Macroscopique …….
mais quantique!

Un hommage à Fritz London

Institut Henry Poincaré

11 mai 2005

Mésoscopique …….
mais pas quantique!

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<u>Soutien:</u>
GDR PRIMA & IMCODE (CNRS)
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NSF/CNRS
ESA (programme Ariadne)

- Mésoscopie quantique et classique
- Conduction photonique sous champ magnétique
- Vorticités optiques
- Mésoscopie des ondes sismiques

Mésoscopie quantique:

$$i\hbar\partial_t \Psi = H\Psi \;\Rightarrow\; \Psi(t) = \Psi(0)\exp\left(-\frac{i}{\hbar}Ht\right)$$

$$H = \underbrace{H_{\mathrm{sys}}} + \underbrace{H_{\mathrm{int}}} + \underbrace{H_{\mathrm{disorder}}}$$

The good… the bad… and the ugly

$\int d\mathbf{r}\ \left\vert\Psi(\mathbf{r},t)\right\vert^2 = 1$	probabilité
$\left\langle \Psi(\mathbf{r},t)\right\rangle_{\text{désordre}} \propto \exp\left(-t/\overline{\tau}\right)$	déphasage
$\left\langle \left\vert\Psi(\mathbf{r},t)\right\vert^2\right\rangle_{\text{désordre}} \propto \exp\left(-r^2/4Dt\right)$	diffusion
$\left\langle \Psi(\mathbf{r},0)\ \Psi^*(\mathbf{r}',t)\right\rangle_{\text{env}} \propto \exp\left(-t/\tau_\phi(\mathbf{r}-\mathbf{r}')\right)$	décohérence

$\mathrm{H} = \mathrm{H}_1 \otimes \mathrm{H}_2$ $\Longleftrightarrow$ enchevêtrement $\Longleftrightarrow$ décohérence

Mésoscopie

Classique:

$$\varepsilon(\mathbf{r})\partial_{t^2}^2 \Psi = c_0^2 \nabla^2 \Psi + S(\mathbf{r},t)$$

$$S = S_{\text{source}} + \delta S_{\text{noise}}$$

$$\varepsilon = \underbrace{\varepsilon_{\text{sys}}} + \underbrace{i\,\varepsilon_{\text{abs}}} + \underbrace{\varepsilon_{\text{disorder}}}$$

The good… the bad… and the ugly

$$\int d\mathbf{r}\, \varepsilon(\mathbf{r}) \left|\Psi(\mathbf{r},t)\right|^2 = \exp(-t/\tau_{\text{abs}})$$ absorption de l'énergie

$$\left\langle \Psi(\mathbf{r},0)\, \Psi^*(\mathbf{r}',t) \right\rangle_{\text{env}} \propto \exp(-t/2\tau_{\text{abs}})$$ ….+ bruit

$$\left\langle \Psi(\mathbf{r},t) \right\rangle_{\text{disorder}} \propto \exp(-t/\bar{\tau})$$ déphasage

$$\left\langle \left|\Psi(\mathbf{r},t)\right|^2 \right\rangle_{\text{disorder}} \propto \exp(-r^2/Dt)$$ diffusion

$H = H_1 \oplus H_2$ Pas de enchevêtrement pas de décohérence

Mésoscopie

Critère

$$\bar{\tau} \quad < \quad \Delta T(L) \quad < \quad \tau^{\max} = \begin{cases} \tau^{\text{abs}} \quad \log \dfrac{\text{source}}{\text{noise}} \\ \tau_{\phi} \end{cases}$$

$\Delta T(L) \approx \dfrac{L^2}{2d\,D}$: Temps de diffusion $= \dfrac{\hbar}{E_{\text{Thouless}}}$

