langle \mathbf{R}|e^{-\beta H}|\mathbf{R}'\rangle \equiv \int d\mathbf{R}_2\, \langle \mathbf{R}|e^{-\beta H/2}|\mathbf{R}_2\rangle\langle \mathbf{R}_2|e^{-\beta H/2}|\mathbf{R}'\rangle$$

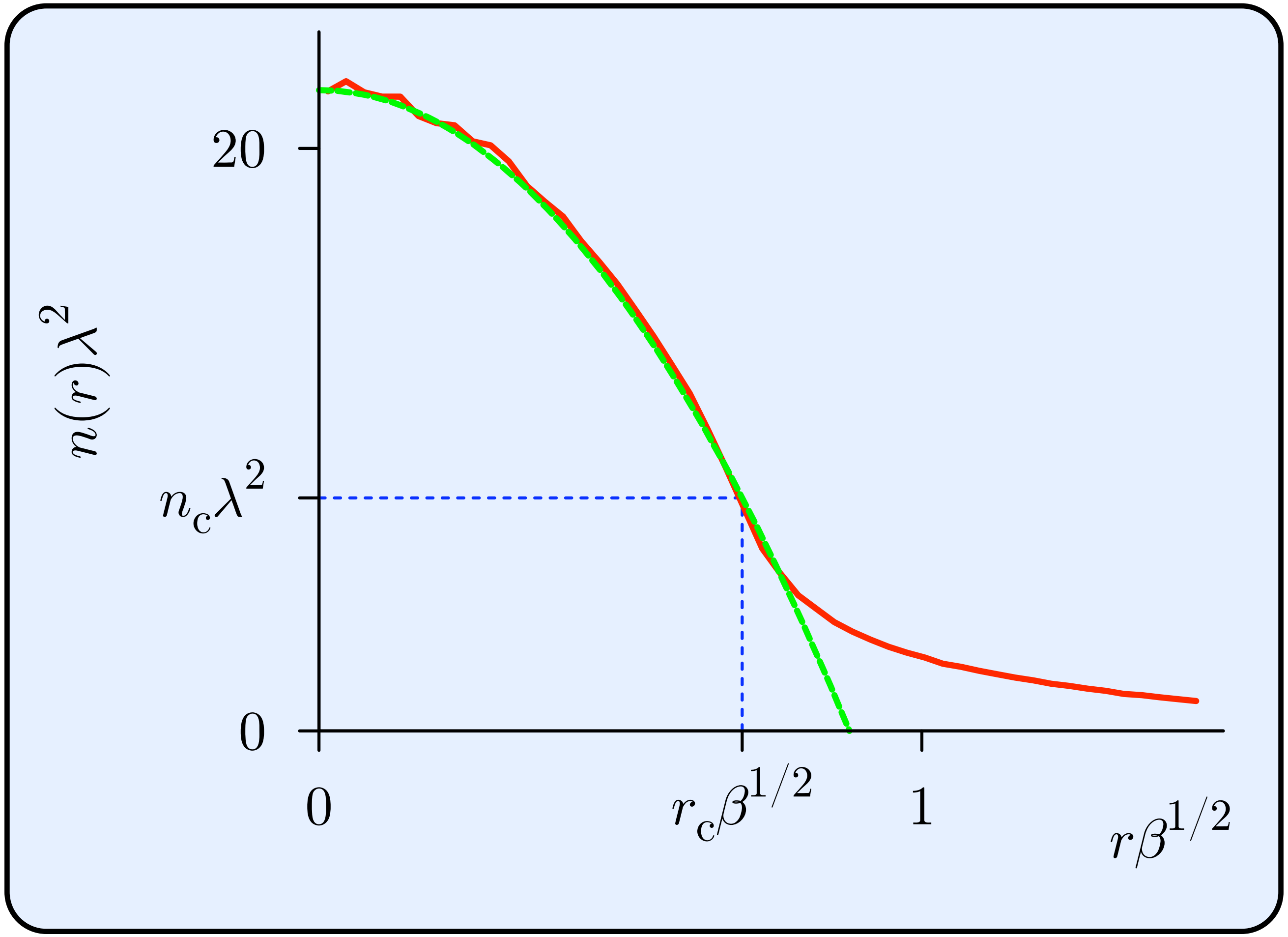
Bosons: $$\rho_B(\mathbf{R},\mathbf{R}') = \frac{1}{Z_B}\frac{1}{N!}\sum_P (-1)^{|P|}\rho(\mathbf{R},P(\mathbf{R}')) \qquad \mathbf{R} \equiv (\mathbf{r}_1,\mathbf{r}_2,\ldots \mathbf{r}_N)$$

Parameters of the simulation

- Full 3D path-integral simulation in quasi 2D geometry
- 3D hard-core interaction (s-wave scattering length a)
- 2D interaction strength at T=0: $g=(8\pi\omega_z)^{1/2}a = 0.13$
- quasi 2D scaling: $\omega_z/T_{BEC}=0.55$
- Number of Bosons N=2 250 up to N=576 000
- Temperature range from $T=0.5\,T_{BEC}$ to $T=T_{BEC}$

QMC density profiles: $T<T_{KT}$

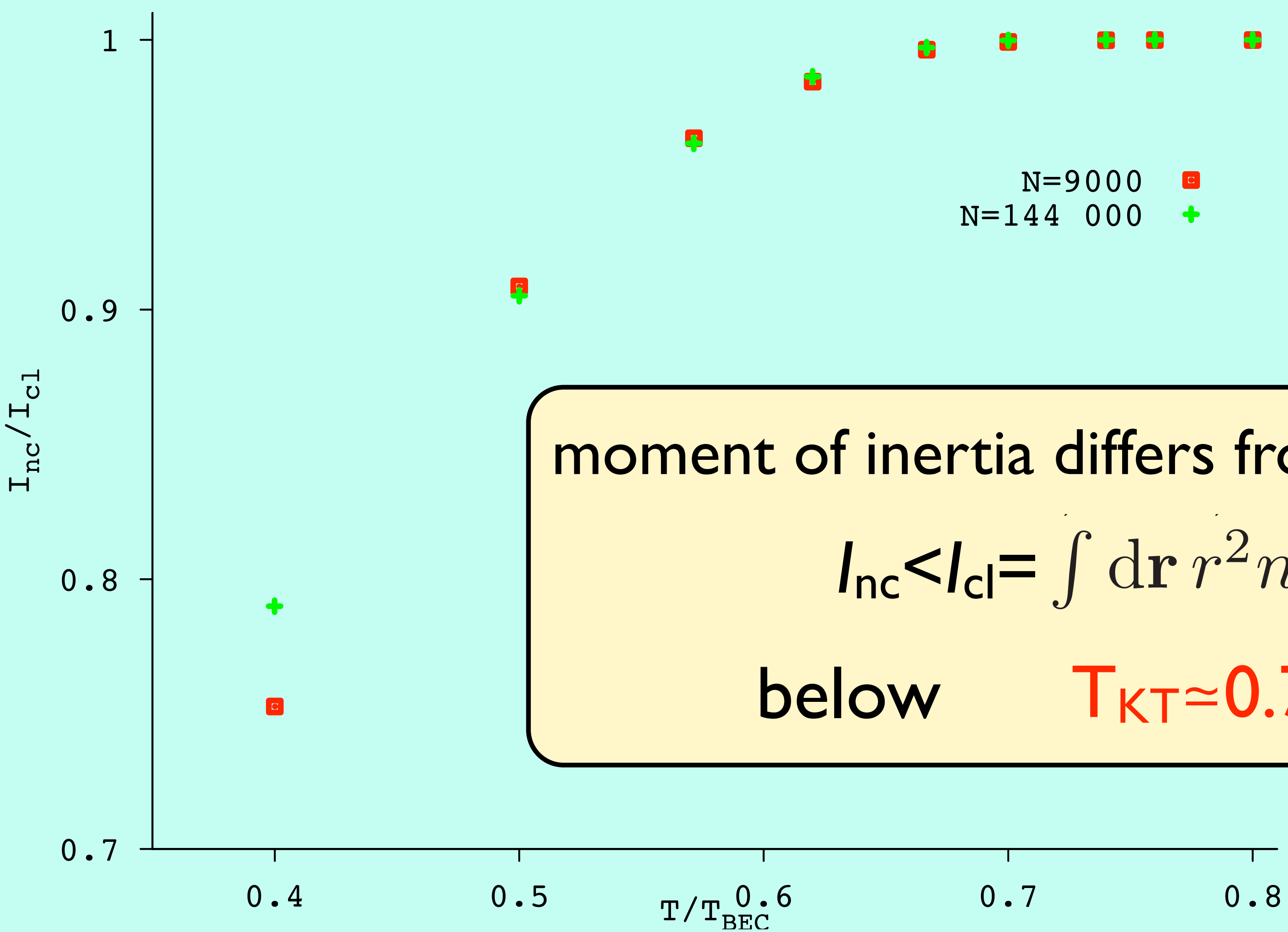
$T=0.5\ T_{BEC}$



Thomas-Fermi shape of the superfluid density

with effective quasi-2D interaction g

QMC superfluidity: non-classical moment of inertia



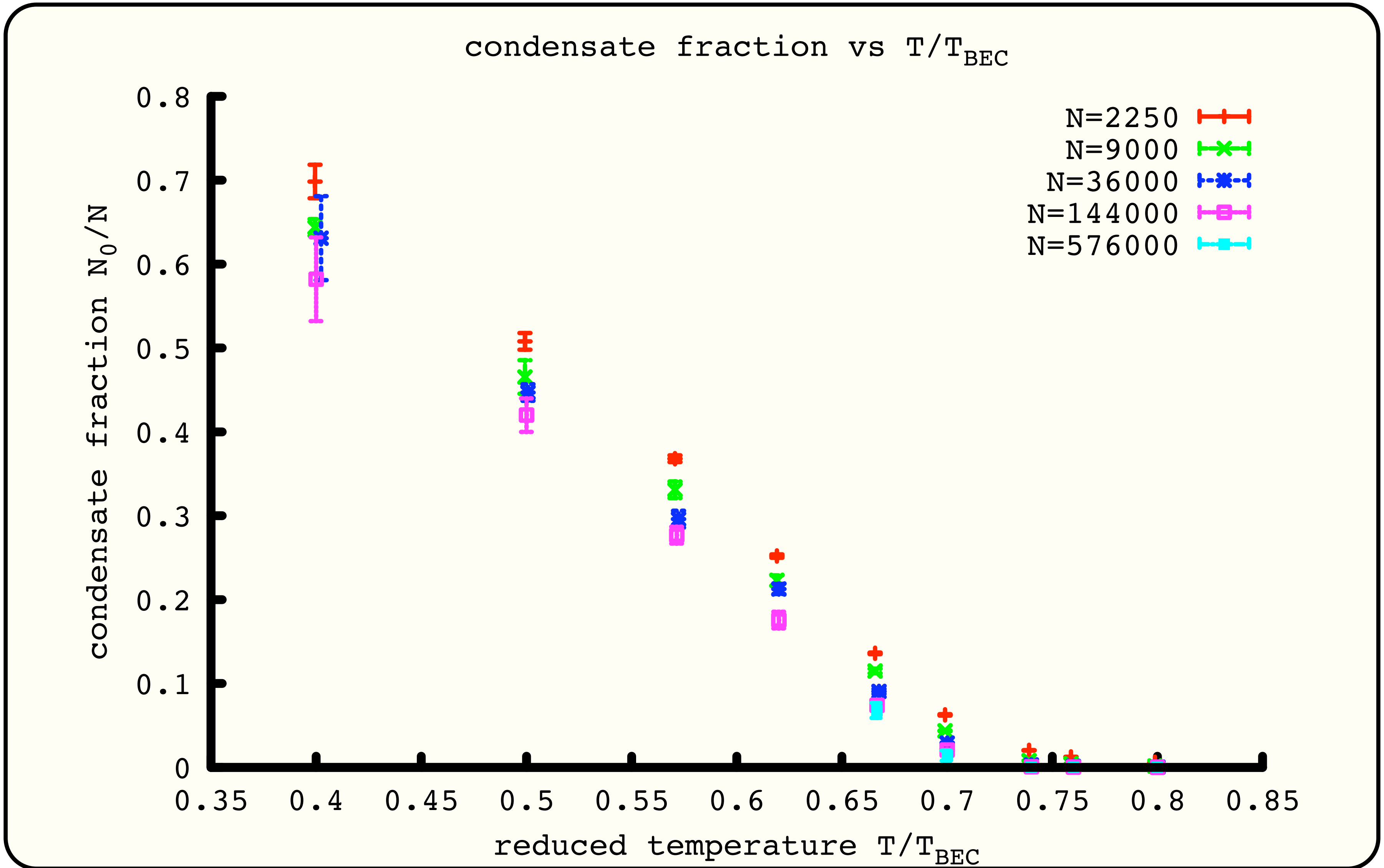
moment of inertia differs from classical value

