# Radiative Transfer of Seismic Waves

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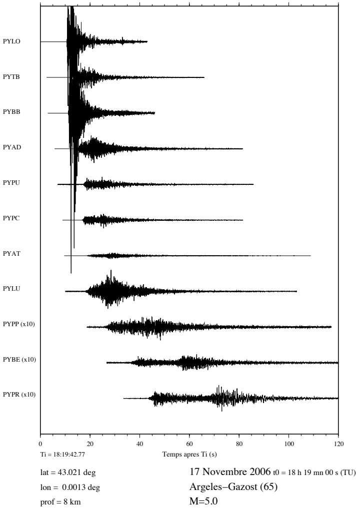
# Content

# Radiative Transfer in Seismology: Why?




Crustal earthquake in the Pyrenees

## Observations

- Duration of signal $\gg$ travel time of ballistic waves
- Rapid attenuation of direct waves
- "Cigar"-shaped envelope
- Pronounced Coda

Role of Scattering

# What is Radiative Transfer?

**Definition**

Radiative Transfer is a theory aimed at predicting the spatio-temporal distribution of energy in a scattering medium

**The physical concepts of Radiative Transfer**

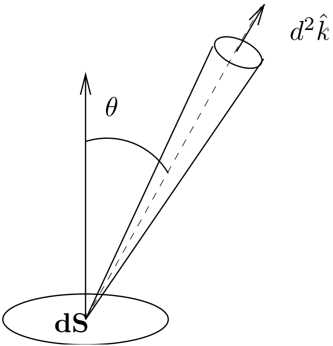
- Local Energy Balance
- Angularly resolved energy flux: Specific intensity
- Scattering strength: mean free path

# Some references in seismology

- Introduced by Wu (1985) and developed by Aki, Zeng, Sato
- Monte Carlo simulations: Gusev and Abubakirov, Hoshiba
- Sato and Fehler (Wave propagation and scattering in the heterogeneous Earth, Academic Press, 1998)
- Dmowska, Sato and Fehler, Eds, Vol. 50 of Advances in Geophysics, Academic Press, 2008
- IASPEI Task group: scattering and heterogeneity in the earth: http://www.scat.geophys.tohoku.ac.jp/index.html

# Content

# The specific intensity




Angularly-resolved energy flux through a surface

## Definition

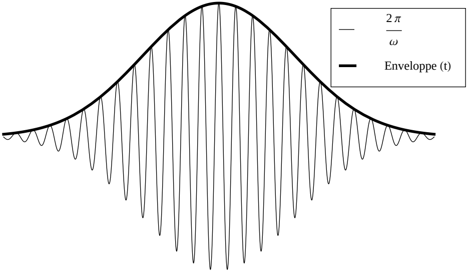
$\mathcal{I}(\omega, t, \mathbf{r}, \hat{\mathbf{k}}) \times dS \times \cos(\theta) \times dt \times d\omega =$ Amount of energy within the frequency band $[\omega, \omega + d\omega]$ flowing through $dS$ around direction $\hat{\mathbf{k}}$ during time $dt$

# Relation to the wavefield

## Wigner-Ville Spectrum

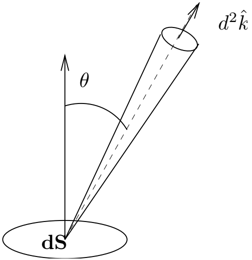
Consider a time-dependent field $\psi(t)$:

$$W_\psi(t,\omega) = \int_{-\infty}^{+\infty} \langle \psi(t-\tau/2)\psi(t+\tau/2)^* \rangle \, e^{-i\omega\tau} d\tau$$




- $\langle\cdot\rangle$: Ensemble Average
- Separation of Time Scales
- $W_\psi$: Instantaneous Energy Spectrum

# Wave content of the specific intensity




Angular Energy Spectrum

## Spatial Wigner-Ville

$$W_\psi(\mathbf{x}, \mathbf{k}) = \iiint \langle \psi(\mathbf{x} - \mathbf{r}/2)\psi(\mathbf{x} + \mathbf{r}/2)^* \rangle \, e^{i\mathbf{k}\cdot\mathbf{r}} d^3 r$$

- Slowly-modulated wave packet
- Specific intensity contains information on the correlation properties of the wavefield