$D \propto \dfrac{1}{d} v \ell$ Constante de diffusion

$\ell \equiv \bar{\tau}\, v$ Libre parcours moyen

$\sqrt{D \tau^{\max}} \equiv L_{\max}$ Longueur d'absorption /de décohérence

$$\ell \quad < \quad L \quad < \quad L_{\max}$$

L. Brillouin, 1960

photons sous champ magnétique

Transfert radiatif **macroscopique**

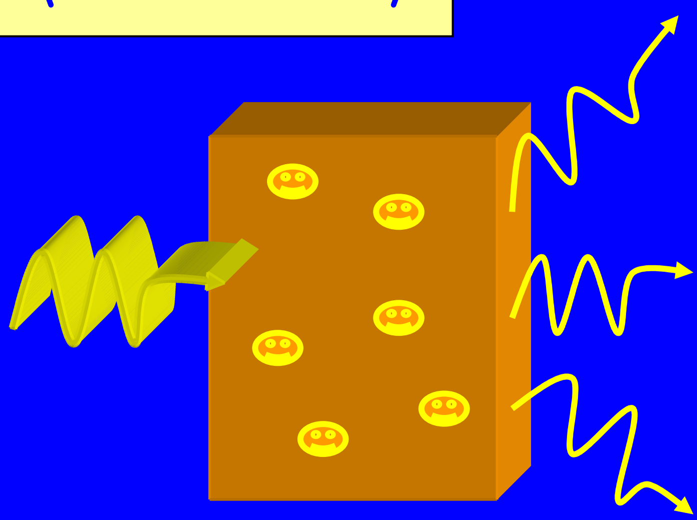
$$\rho(\mathbf{r},t) = \tfrac{1}{2}\mathbf{E}^{*}\cdot\boldsymbol{\varepsilon}(\mathbf{r})\cdot\mathbf{E} + \tfrac{1}{2}\mathbf{B}^{*}\cdot\mathbf{B} \qquad \mathbf{J}(\mathbf{r},t) = \frac{1}{4\pi}\mathbf{E}^{*}\times\mathbf{H}$$

$$\left\langle \mathbf{J}(\mathbf{r},t)\right\rangle = -\,D\,\nabla\left\langle \rho(\mathbf{r},t)\right\rangle$$

Loi d'Ohm:

→ **symétrie par renversement ≠ OK**

→ **Parité =OK**



photons sous champ magnétique

Transfert radiatif **mésoscopique**

$$\rho(\mathbf{r},t) = \tfrac{1}{2}\mathbf{E}^* \cdot \boldsymbol{\varepsilon}(\mathbf{r}) \cdot \mathbf{E} + \tfrac{1}{2}\mathbf{B}^* \cdot \mathbf{B} \qquad \mathbf{J}(\mathbf{r},t) = \frac{1}{4\pi}\mathbf{E}^* \times \mathbf{H}$$

$$\mathbf{J}(\mathbf{r},t) = -\left(D_B \pm \cdots\right)\nabla\langle\rho(\mathbf{r},t)\rangle + D_H\, V\, \mathbf{B}_0 \times \nabla\langle\rho(\mathbf{r},t)\rangle$$

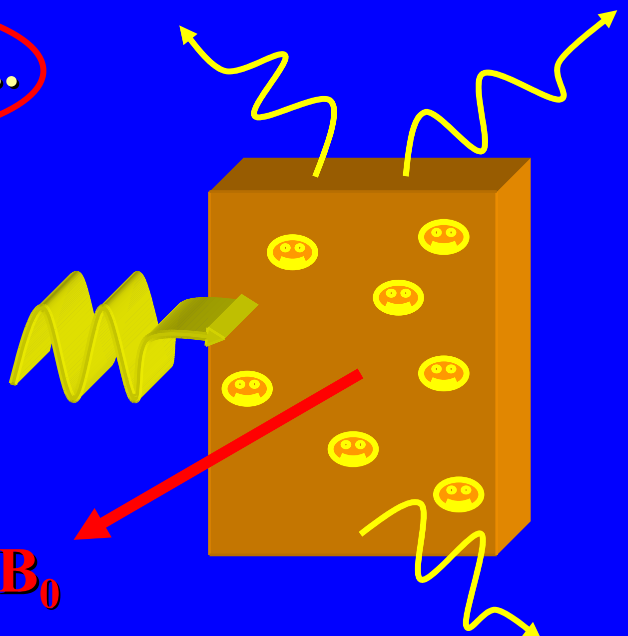
+ tavelures .....

Effet Hall photonique:

→ **symétrie par renversement T =OK**

→ **Parité =OK ;**

→ **Conjugaison de charge = OK**



Rikken & BAvT, Nature, 1996

photons sous champ magnétique

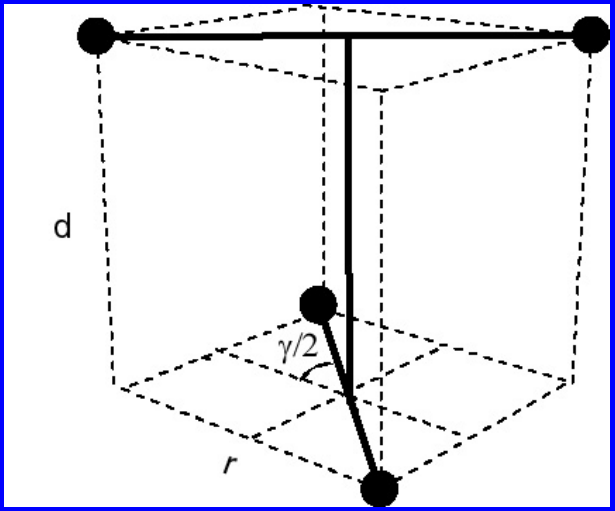
Supercourant de London photonique ?