$$I_{nc} < I_{cl} = \int \mathrm{d}\mathbf{r}\, r^2 n(r)$$

below $T_{KT} \simeq 0.70\, T_{BEC}$

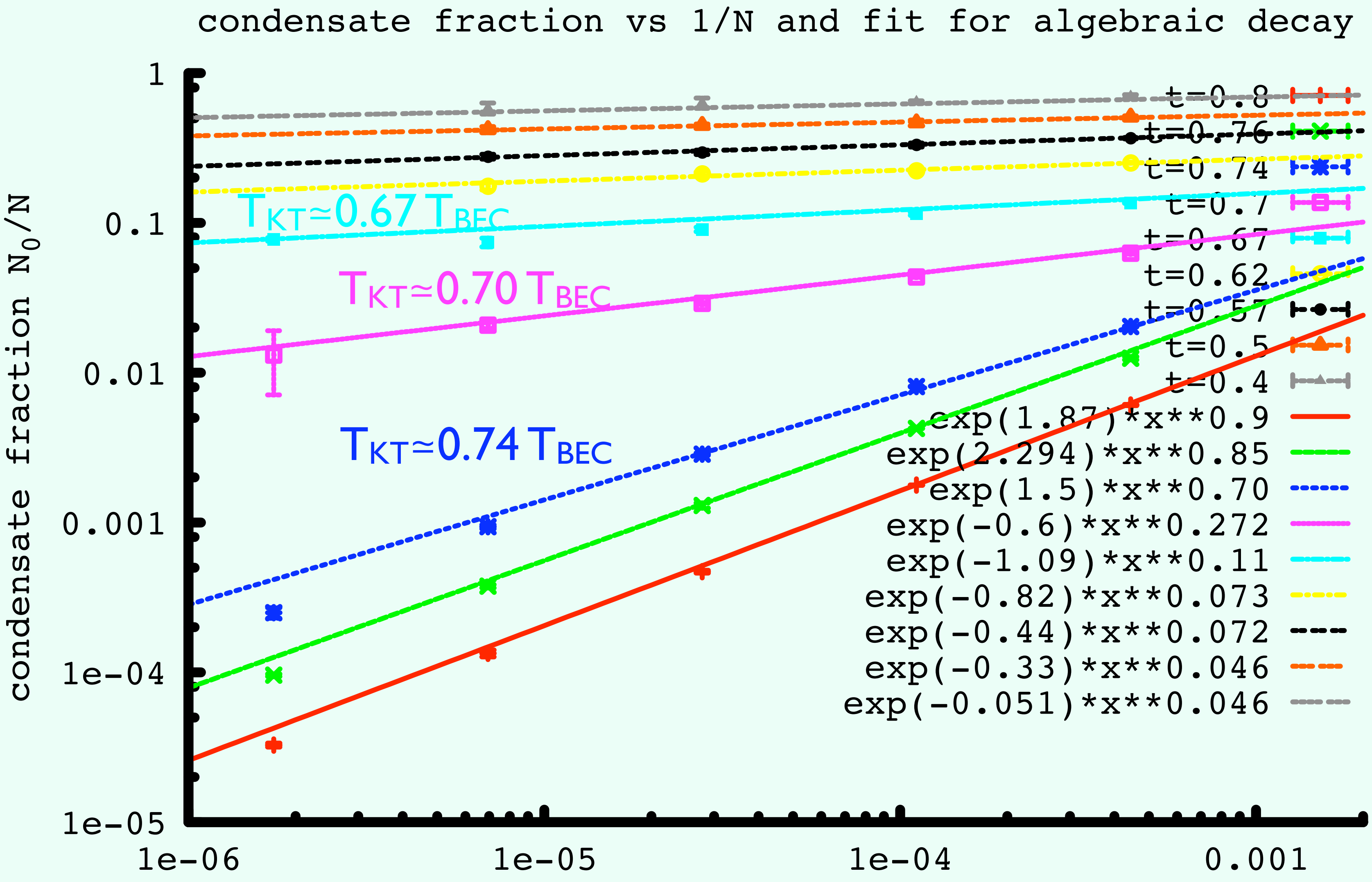
QMC condensate fraction (I): BEC dominates...

strong finite-size effects !



QMC condensate fraction: (II) KT behaviour

$N_0/N \sim N^{-\eta(T)/2}$ $\eta(T_{KT})=1/4$ $T_{KT} \simeq 0.70\, T_{BEC}$



Fluctuation and correlation region

M. H., M. Chevallier, W. Krauth, PRA 81, 043622 (2010)

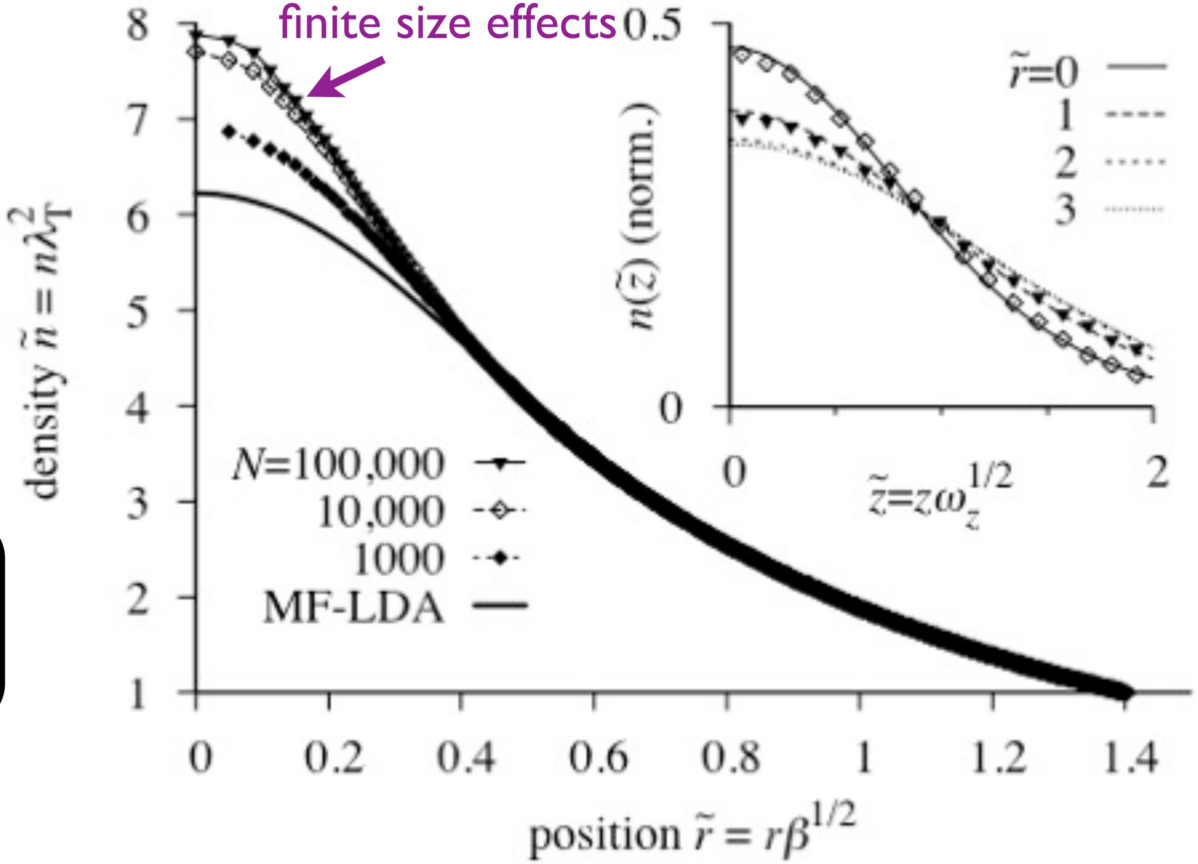
Phase space density profile $n(r)\lambda^2$ close to T_{KT}

Correlation density {

How to quantify correlations?

Can the local density approximation be used?

strong finite size effects



Quantitative comparisions with approximations based on classical field theory

(N. Prokof'ev, O. Ruebenacker, B. Svistunov, PRL 87, 270402 (2001)
L. Giorgetti, I. Carusotto, Y. Castin, PRA 76, 013613 (2007)
R.N. Bisset, M.J. Davis, T.P. Simula, P.B. Blakien PRA 79, 033626 (2009))

Universal leading order corrections to mean-field: homogeneous system

- homogeneous 2D system: scale invariance of the phase space density

$n\lambda^2$ is a function of Δ=-$\beta\mu$ and g

➪ ideal gas: $n_{id}\lambda^2 = -\log[1-\exp(-\Delta)]$

➪ mean field: $n_{mf}\lambda^2 = -\log[1-\exp(-\Delta_{mf})]$

mean field «gap»: $\Delta_{mf}=\Delta+g n_{mf}\lambda^2/\pi$

➪ correlation corrections: leading order by classical field theory

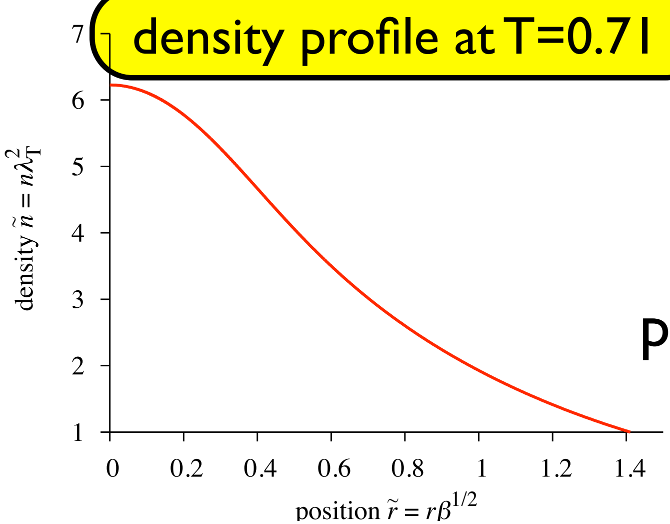
$_\Delta n\lambda^2 = (n-n_{mf})\lambda^2$ is a universal function of Δ_{mf}/g

function calculated and tabulated in N. Prokof'ev, B. Svistunov PRA 66, 043608 (2002).

Quasi2D trapped system:
Local density approximation (LDA)

local density approximation (LDA) for the mean-field gap:

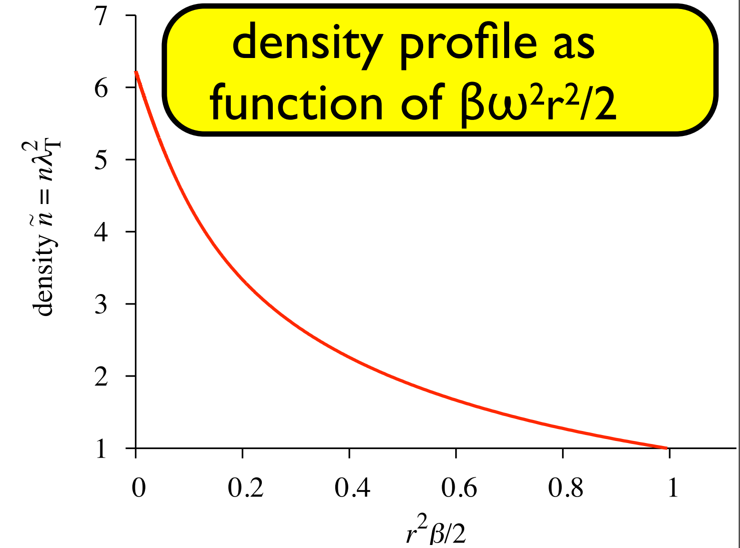
density profile at T=0.71



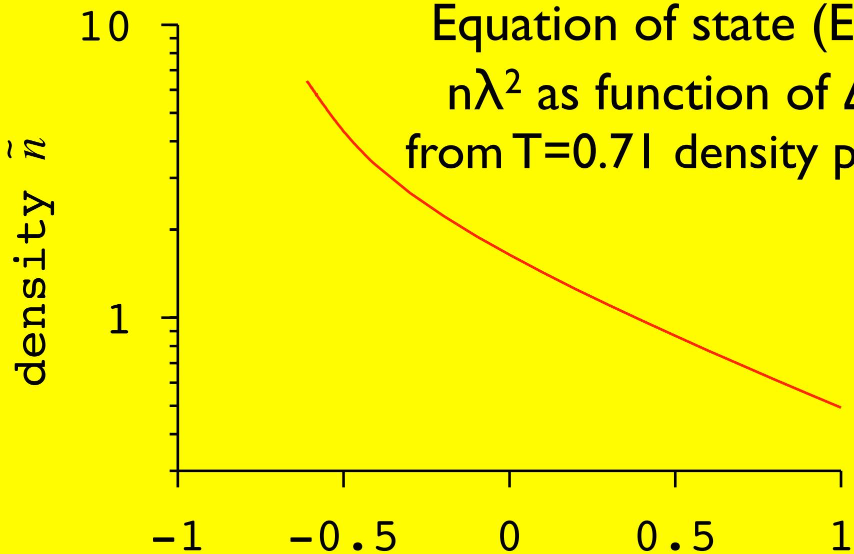
$\Delta_{mf}(r) = \Delta_{mf}(0) + \beta\omega^2 r^2/2$

provides mapping $r \leftrightarrow \Delta_{mf}$

density profile as function of $\beta\omega^2 r^2/2$



Equation of state (EOS)
$n\lambda^2$ as function of Δ_{mf}
from T=0.71 density profile