# The Equation of Radiative Transfer

## Scalar Case

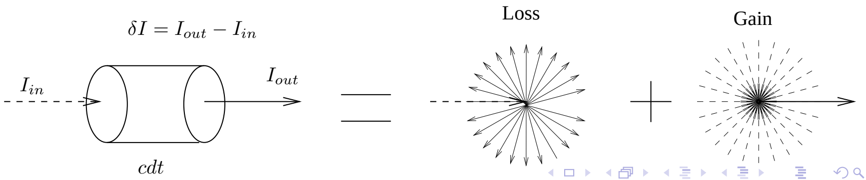
$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \hat{\mathbf{k}}\cdot\nabla_{\mathbf{x}}\right)\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}) = -\left(\frac{1}{l^s}+\frac{1}{l^a}\right)\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}) + \frac{1}{l^s}\oint p(\hat{\mathbf{k}},\hat{\mathbf{k}}')\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}')d^2\hat{k}'$$

$l^s$: scattering mean free path

$l^a$: absorption length

$c$: wave speed

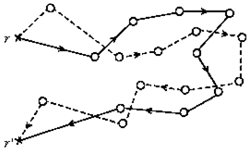
$p(\hat{\mathbf{k}},\hat{\mathbf{k}}')$: scattering anisotropy



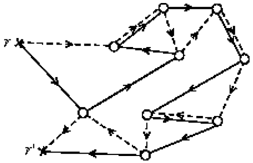
# Assumptions in radiative transfer

## Averaging the products of 2 Green functions

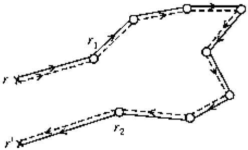
Example of scattering path



Visit the same scatterers



Pairing of trajectories



*From Akkermans & Montambaux (2005)*

# The case of vector waves

## The correlation tensor

$$\Gamma_{ij}(\mathbf{x},\mathbf{k}) = \iiint \langle \psi_i(\mathbf{x}-\mathbf{r}/2)\psi_j(\mathbf{x}+\mathbf{r}/2)^* \rangle \, e^{i\mathbf{k}\cdot\mathbf{r}} d^3r$$

Rotate from the frame $(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3)$ onto the frame $(\hat{\mathbf{k}}_1 = \hat{\mathbf{k}}, \hat{\mathbf{k}}_2, \hat{\mathbf{k}}_3)$:

$$\Gamma(\mathbf{x},\mathbf{k}) = \begin{pmatrix} \Gamma_{\hat{\mathbf{k}}_1\hat{\mathbf{k}}_1} & \Gamma_{\hat{\mathbf{k}}_1\hat{\mathbf{k}}_2} & \Gamma_{\hat{\mathbf{k}}_1\hat{\mathbf{k}}_3} \\ \Gamma_{\hat{\mathbf{k}}_2\hat{\mathbf{k}}_1} & \Gamma_{\hat{\mathbf{k}}_2\hat{\mathbf{k}}_2} & \Gamma_{\hat{\mathbf{k}}_2\hat{\mathbf{k}}_3} \\ \Gamma_{\hat{\mathbf{k}}_3\hat{\mathbf{k}}_1} & \Gamma_{\hat{\mathbf{k}}_3\hat{\mathbf{k}}_2} & \Gamma_{\hat{\mathbf{k}}_3\hat{\mathbf{k}}_3} \end{pmatrix}$$

## Interpretation

$\Gamma_{\hat{\mathbf{k}}_1\hat{\mathbf{k}}_1}$: $P$-wave intensity

$\Gamma_{\hat{\mathbf{k}}_2\hat{\mathbf{k}}_2}$, $\Gamma_{\hat{\mathbf{k}}_3\hat{\mathbf{k}}_3}$: $S$-wave intensity along $\hat{\mathbf{k}}_2$, $\hat{\mathbf{k}}_3$

$\Gamma_{\hat{\mathbf{k}}_2\hat{\mathbf{k}}_3}$, $\Gamma_{\hat{\mathbf{k}}_3\hat{\mathbf{k}}_2}$: Stokes-like parameters $\rightarrow$ polarization

Remaining terms: correlation between $P$ and $S$ waves

# The equation of radiative transfer for elastic waves

$$\left(\mathbf{c}^{-1}\frac{\partial}{\partial t} + \hat{\mathbf{k}}\cdot\nabla_{\mathbf{x}}\right)\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}) = -\left(\mathbf{l}^{-1} + \mathbf{l^a}^{-1}\right)\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}) + \mathbf{l}^{-1}\int \mathbf{p}(\hat{\mathbf{k}},\hat{\mathbf{k}}')\mathcal{I}(t,\mathbf{x},\hat{\mathbf{k}}')d^2\hat{k}'$$