~~$$\langle \mathbf{J}(\mathbf{r},t)\rangle = D_L\, e\mathbf{A}_0 \langle \rho(\mathbf{r},t)\rangle$$~~

T = OK P= OK C=OK
Jauge = OK

$$\langle \mathbf{J}(\mathbf{r},t)\rangle = D_L\, gV\, \mathbf{B}_0 \langle \rho(\mathbf{r},t)\rangle$$

T = OK P= OK C=OK
Jauge = OK



$D_L = \ldots.. = 0$

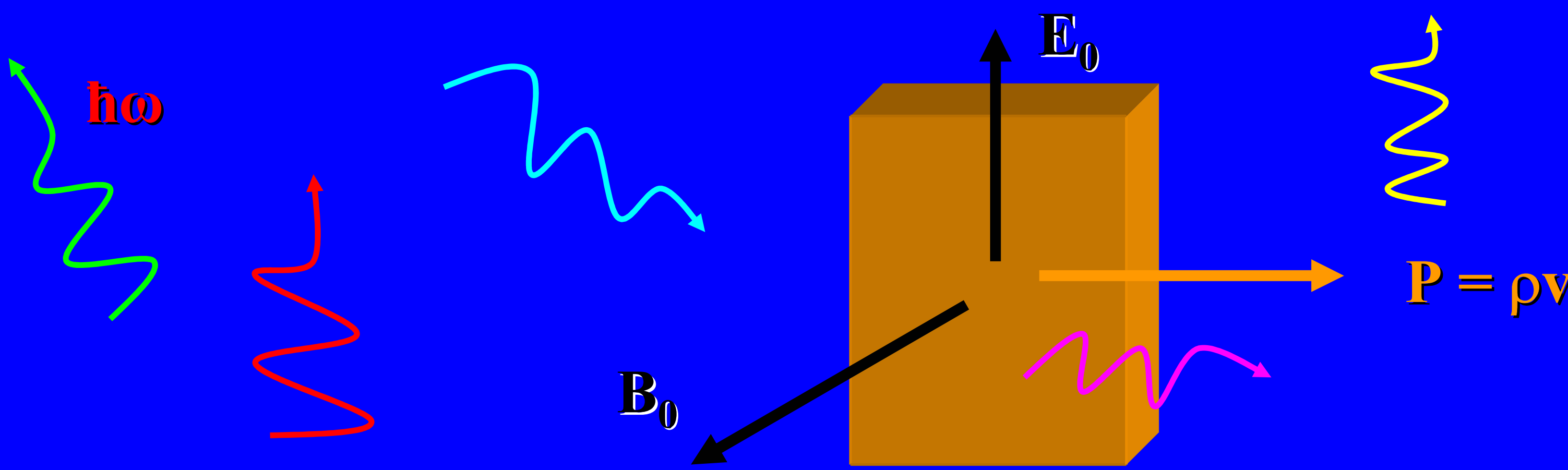
Pinheiro & BAvT, J. Opt. Soc. Am. A 20, 99-105 (2003)

The Feigel process:
Quantité de mouvement venant de nul part ?

A. Feigel, Phys. Rev. Lett. **92**, 020404 (2004)

Milieux bi-anisotropes:

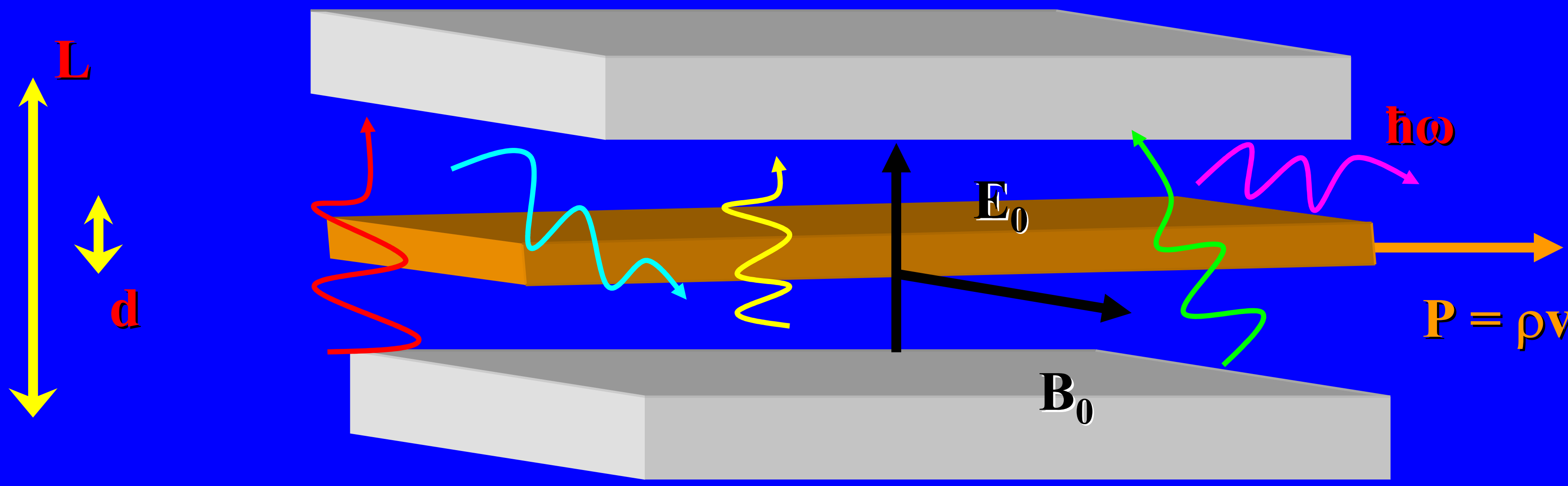
$$\begin{cases} \mathbf{D} = \boldsymbol{\varepsilon}\cdot\mathbf{E} + \boldsymbol{\chi}\cdot\mathbf{B} \\ \mathbf{H} = \mathbf{B} - \boldsymbol{\chi}\cdot\mathbf{E} \end{cases}$$