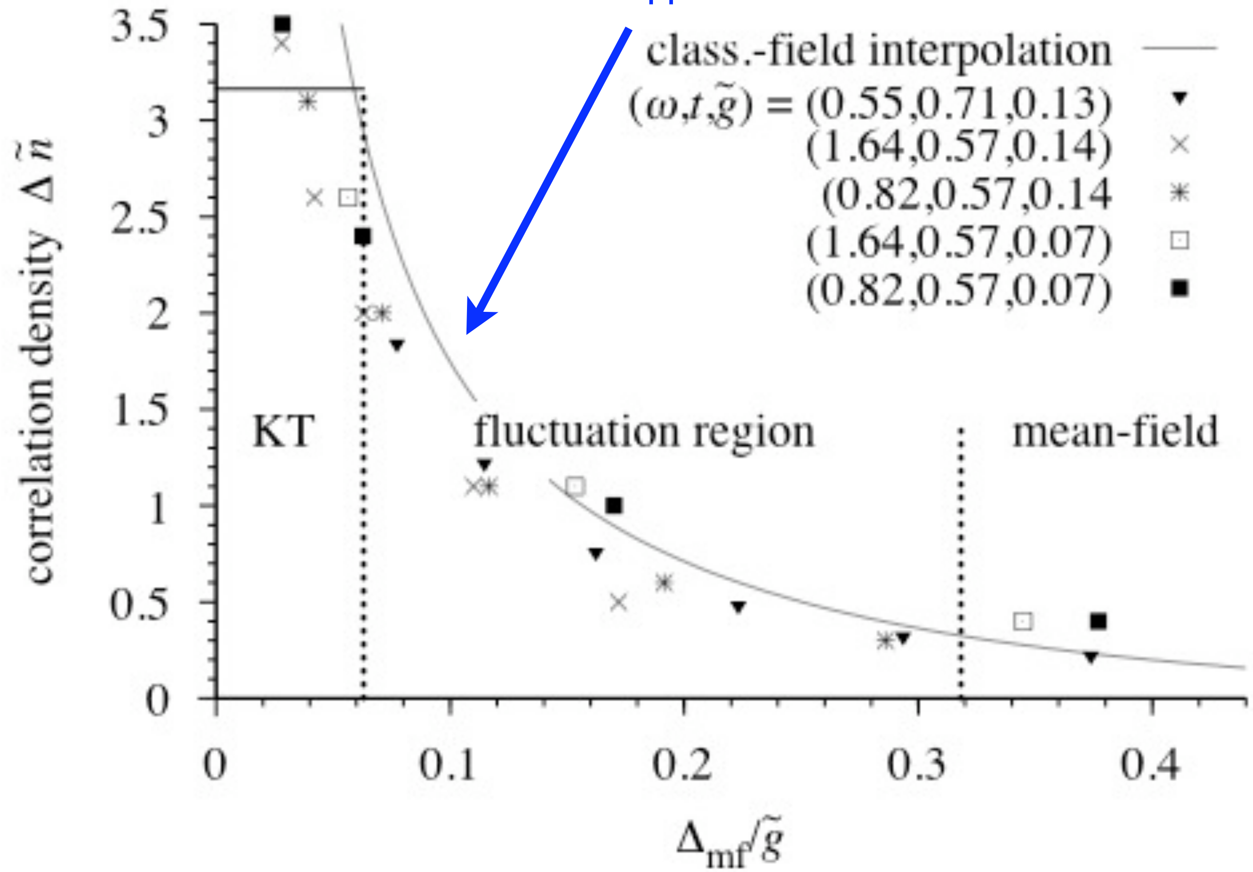


Correlation density and Universality: quasi2D trapped system

correlation density $\Delta n\lambda^2 = (n-n_{mf})\lambda^2$ vs LDA-gap Δ_{mf}
from many systems with different ω_z, T, g

$\Delta n\lambda^2 = (n-n_{mf})\lambda^2$

classical field approximation: N. Prokof'ev, B. Svistunov PRA 66, 043608 (2002).

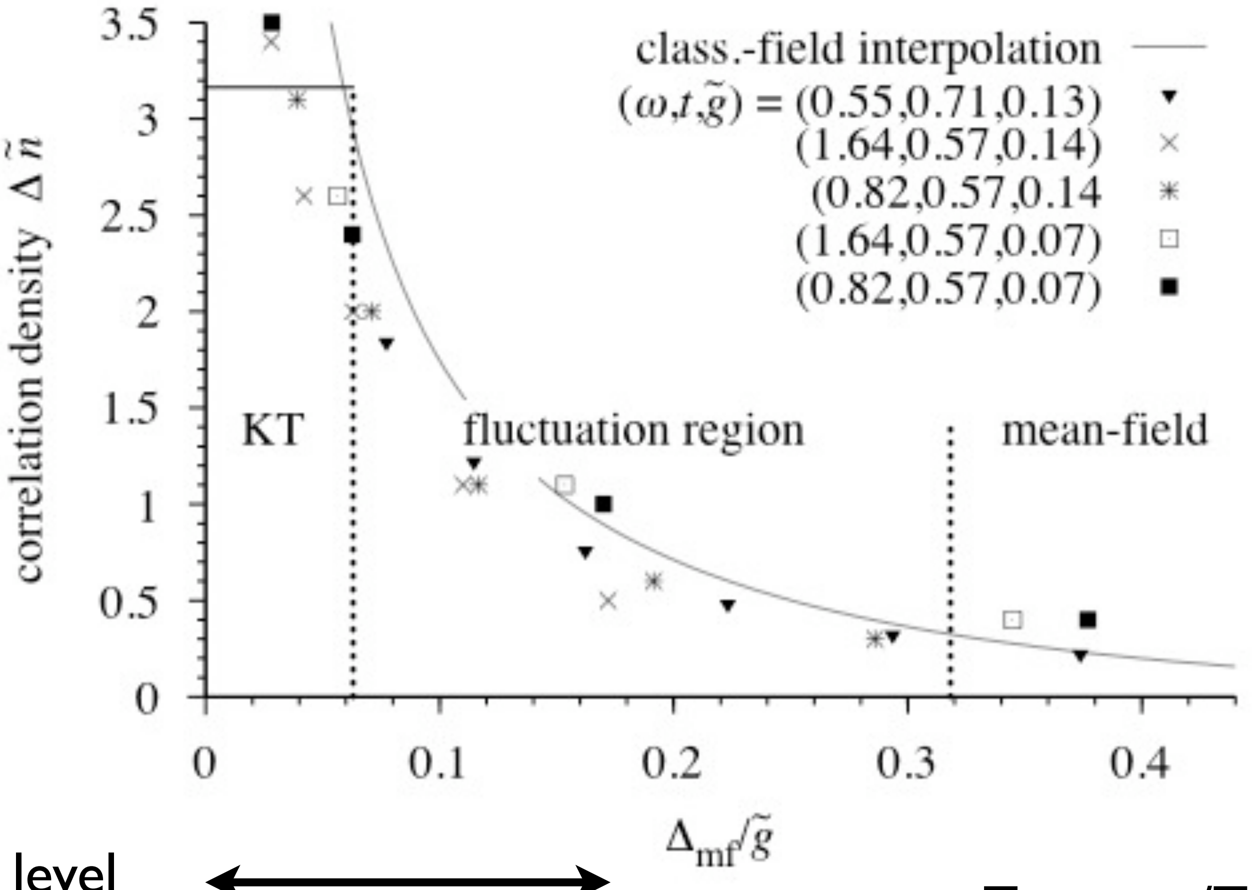


Δ_{mf} /g

full QMC calculations in quasi2D trap

M. H., M. Chevallier, W. Krauth, Phys. Rev. A 81, 043622 (2010)

Finite size effects: Cross-over to BEC for small N



finite system: level spacing in trap: ω

$\omega/(gT)$

at T_{BEC}: $\omega/T_{BEC} = \pi\,(6\,N)^{-1/2}$

Cross-over to mean-field physics for $\omega/T_{BEC} > g/\pi$

mean-field (with BEC) for $N < \pi^4/(6\,g^2)$

Theory (QMC) ↔ Experiment: density profile

Scale invariance in time-of-flight expansion in 2D

S.P. Rath, T. Yefsah, K.J. Günter, M. Cheneau, R. Desbuquois, M.H., W. Krauth, J. Dalibard,
Phys. Rev. A 82, 013609 (2010)

density profile after 2D time-of-flight given by scaling transform $n(\boldsymbol{r},t) = \eta_t^2\, n_{\text{eq}}(\eta_t \boldsymbol{r})\ , \qquad \eta_t = (1+\omega^2 t^2)^{-1/2}$

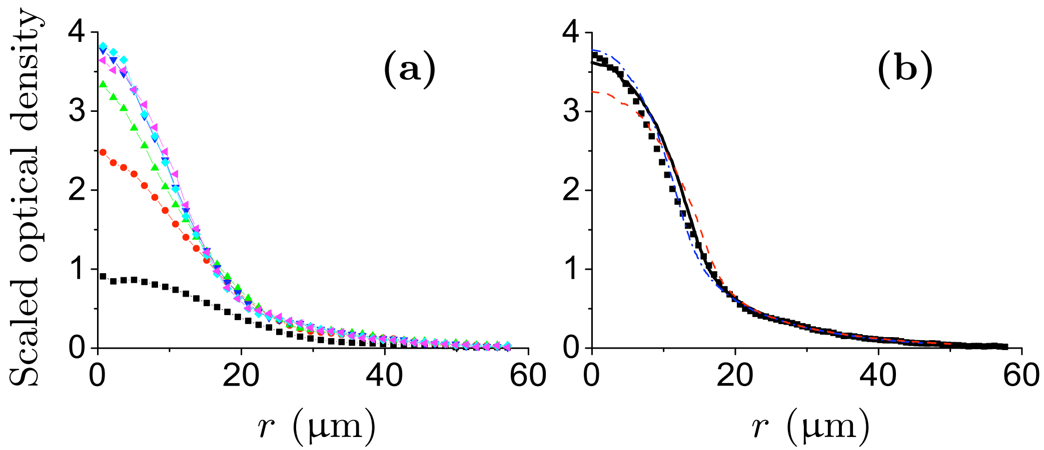
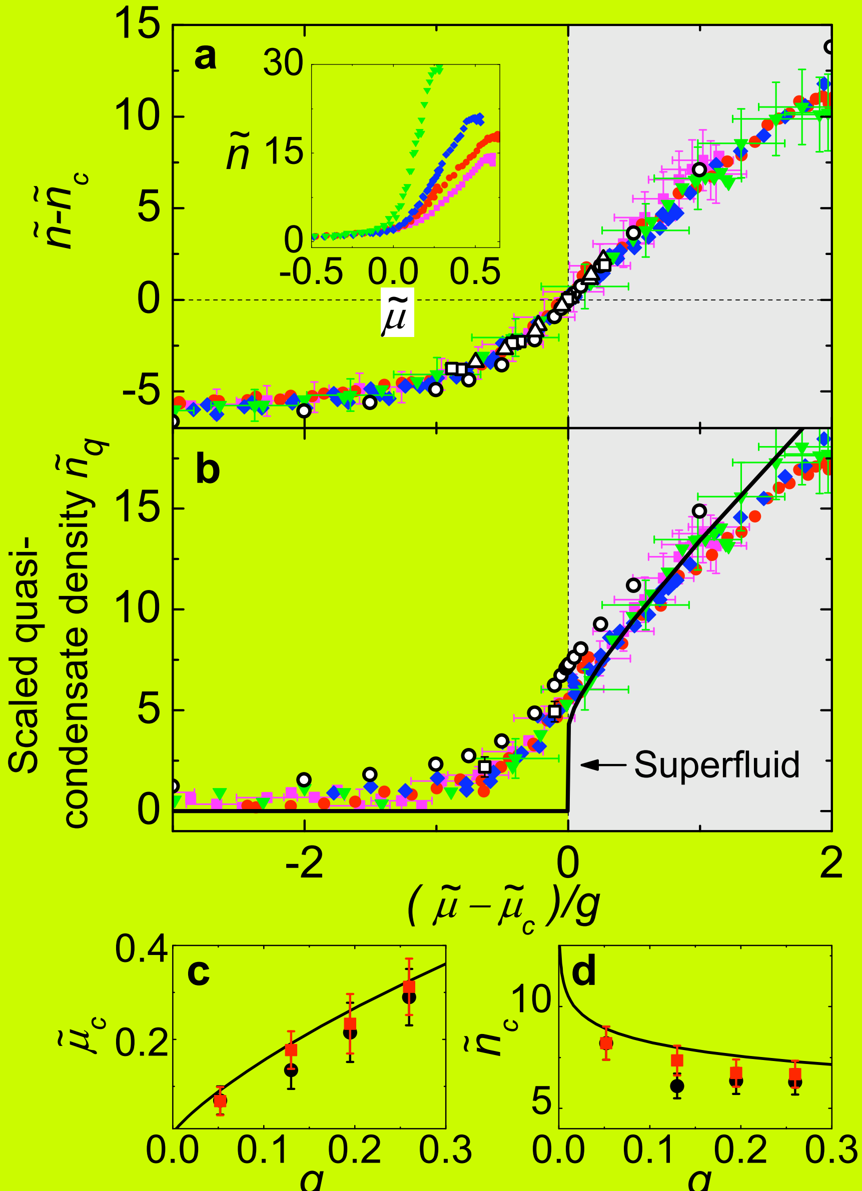


FIG. 4: (Color online) (a) Optical density profiles obtained for a TOF duration $t = 0$ (black), 3 (red), 6 (green), 10 (blue), 12 (cyan), 14 (magenta) ms and rescaled to their in-situ value according to (1). (b) Squares: Optical density profile obtained by averaging the results of (a) for $10 \leq t \leq 14$ ms, yielding fit parameters $T = 94$ nK, $\alpha = 0.36$ for $\xi = 0.63$. Lines: QMC results for the same fit parameters (continuous, $N = 42000$), and for those deduced assuming $\xi = 0.47$ [dashed red, $(T\ (\text{nK}), \alpha, N) = (104, 0.39, 57600)$] and $\xi = 0.79$ [dash-dotted blue, $(87, 0.33,\ 32100)$].

QMC and experiment in good agreement!

Measurement of Critical Densities/ Temperature and Universality

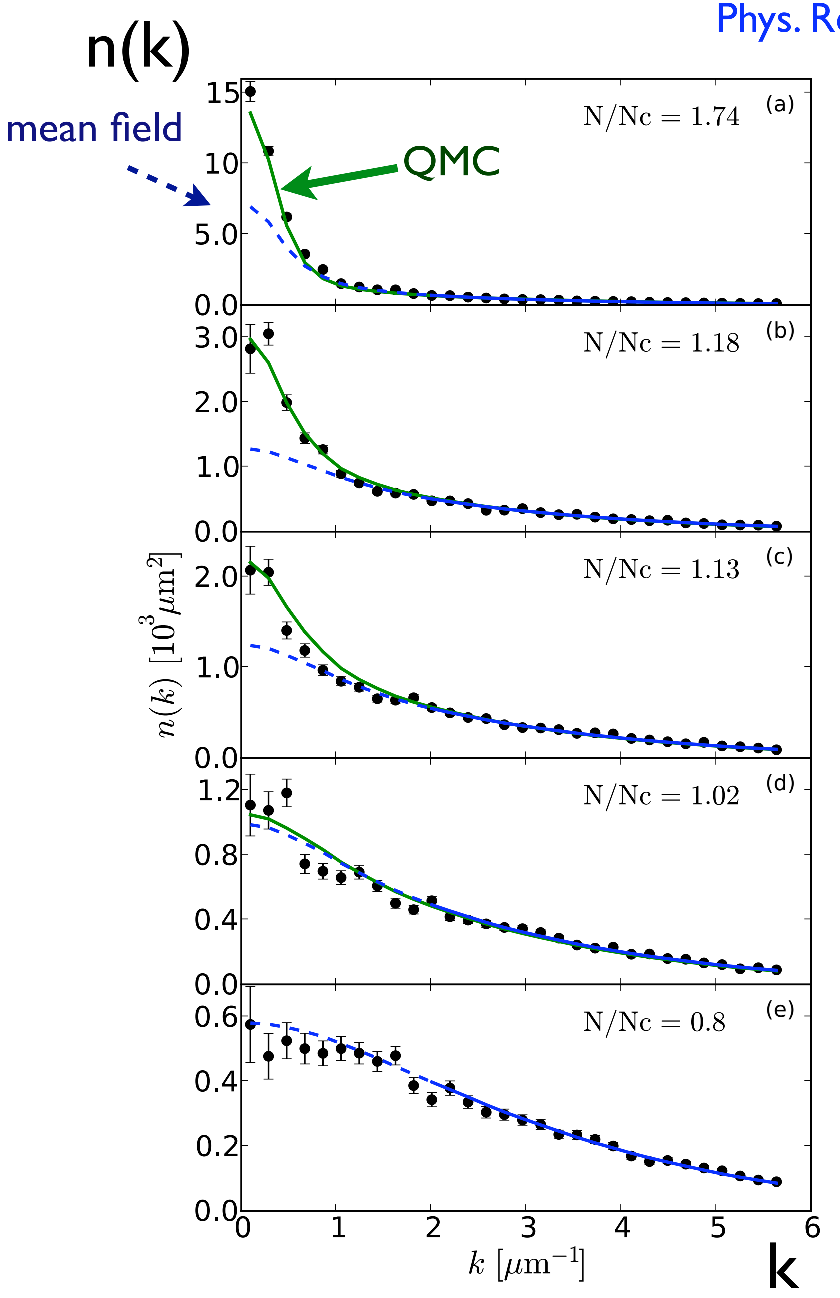


C.-L. Hung, X. Zhang, N. Gemelke , and C. Chin, Nature (London) 470, 236 (2011)

FIG. 3: **Universal behavior near the BKT critical point.** **a**, Rescaled density profiles $\tilde{n} - \tilde{n}_c$ measured at various coupling strengths, $g = 0.05$ (green triangles), 0.13 (blue diamonds), 0.19 (red circles), and 0.26 (magenta squares). Inset shows the original equation of state $\tilde{n}(\tilde{\mu})$. **b**, scaled quasi-condensate density $\tilde{n}_q = \sqrt{\tilde{n}^2 - \delta\tilde{n}^2}$ at different interaction strengths. In both plots, MC calculations from Ref. [9] (open circles) and Ref. [10] (**a**, open squares for $g = 0.07$ and open triangles for $g = 0.14$; **b**, open squares) are plotted for comparison [23]. The shaded area marks the superfluid regime and the solid line in **b** shows the superfluid phase space density calculation [9]. **c** and **d**, critical values $\tilde{\mu}_c$ and $\tilde{n}_c$ determined from the following methods: universal scaling as shown in **a** (red squares), density fluctuation crossover (see text, black circles), and MC calculation from Ref. [8] (solid line). Error bars show the standard deviation of the measurement.

Theory (QMC) ↔ Experiment: Momentum profile n(k)

T. Plisson, B. Allard, M. H., G. Salomon, A. Aspect, P. Bouyer, and T. Bourdel
Phys. Rev. A 84, 061606(R) (2011)



3D Time of flight expansion (TOF):

- time evolution of the density operator

$$\rho(t) = e^{-iHt/\hbar}\rho(t=0)e^{iHt/\hbar}$$

- strong confinement in z (quasi 2D)
  - ⇒ large initial momentum in z
  - ⇒ rapid expansion in z
- slow expansion for in-plane density x/y

$$H_{TOF} \approx \sum_i \frac{p_{ix}^2 + p_{iy}^2}{2m}$$

⇒ TOF density ≅ momentum dist., n(k),

for long TOF-time t

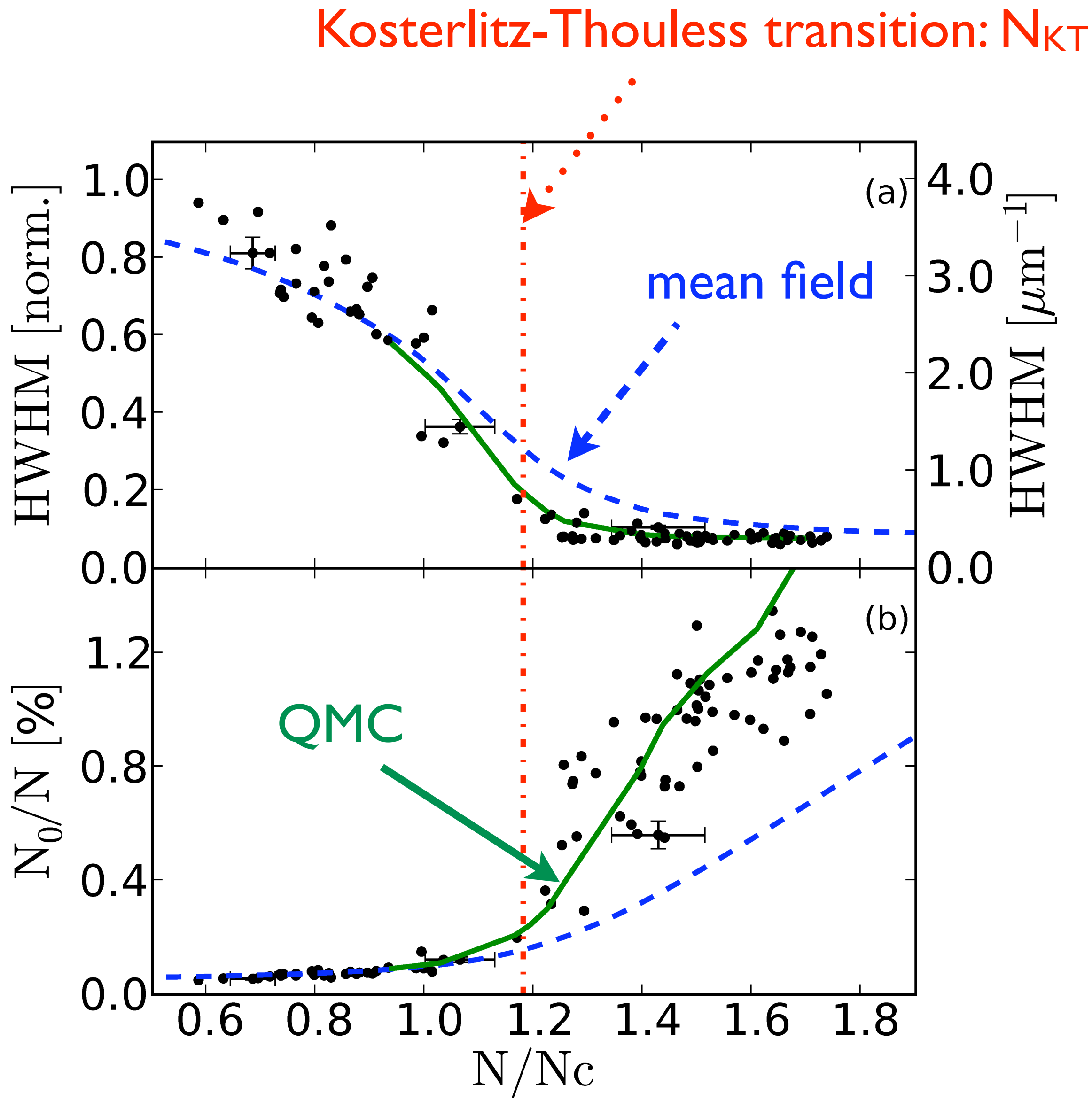
Theory (QMC) ↔ Experiment: Coherence properties

Characterization of Coherence (Peak around k=0):

Width of the peak: HWHM

Height of the peak:

Fraction of particles in k=0 peak: N_0/N



$N/N_c = (T/T_c)^{-2}$

Coherence properties: Where is Kosterlitz-Thouless?

Infinite (homogeneous) system:

Low T-phase with algebraic order: $n_k \sim k^{-[2-\eta(T)]}$ for $k \to 0$

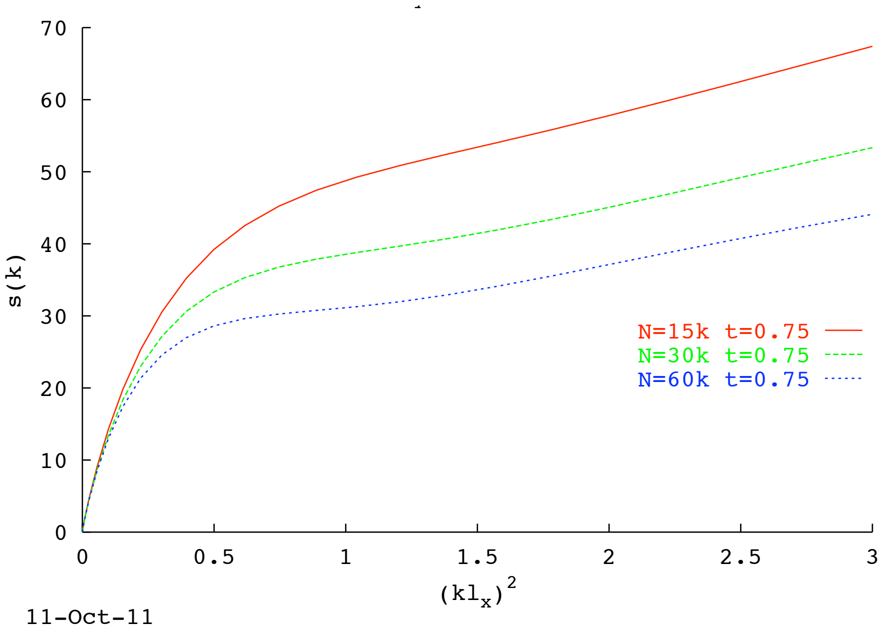
with $1/4 \leq \eta(T) \leq 2$

High T-phase normal: $n_k \sim 1$

Kosterlitz-Thouless transition at η=1/4

⇒ plot $s(k) = n_k k^{2-1/4}$

s(k) for finite trapped system:

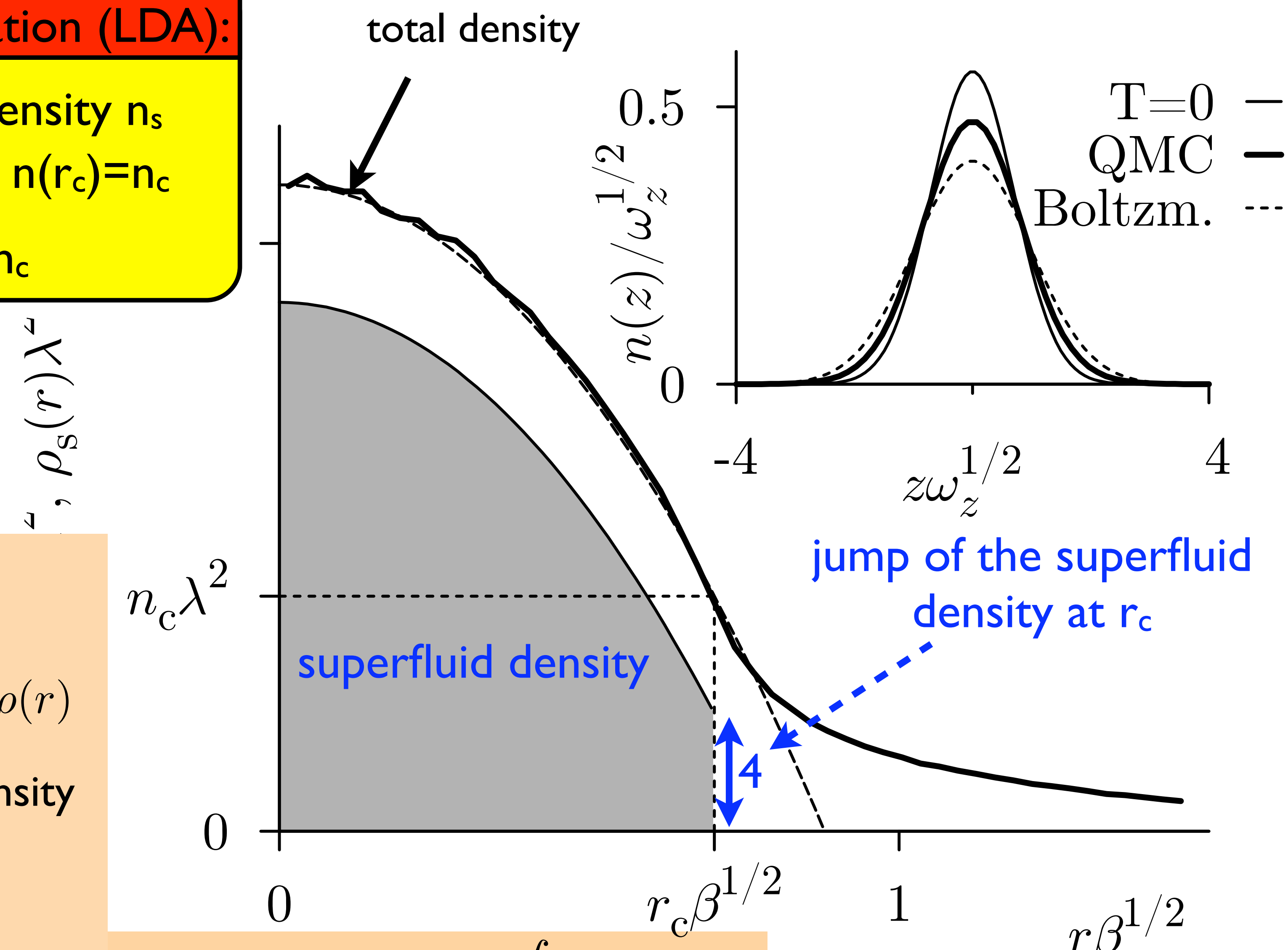


Superfluid properties (LDA)

Local density approximation (LDA):

jump in the superfluid density n_s where density is critical $n(r_c)=n_c$

KT: $n_s\lambda^2=4$ at n_c



moment of inertia I:

classical: $I_{cl}=\int d\mathbf{r}\, r^2\rho(r)$

below T_c: only normal density

$$I=\int d\mathbf{r}\, r^2\rho_n(r)$$

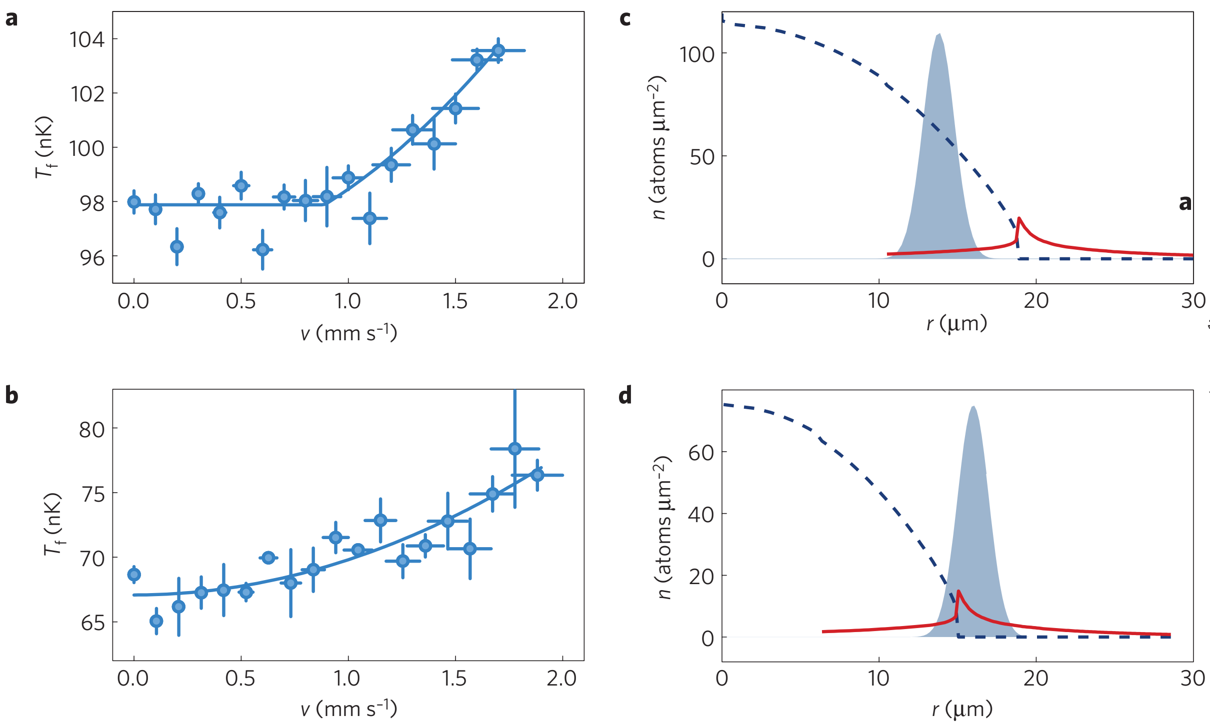
non-classical moment of Inertia (NCMI): $I_{cl}-I=\int d\mathbf{r}\, r^2\rho_s(r)$

M. H., W. Krauth, PRL 100, 190402 (2008)

Local superfluid probe: experiment

R. Desbuquois, L. Chomaz, T. Yefsah, J. Léonard, J. Beugnon, C. Weitenberg, and J. Dalibard,
Nature Physics 8, 645 (2012)

Step-like behavior in the local critical velocity as a function of the local chem. potential



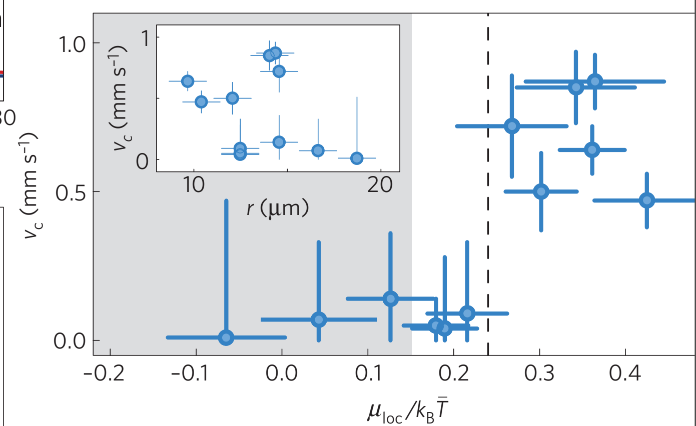


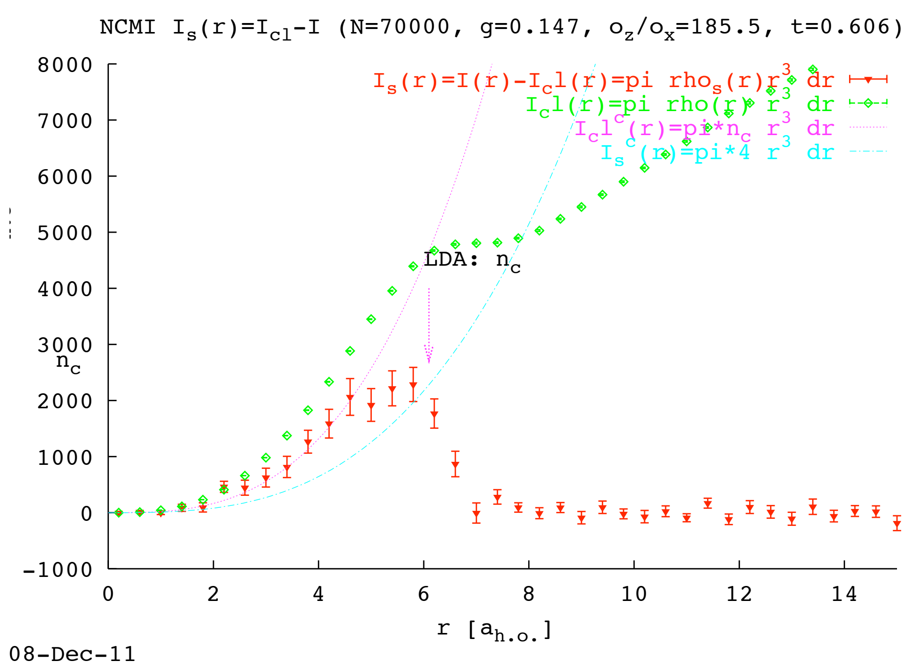
Figure 2 | Evidence for a critical velocity. Two typical curves of the temperature after stirring the laser beam at varying velocities. **a**, In the superfluid regime, we observe a critical velocity (here $v_c = 0.87(9)$ mm s^{-1}), below which there is no dissipation. **b**, In the normal regime, the heating is quadratic in the velocity. The fitted heating coefficients are $\kappa = 18(3)$ nK s mm^{-2} and $\kappa = 26(3)$ nK s mm^{-2} in **a** and **b**, respectively. The experimental parameters are $(N, \bar{T}, \mu, r) = (87{,}000, 89$ nK, $k_B \times 59$ nK, 14.4 μm) and (38,000, 67 nK, $k_B \times 39$ nK, 16.6 μm) for **a** and **b**, respectively, yielding $\mu_{loc}/k_B\bar{T} = 0.36$ and $\mu_{loc}/k_B\bar{T} = 0.04$. The data points are the average of typically ten shots. The *y* error bars show the standard deviation. The *x* error bars denote the spread of velocities along the size of the stirring beam ($1/\sqrt{e}$ radius). The solid line is a fit to the data according to equation (1). The stirring time is 0.2 s for all data points. Note that the three low-lying data points in **a** correspond to the completion of an odd number of half turns. For these data points, where we see a downshift of the temperature by approximately 1.5 nK, we also observe a displacement of the centre of mass of the cloud by a few micrometres.
c,**d**, Calculated radial density distribution for the clouds in **a** and **b**, respectively. The dashed blue curve shows the superfluid density, the solid red curve shows the normal density. The stirring beam potential is indicated by the grey shaded area (in arbitrary units). The densities are calculated via the local density approximation from the prediction for an infinite uniform system[16]. The jump of the superfluid density from zero to a universal value of $4/\lambda_{dB}^2$ (where λ_{dB} is the thermal de Broglie wavelength) is a prominent feature of the BKT transition. The normal density makes a corresponding jump to keep the total density continuous.

Local superfluid density: QMC

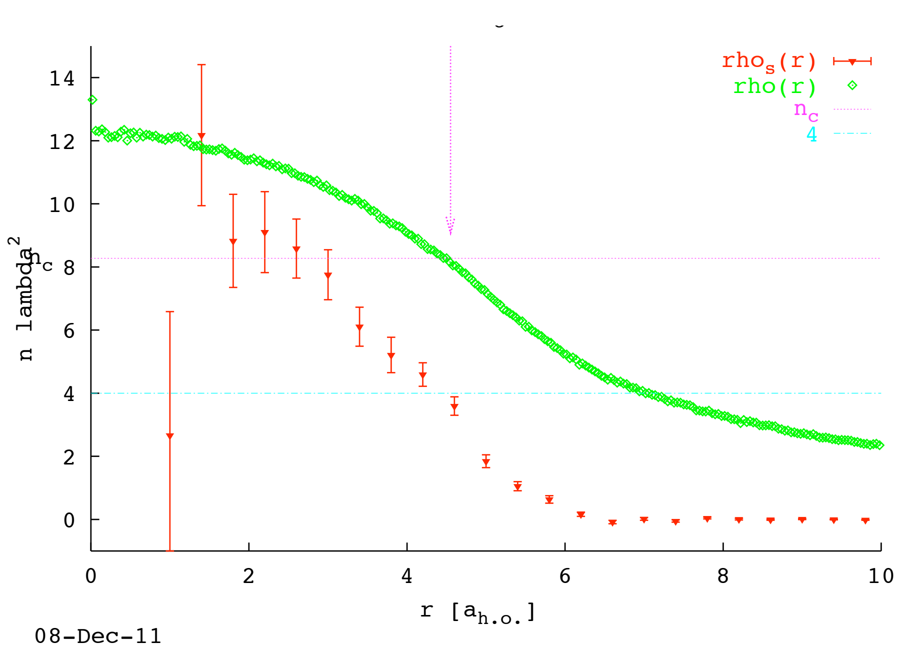
«local» moment of inertia $I(r)$
from linear response to local field coupled to momentum density