$$\langle 0 | \rho\, \mathrm{v}_n | 0 \rangle = \frac{2}{3}\frac{\hbar\omega_c^4}{\pi^3 c^4}(1+\varepsilon)\,\varepsilon_{nkl}\chi^{kl} \propto \hbar\omega_c^4\ \mathbf{E}_0 \times \mathbf{B}_0$$

Invariance de Lorentz? divergence….?

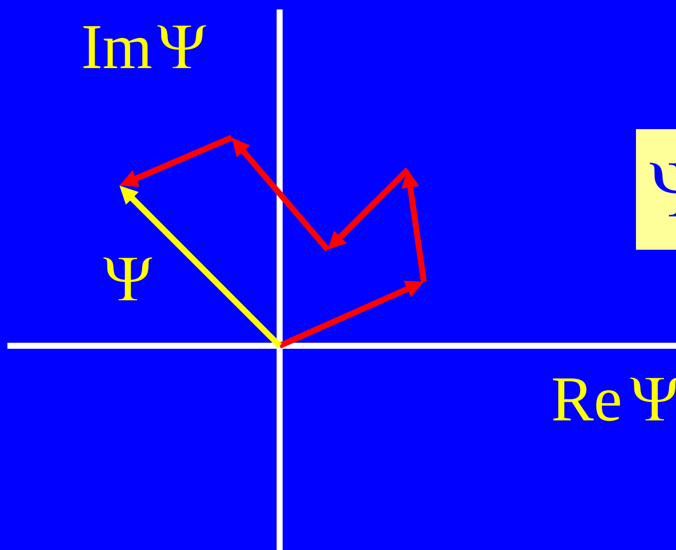
The Feigel process:
Quantité de mouvement venant de nul part ?



BAvT & G. Rikken
En préparation

$$\langle 0|\rho\, \mathbf{v}|0\rangle = -\frac{\pi^3}{L^4}\ \hbar c_0\ \chi\, \mathbf{E}_0 \times \mathbf{B}_0 \left(1 - \frac{30\,L}{\pi d}\,\frac{\sin\frac{\pi d}{2L}}{\cos^3\frac{\pi d}{2L}}\right)$$