classical «local» moment of intertia $I_{cl}(r)$ from local total density

non-classical «local» moment of intertia $I_{ncmi}(r)=I_{cl}(r)-I(r)$



«local» superfluid density
from $I_{ncmi}(r)= n_s(r)\, r^2$

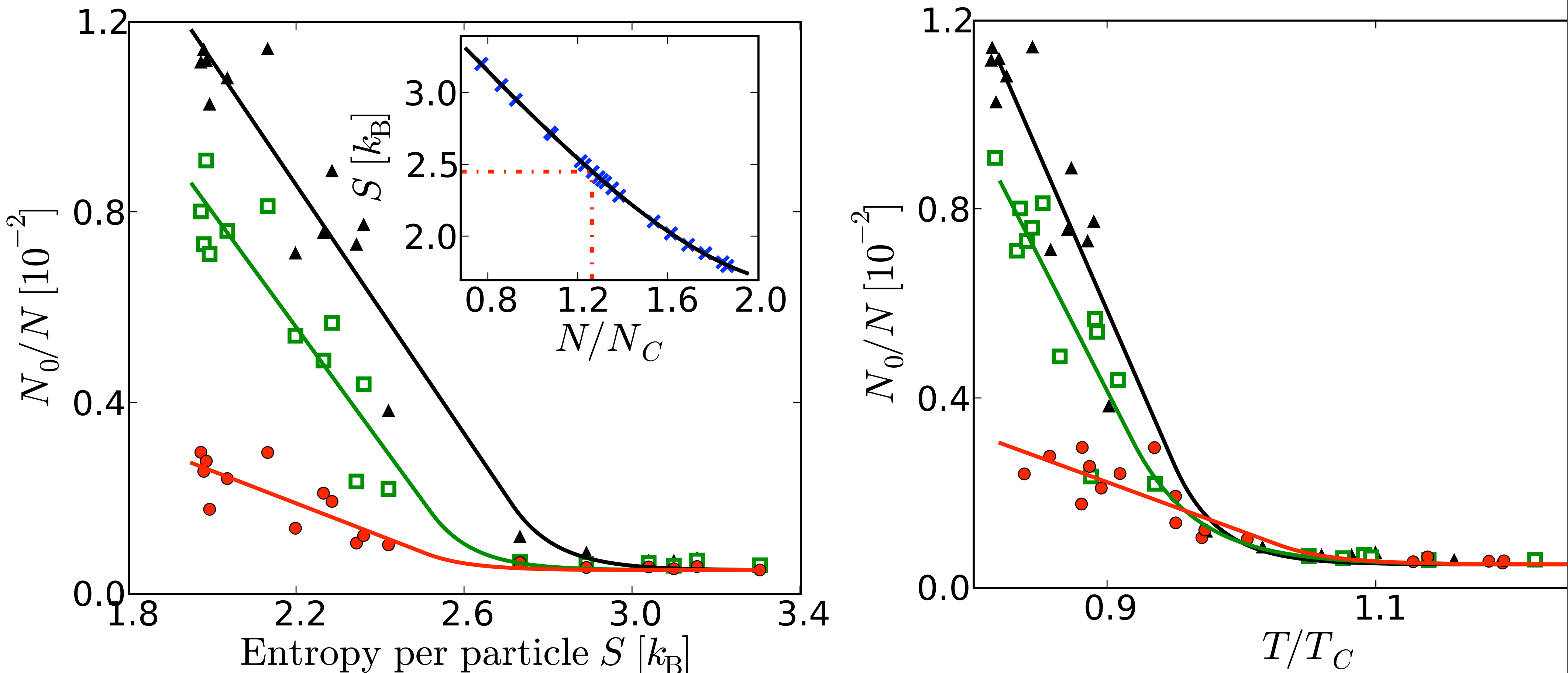


Influence of disorder

B. Allard, T. Plisson, M. H., G. Salomon, A. Aspect, P. Bouyer, and T. Bourdel
Phys. Rev. A 85, 033602 (2012)

adiabatic ramping of correlated disorder potential (speckle)

supression of peak density N_0 observed



Summary-Outlook

- Quasi2D show Kosterlitz-Thouless physics
- Density profiles in experiments are quantitatively descrived by quasi2D mean-field + classical field correlations in LDA
- Small systems with $N < g^{-2}$ behave like a mean-field gas with a BEC transition!
- Coherence properties show no abrupt sign at the Kosterlitz-Thouless transition
- «Local» superfluid density shows sharp features at the local critical density
- Disorder potential suppresses coherence

  Possible normal (insulator) phase at T=0?

  Metal-Insulator transition at finiteT?

Pair-correlation function and quasi-condensate

strictly 2D mean-field: pair-correlation function g(0)=2 (bosonic bunching)

quasi 2D: interference effects due to z-excitations visible => g(0)<2 even in mean-field

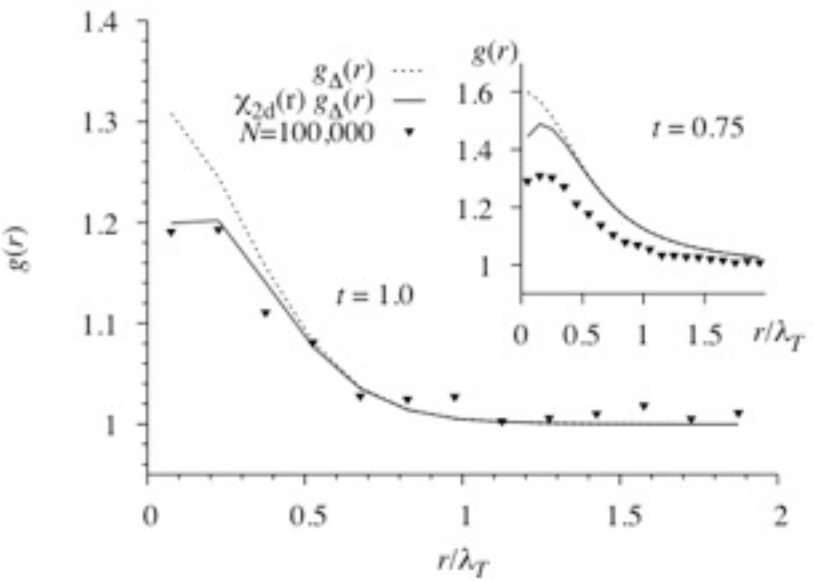


FIG. 9: Central pair correlations, $g(r) = n^{(2)}(r,0)/[n(r)n(0)]$, of the quasi-two-dimensional trapped Bose gas at temperature $T = T_{\rm BEC}^{\rm 2d}$ (main figure) and $T = 0.75\,T_{\rm BEC}^{\rm 2d}$ (inset) for $N = 100,000$ atoms (ENS parameters), together with the prediction of the mean-field gap model, $g_\Delta(r) = n_\Delta^{(2)}(r,0)/n(r)n(0)$, and the short-range improved mean-field model, $\chi_{2d}(r)g_\Delta(r)$.

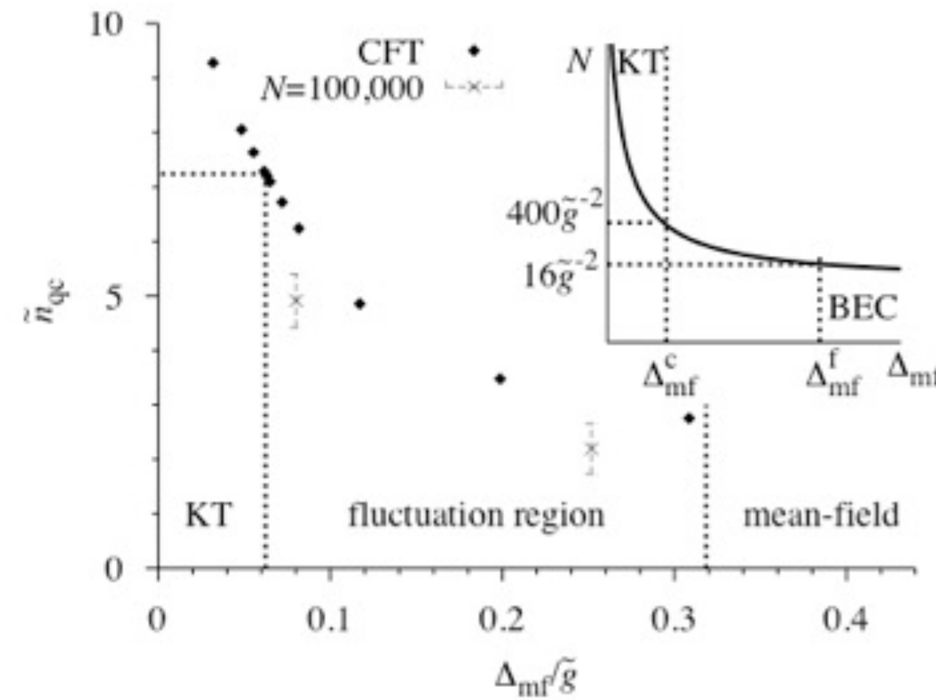


FIG. 10: Quasi-condensate density $n_{\rm qc}$ obtained by QMC from the renormalized pair-correlation function at $T = 0.71\,T_{\rm BEC}^{\rm 2d}$, and $T = 0.75\,T_{\rm BEC}^{\rm 2d}$ (see Fig. 9) for ENS parameters, plotted as a function of $\Delta_{\rm mf}$, and compared to classical-field simulations [12]. The inset shows the boundary of the region with strong finite-size effects (see Eq. (39)).

Quasi2D mean field local density approximation (LDA)

Determine mean-field eigenmodes in the confined z-directions

3D density in LDA: $$\tilde{n}^{\rm mf}_{3d}(\tilde{r},\tilde{z}) = \sum_\nu \tilde{\phi}^2_\nu(\tilde{r},\tilde{z})\tilde{n}^{\rm mf}_\nu(\tilde{r})$$

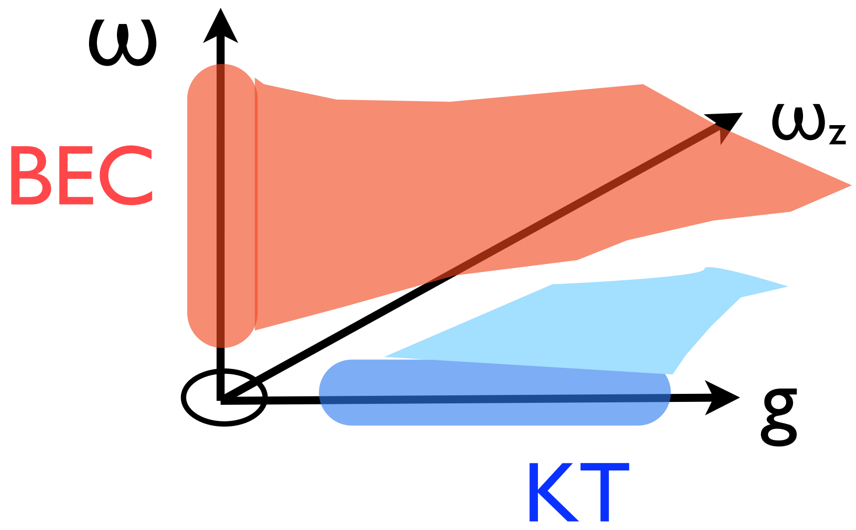
$$\tilde{n}^{\rm mf}_\nu(\tilde{r}) = -\log\left[1-\exp\left(\tilde{\mu}(\tilde{r})-\tilde{\epsilon}_\nu(\tilde{r})/t\right)\right].$$

radial dependence only parametrically through chemical potential: $$\tilde{\mu}(\tilde{r}) = \tilde{\mu} - \frac{\tilde{r}^2}{2}$$

ground state occupation in z: $$\tilde{n}^{\rm mf}_0(\tilde{r}) = -\log\left(1-{\rm e}^{-\Delta_{\rm mf}(\tilde{r})}\right)$$

depends only on local gap: $$\Delta_{\rm mf}(\tilde{r}) = \tilde{\epsilon}_0(\tilde{r})/t - \tilde{\mu}(\tilde{r})$$

Schematic phase diagram in quasi 2D...



3D inter-atomic collisions: $g_{3D}=4\pi a_s/m$
s-wave scattering length a_s

$$\frac{g_{3D}}{2}\int d^2\mathbf{r}\int dz|\Psi_{3D}(\mathbf{r},z)|^4$$

quasi 2D

$$\frac{g_{3D}}{2}\underbrace{\int dz|\phi_0(z)|^4}\int d^2\mathbf{r}\Psi_{2D}(\mathbf{r})|^4$$

$g=(8\pi m\omega_z)^{1/2}a_s$

D. Petrov, M. H., G. Shlyapnikov, PRL 84, 2551 (2000)

quasi2D thermodynamic limit to simplify and define phases...