vorticité et phase



$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 + \ldots$$

probability distribution

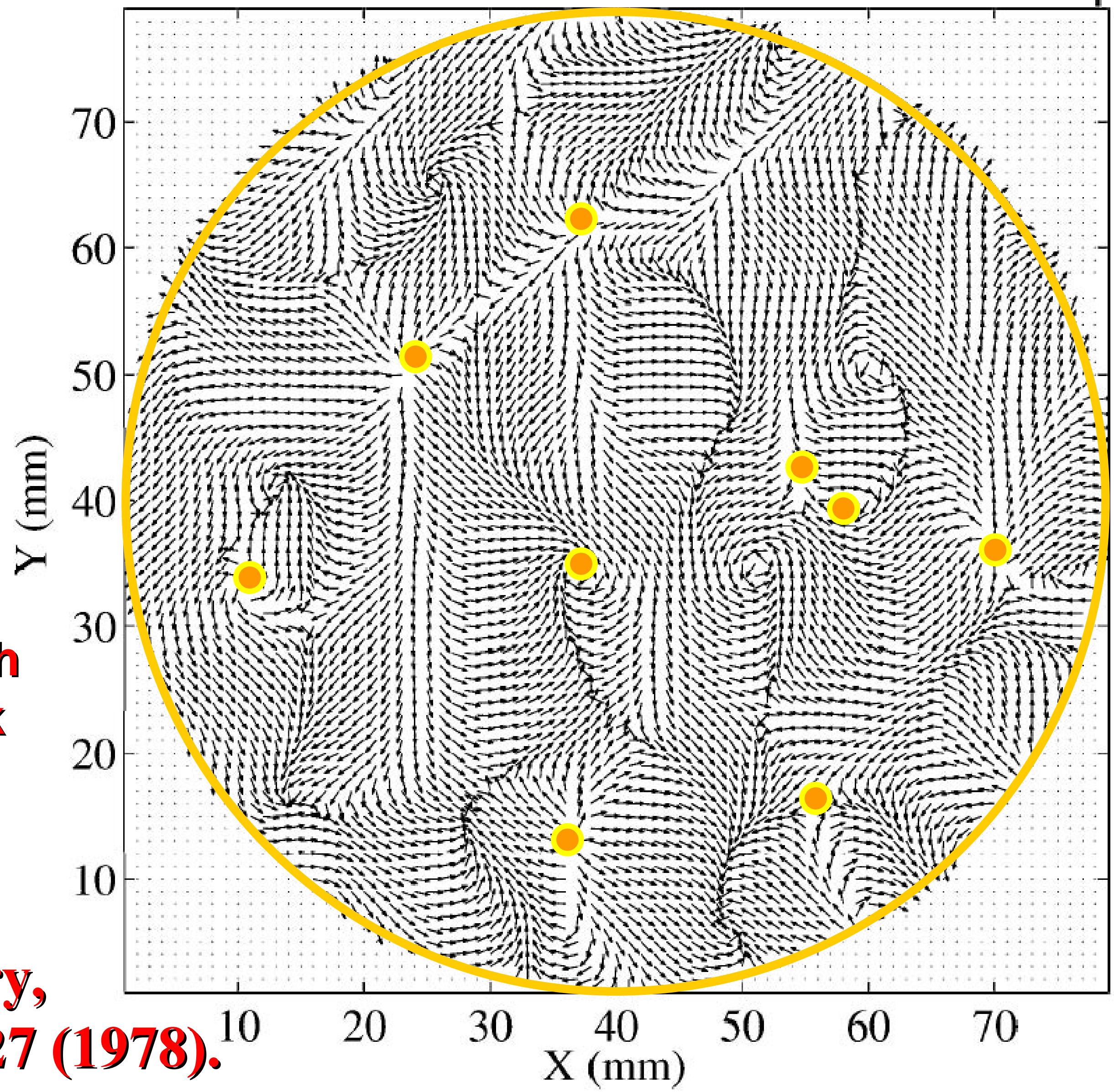
$$P(\Psi_1, \Psi_2, \ldots \Psi_N) = \frac{1}{\pi^N \det \mathbf{C}} \exp\left(-\mathbf{\Psi}^* \cdot \mathbf{C}^{-1} \cdot \mathbf{\Psi}\right) \qquad C_{ij} \equiv \langle \Psi_i \Psi_j^* \rangle$$

diffusion equation

$$\Psi = A e^{i\phi}$$

Genack, van Tiggelen, Sebbah, > 1998

vorticité et phase



Patrick Sebbah
Azriel Genack

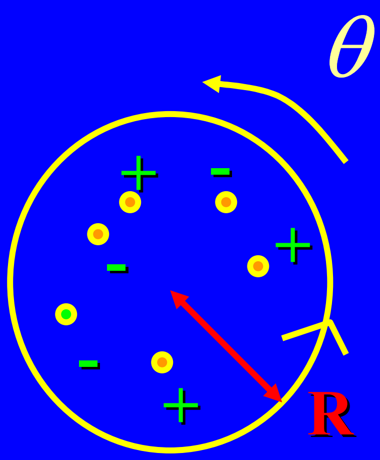
M. Berry,
J. Phys.A. 11, 27 (1978).

vorticité et phase

théorème

$$\oint d\mathbf{l}\cdot\nabla\phi(\mathbf{r}) = 2\pi Q$$

$$Q = \sum_{zero\ i} q_i$$



$$\langle Q \rangle = 0$$

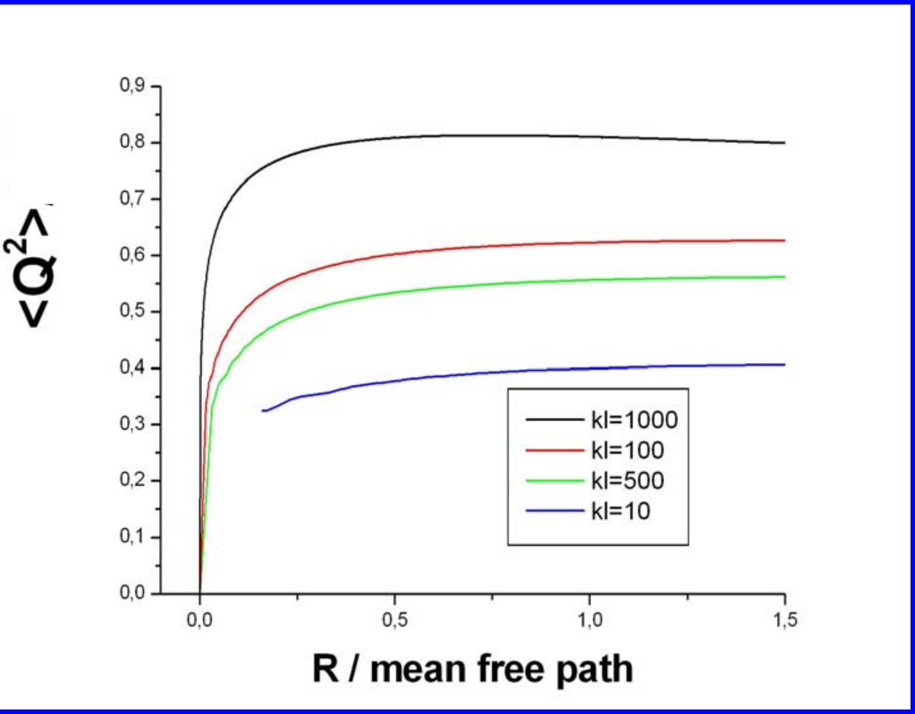
$$\langle Q^2(\mathrm{circle}) \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

Compter le libre parcours moyen?

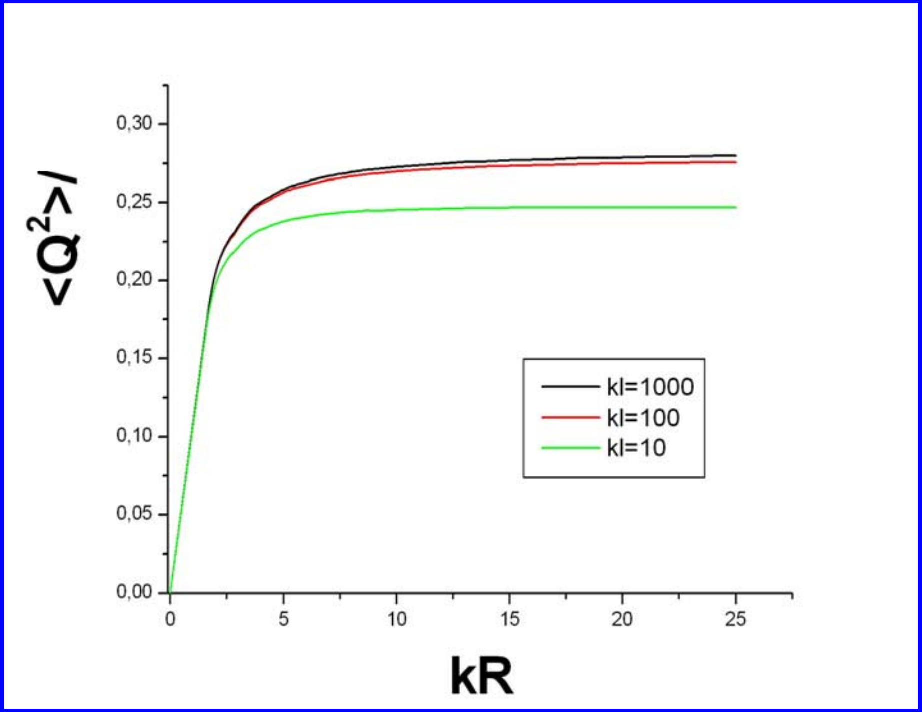
$$\langle Q^2(\text{circle})\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

$$P\left[\psi(\mathbf{r}_1),\psi(\mathbf{r}_2),\psi(\mathbf{r}_3),\psi(\mathbf{r}_4)\right]$$

$$\langle\psi(\mathbf{r})\psi^*(\mathbf{r}')\rangle = J_0(k\Delta r)\exp(-\Delta r/2\ell)$$