$N\rightarrow\infty,\omega\rightarrow 0$ with ω_z/T, $N\omega^2$, and g constant

M. H., M. Chevallier, W. Krauth, EPL 82, 30001 (2008)

semiclassical quasi2D density profile:

discrete sum in z

$$n(r)=\sum_\nu\int_0^\infty \frac{\mathrm{d}k^2}{4\pi}\frac{1}{\exp\left(\beta(\frac{\hbar^2k^2}{2m}+v(r)+\nu\hbar\omega_z-\mu)\right)-1}$$

Thermal occupation of excited states in z important for quantitative comparision between theory and experiment

Quasi2D mean field local density approximation (LDA)

- Determine mean-field eigenmodes in the confined z-directions

  from mean-field potential: $$V_{eff}(r,z) = \frac{1}{2}m\omega r^2 + \frac{1}{2}m\omega_z z^2 + 2g_{3D}n(r,z)$$

- simplification from quasi2D thermodynamic limit ➠ LDA

  M. H., M. Chevallier, W. Krauth, Phys. Rev. A 81, 043622 (2010)

$$\tilde{n}^{\mathrm{mf}}_{3d}(\tilde{r},\tilde{z}) = \sum_{\nu} \tilde{\phi}^2_{\nu}(\tilde{r},\tilde{z})\tilde{n}^{\mathrm{mf}}_{\nu}(\tilde{r})$$

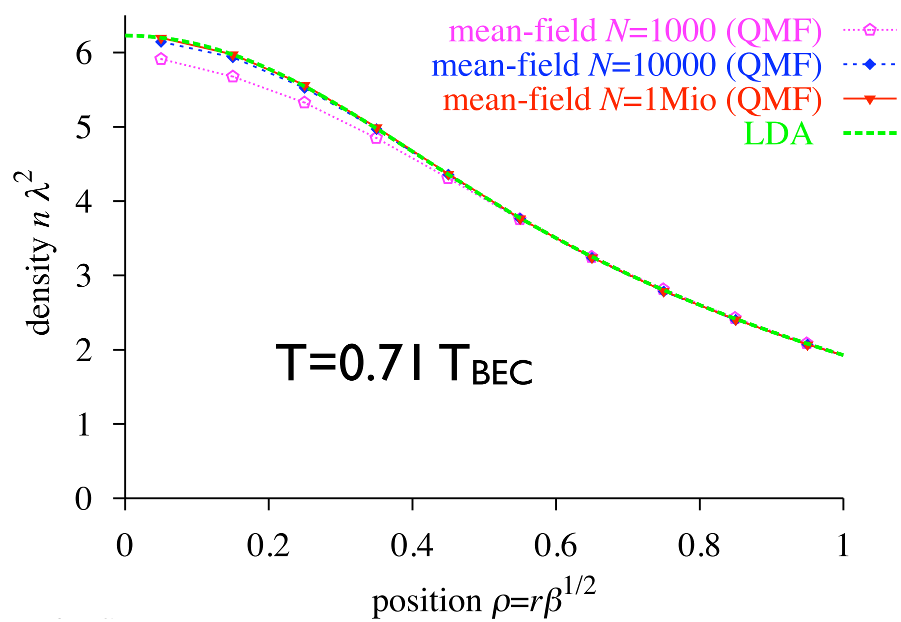
$$\tilde{n}^{\mathrm{mf}}_{\nu}(\tilde{r}) = -\log\left[1 - \exp\left(\tilde{\mu}(\tilde{r}) - \tilde{\epsilon}_{\nu}(\tilde{r})/t\right)\right].$$

LDA: radial dependence only parametrically through chemical potential: $$\tilde{\mu}(\tilde{r}) = \tilde{\mu} - \frac{\tilde{r}^2}{2}$$

- numercial convergence test:

solve mean-field equations exactly for finite N (by QMC)

mean-field finite size corrections to LDA vanish rapidely



Density profile in trap for weak interactions ($g \to 0$)

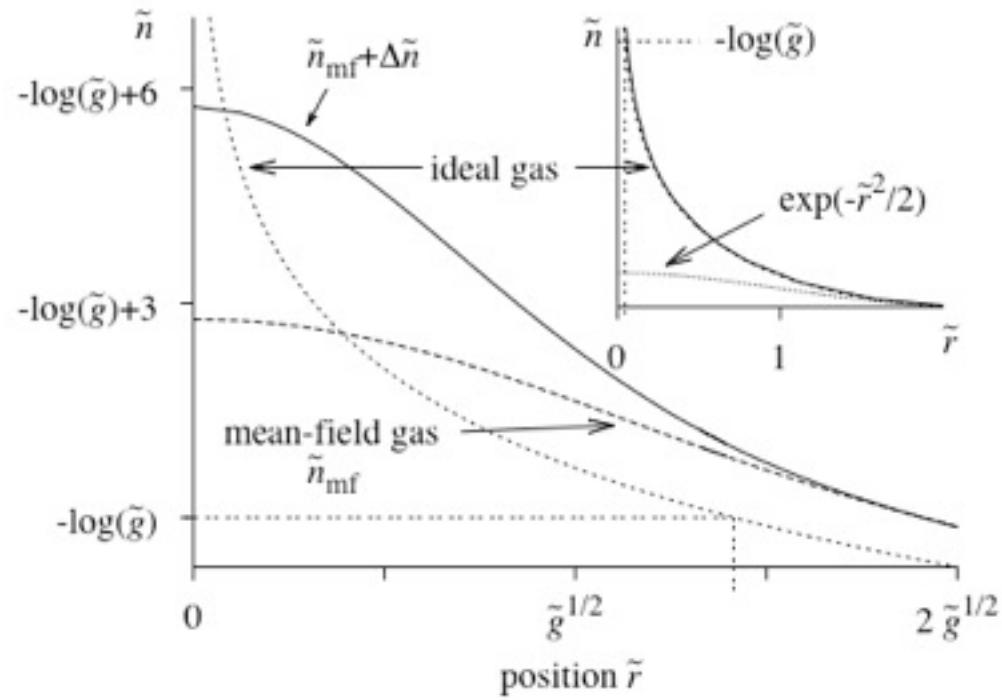
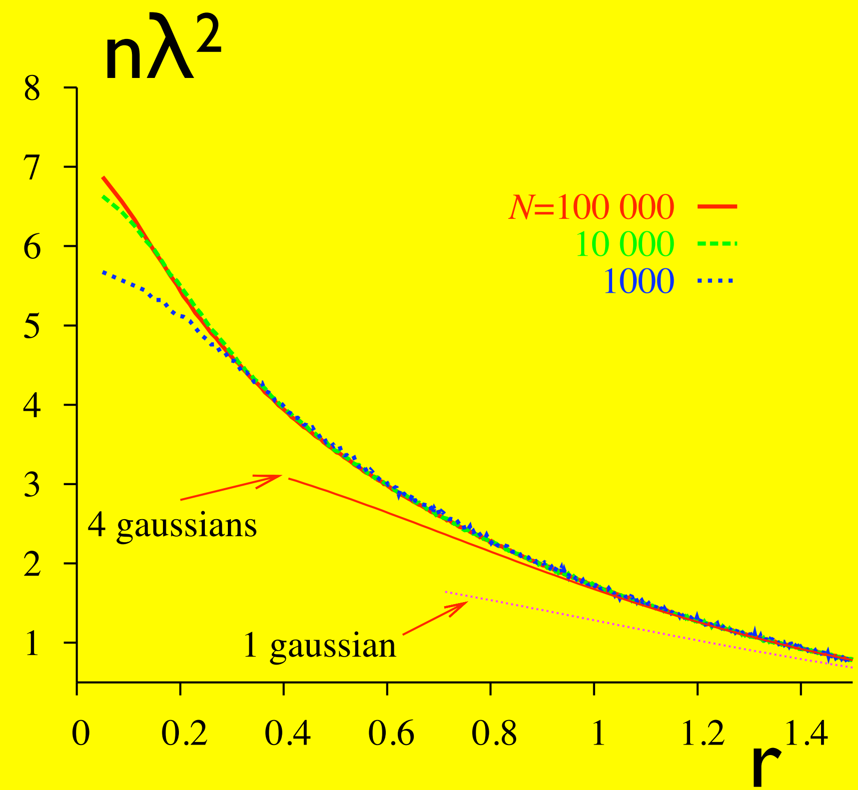


FIG. 5: Schematic density profile of a strictly two-dimensional trapped Bose gas at T_{KT} for $\tilde{g} \to 0$. For $\tilde{r} \gg \sqrt{\tilde{g}}$, $\tilde{n}$ coincides with the ideal gas at T_{BEC}^{2d} (inset, the classical Boltzmann distribution $e^{-\tilde{r}^2/2}$ is given for comparison). In the fluctuation regime, for $\tilde{r} \lesssim 1.25\sqrt{\tilde{g}}$, mean-field and correlation effects become important. The density diverges as $\sim \log(1/\tilde{g})$, yet the correlation contribution $\Delta\tilde{n}$ remains finite.

NIST parameters: g=0.02

P. Cladé, C. Ryu, A. Ramanathan, K. Helmeron, W. D. Phillips, PRL 102, 170401 (2009).

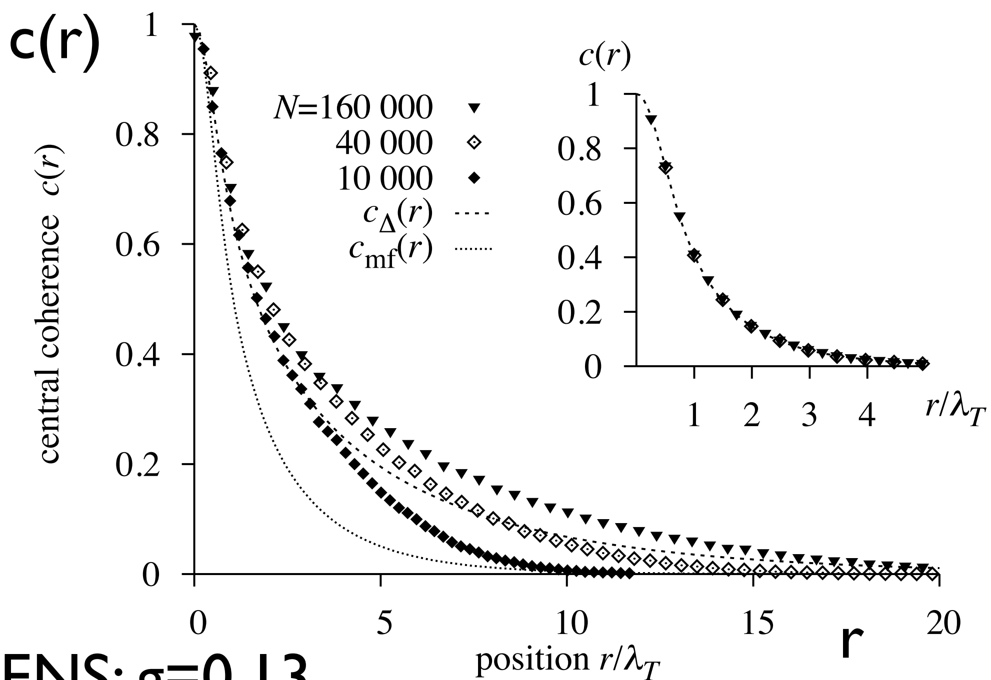
phase space density profile close to T_{KT}



One-particle coherence: Finite size effects at $T>T_{KT}$

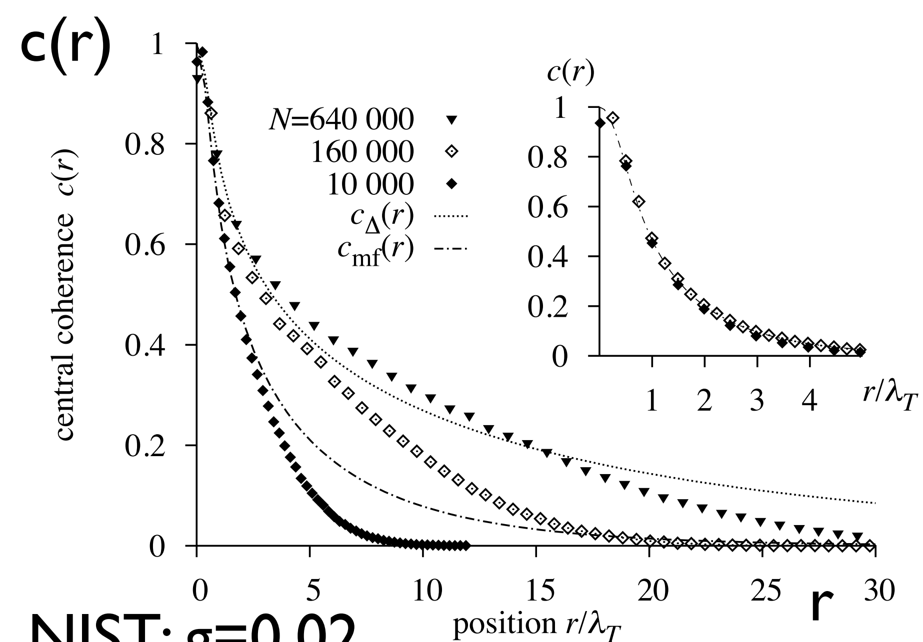
Central coherence c(r):
one-body density matrix in the center of the trap

$$c(r) = \frac{\int dz \; n_{3d}^{(1)}(r,z;0,0)}{\int dz \; n_{3d}^{(1)}(0,z;0,0)}$$



ENS: g=0.13

FIG. 6: Off-diagonal coherence $c(r)$ for ENS parameters with $t = 0.71$ (main graph, $\tilde{n} > \tilde{n}_f$) and $t = 0.769$ (inset, $\tilde{n} < \tilde{n}_f$) compared to the mean-field prediction $c_{\rm mf}(r)$ and the gap model of Eq. (37). In the fluctuation regime, finite-size effects for off-diagonal correlations are more pronounced than for the density (see Fig. 1).