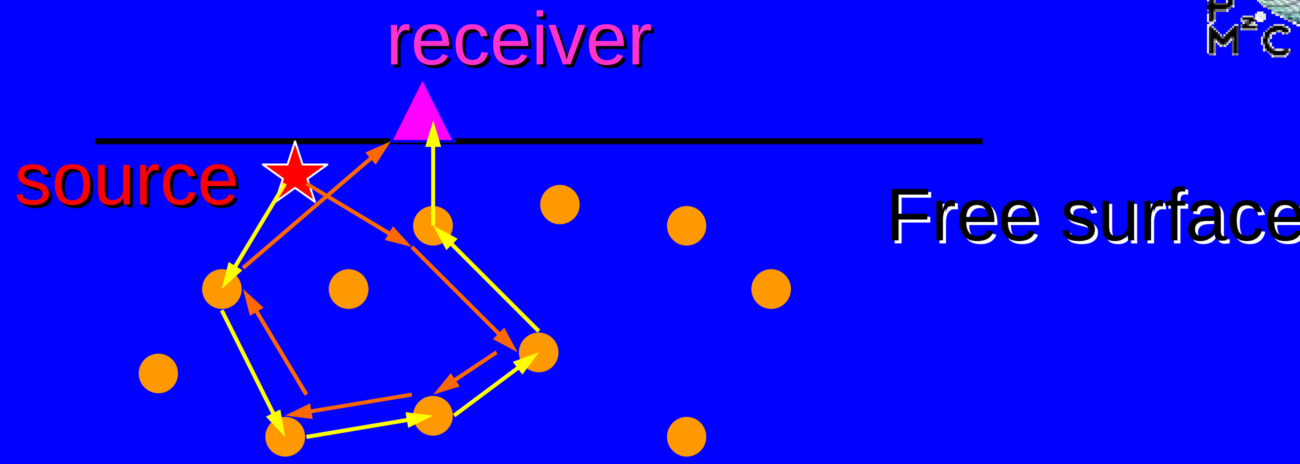


2 dimensions



3 dimensions

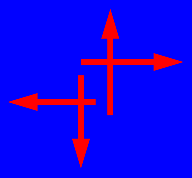
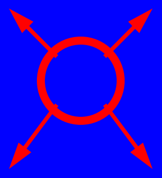
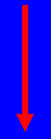
sismologie mésoscopique



1. Distance source receiver < wavelength

$$CBS(r) \propto 1 + J_0^2\left(\frac{2\pi r}{\lambda}\right) \times \left(1 - e^{-t/\tau}\right)$$

2. Symmetry source = symmetry receiver & magnitude

 measure $\left\|\partial_y u_x + \partial_x u_y\right\|^2$ **Earth quake**	 measure $\left\|\text{div}\,\mathbf{u}\right\|^2$ **Explosion**	 measure $\left\|u_z\right\|^2$ **Sledge hammer**