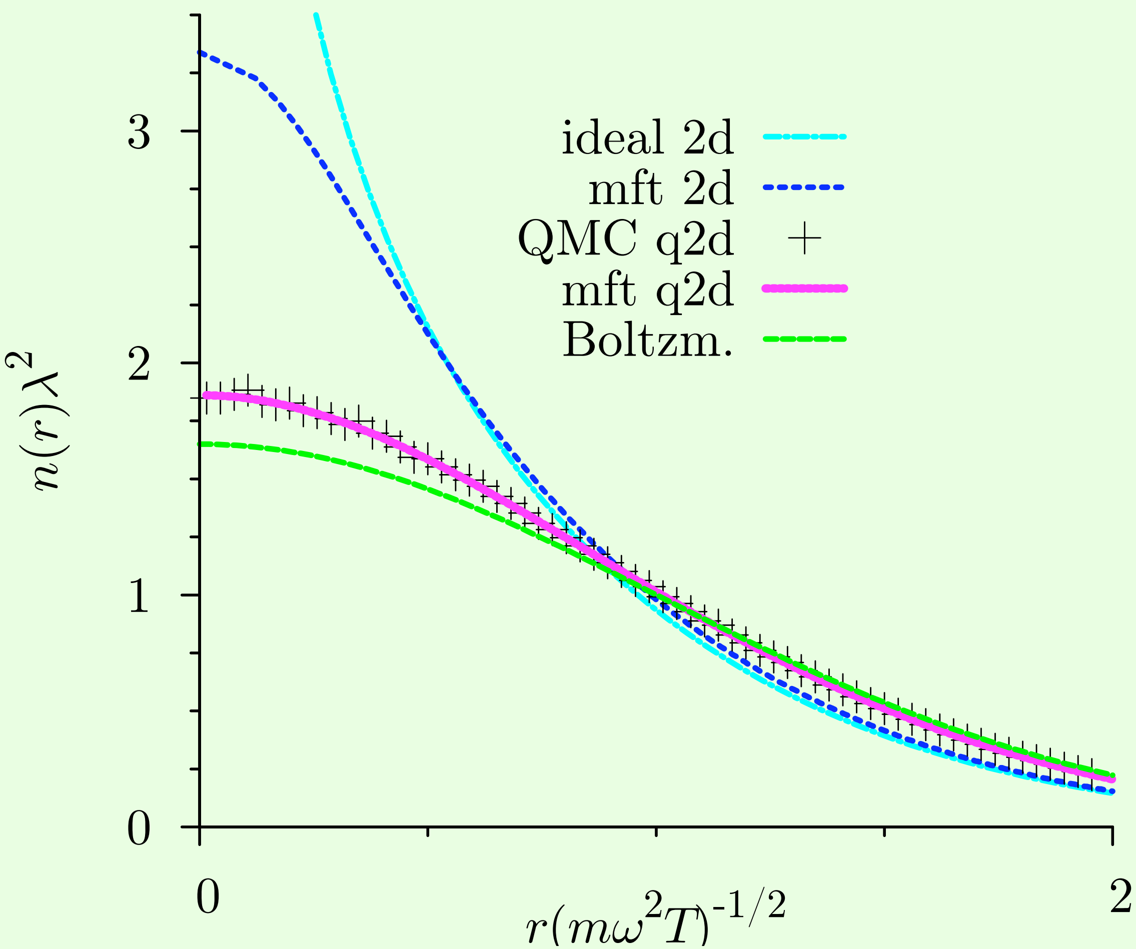
NIST: g=0.02

FIG. 7: Off-diagonal coherence $c(r)$ for NIST parameters with $t = 0.74$ (main graph) and $t = 0.769$ (inset) in comparison with the mean-field prediction, $c_{\rm mf}(r)$, and the gap model, $c_\Delta(r)$, defined in Eq. (37). At $t = 0.769$, the total central density is $\tilde{n}(0) \simeq 5.1 < \tilde{n}_f$, and the system is outside the fluctuation regime. At $t = 0.74$, $\tilde{n}(0) \simeq 10.5 > \tilde{n}_f$, and the system is close to the Kosterlitz–Thouless transition, $\Delta_{\rm mf}/\tilde{g} \simeq 0.08$. Strong finite-size effects are evident in the fluctuation regime.

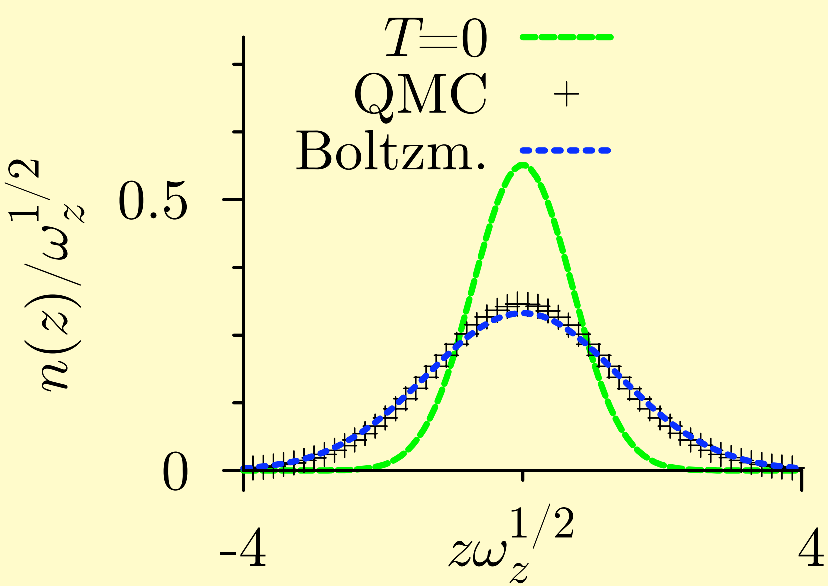
Size effects indicate Bose-condensation for small systems

QMC density profiles: $T > T_{KT}$ (I)

$T = 1.0\, T_{BEC}$



density profile in z



strong quasi 2D effects !

quantitative agreement with experiment:

P. Krüger, Z. Hadzibabic, J. Dalibard, PRL 99, 040402 (2007);
Z. Hadzibabic, P. Krüger, M. Cheneau, S. P. Rath, J. Dalibard, New Journal of Phys. 10, 045006 (2008).

Theory (QMC) ↔ Experiment: direct comparision

S.P. Rath, T. Yefsah, K.J. Günter, M. Cheneau, R. Desbuquois, M.H., W. Krauth, J. Dalibard,
Phys. Rev. A 82, 013609 (2010)

in-situ density measurements

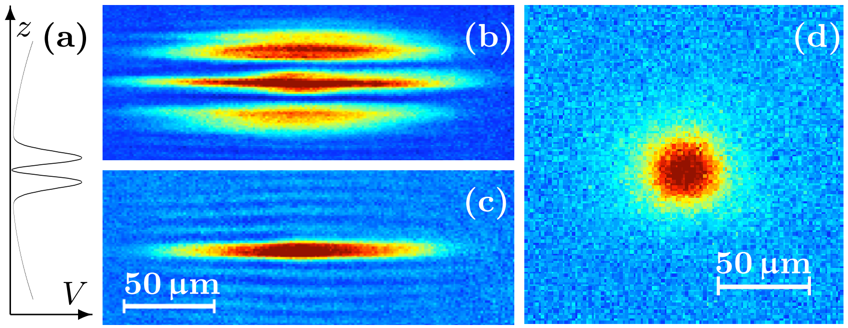


FIG. 1: (Color online) (a) Potential V along the vertical direction z created by the magnetic trap and the laser beam. (b-c) Side view of the cloud before (b) and after (c) depumping atoms in the side wells. The horizontal stripes are due to diffraction. (d) Top view in-situ image yielding fit parameters $T = 132\,\text{nK}$ and $\alpha = 0.29$ (for $\xi = 0.25$).

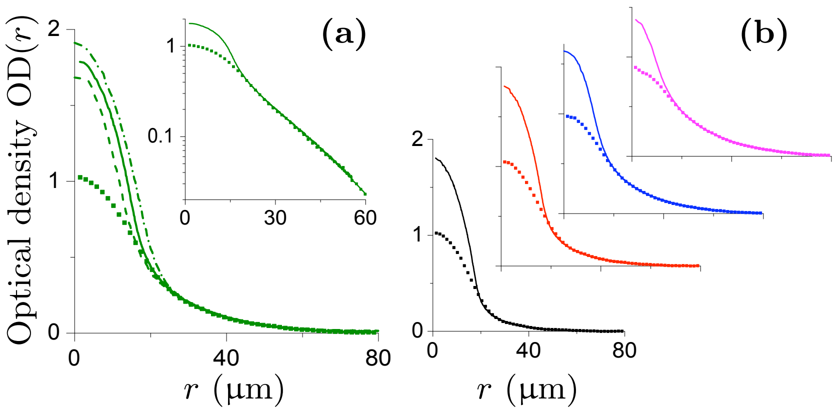


FIG. 2: (Color online) (a) Dots: Measured in-situ profile OD(r). For $\xi = 0.25$ the fit with MFHF theory yields $T = 126\,(6)\,\text{nK}$, $\alpha = 0.34\,(9)$, where uncertainties represent standard deviations obtained by fitting individual images. Continuous line: corresponding QMC simulation with $N = 73900$ atoms (inset: same data in log plot). Upper (dash-dotted) and lower (dashed) lines: QMC results obtained assuming $\xi = 0.21$ [fit parameters (T (nK), α, N)=(130, 0.39, 96300)] and $\xi = 0.29$ (122, 0.29, 57900), respectively. (b) Set of measured density profiles (dots) and corresponding QMC results (lines) for rf evaporation parameters. From bottom to top: (T (nK), α, N)=(87, 0.49, 54100) [black], (109, 0.39, 63800) [red], (142, 0.28, 78400) [blue], (153, 0.23, 79900) [magenta]. Each experimental profile in (a) and (b) is an average of 9 images.

here: $\alpha = \beta\mu$

linear relation between optical density OD and density assumed!

Theory (QMC) ↔ Experiment:

discrepancy of in-situ densities due to multiple scattering

non-linear relation between OD and density

EOS: phase-space density D=nλ as a function of α=βμ in LDA

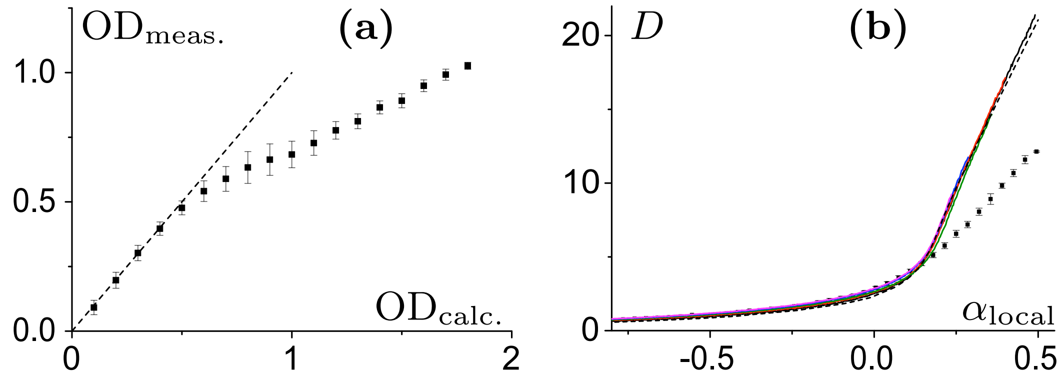


FIG. 3: (Color online) Combination of the experimental and theoretical results of Fig. 2. Error bars indicate the standard deviation of the data. (a) Measured OD as a function of calculated OD, averaged over the data shown in Fig. 2. The dashed line with a slope 1 is a guide to the eye. (b) Continuous lines: QMC results for D as a function of $\alpha_{\text{local}} = \alpha - m\omega^2 r^2/k_B T$ for the data shown in Fig. 2 (same color code). Black dashed line: prediction of [8] for the uniform case. Dots: Measured D, averaged over all experimental data shown in Fig. 2.

Ultracold Bosons in a 2D harmonic trap: Schematic Phase Diagram

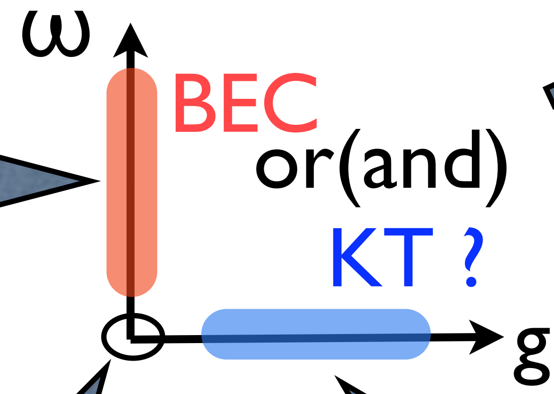
ω: trap frequency
N: number of Bosons

$$T_{KT} = T_{BEC}\left[1 + \frac{3g}{\pi^3}\left(\ln\frac{380}{g}\right)^2\right]^{-1/2}$$

M. H., G. Baym, J.-P. Blaizot, F. Laloë, PNAS 104, 1476 (2007)

$$T_{BEC} = \frac{\sqrt{6N}}{\pi}\omega$$

V. Bagnato, D. Kleppner, PRA 44, 7439 (1991)



$$T_{BEC} = 0$$
$$T_{KT} = 0$$

$$T_{KT} \leftrightarrow n\lambda^2 = \log\frac{380}{g}$$

N. Prokof'ef, O. Ruebenacker, B. Svistunov, PRL 87, 270402 (2001)

g ~$(\log na^2)^{-1}$ interaction for hard disks of diameter a