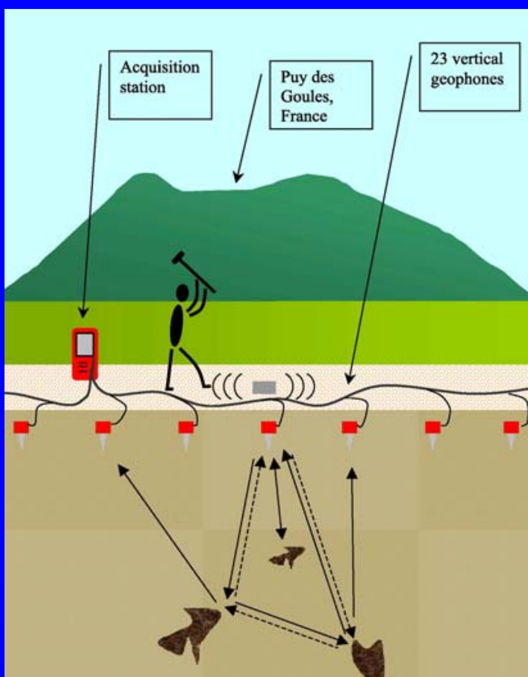
← magnitude

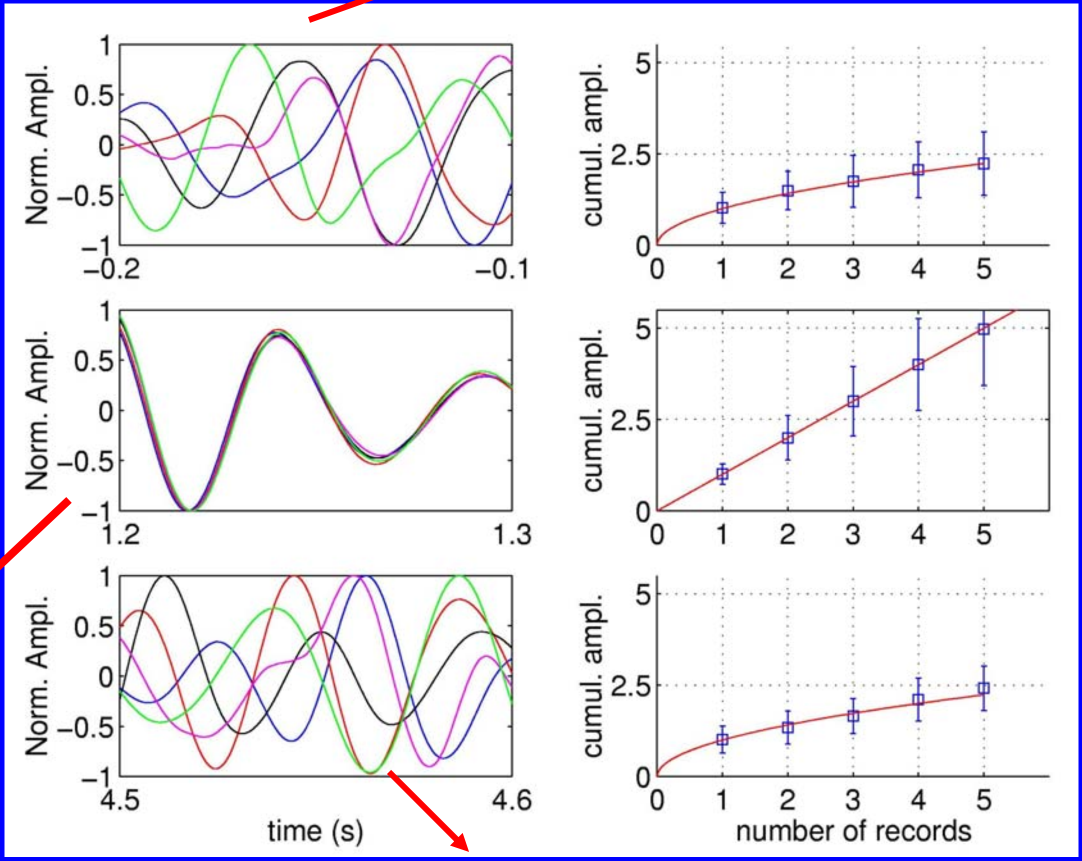
sismologie mésoscopique

Ondes simiques en Auvergne

Eric Larose, Ludovic Margerin, Michel Campillo et Bart van Tiggelen , PRL 2004

Operator noise



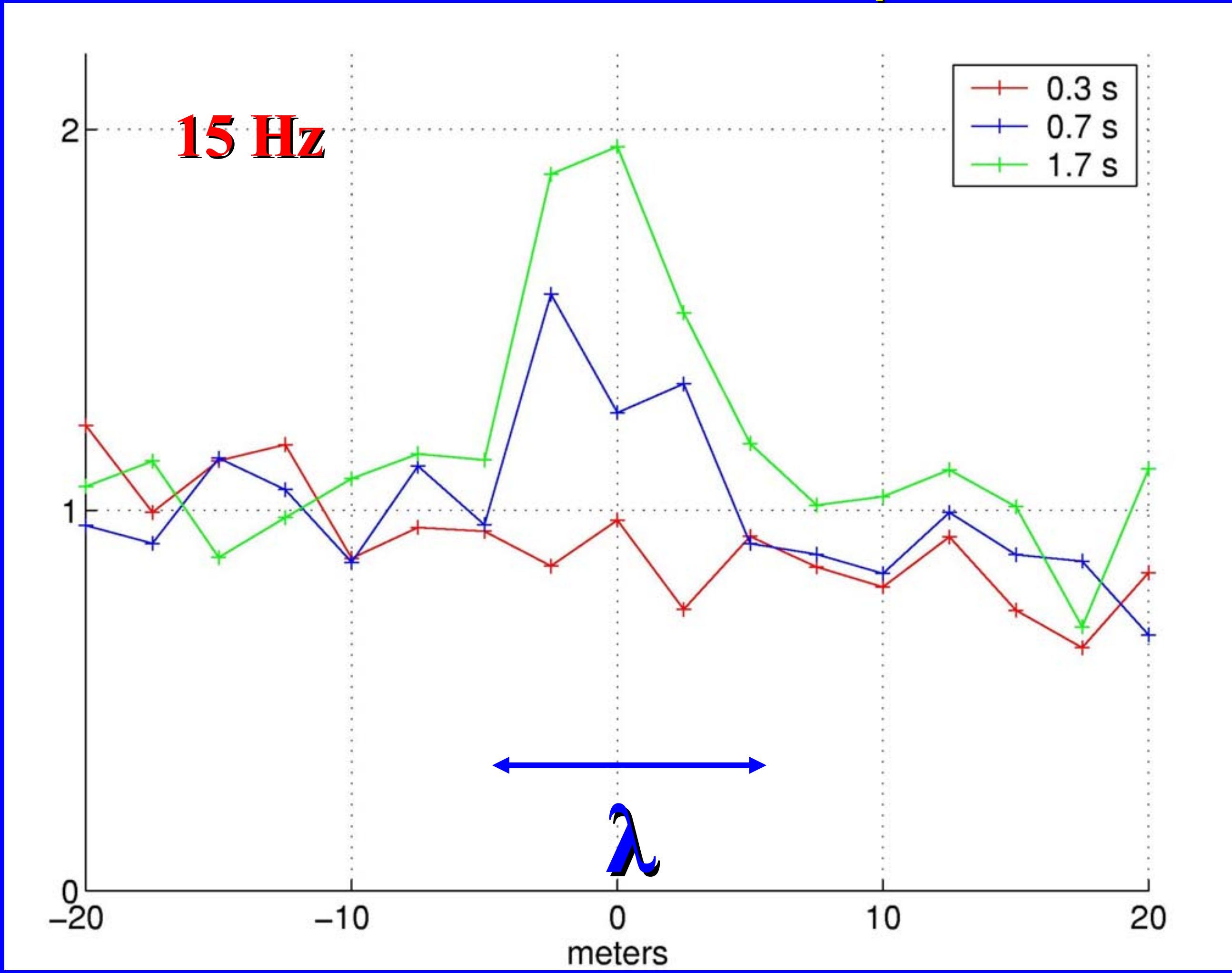


Mesoscopic signal

Background noise

sismologie mésoscopique

Localisation faible des ondes simiques



Mean free time=0.7 seconds

Wavelength= 20 meter $c_{Rayleigh} = 300$ **m/s**

Mean free path = 210 m