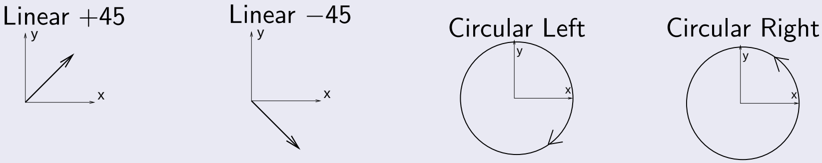
$\mathbf{l} = \mathrm{Diag}(l_p, l_s, l_s, l_s, l_s)$

$\mathbf{l^a} = \mathrm{Diag}(l_p^a, l_s^a, l_s^a, l_s^a, l_s^a)$

$\mathbf{c} = \mathrm{Diag}(v_p, v_s, v_s, v_s, v_s)$

$\mathcal{I} = (I_p, I_x, I_y, U, V)$

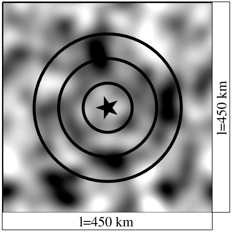
## Interpretation of $U$ and $V$

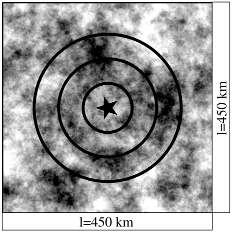


# Content

# Numerical Tests

Models of random media

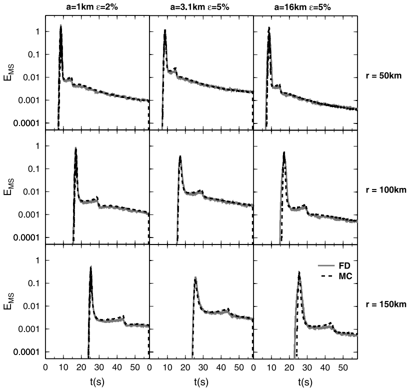



- 2-D Gaussian, exponential media
- Finite-Difference calculations
- $P$ wave source
- Ensemble average
- Elastic R.T.E.
- $\mathbf{p}(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$ from Born approx.
- Monte-Carlo simulations

*Przybilla et al.*, J. Geophys. Res, 111, B04305, 2006

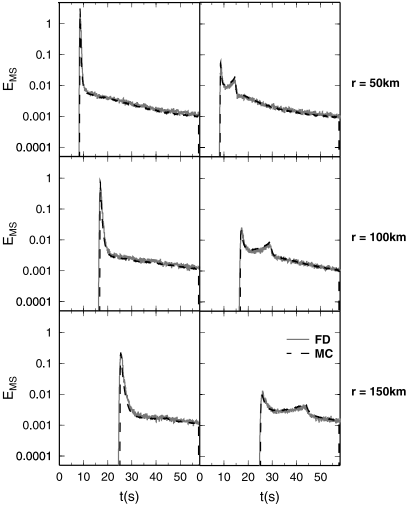
# Comparison between RT and FD simulations




- Left: $t^s = 11.4s$, weak anisotropy, weak scattering
- Middle: $t^s = 0.5s$
- Right: $t^s = 0.1s$, strong anisotropy, strong scattering
- Excellent agreement even for strong forward scattering
- Uniform energy distribution at long time

*Przybilla et al.*, J. Geophys. Res, 111, B04305, 2006

# Comparison between RT and FD simulations



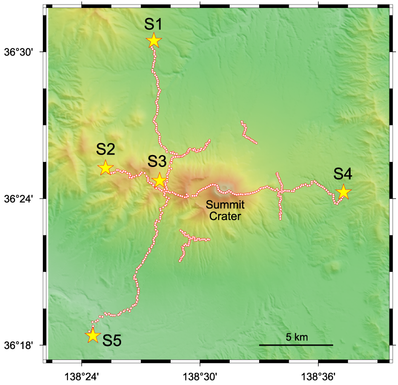

- Excellent prediction of energy partition on radial and transverse components
- Equipartition at large time
- Energy ratios stabilize: $\frac{E_s}{E_p} = \left(\frac{v_p}{v_s}\right)^2$

*Przybilla et al.*, J. Geophys. Res, 111, B04305, 2006

# Content

# Imaging of volcanic heterogeneity

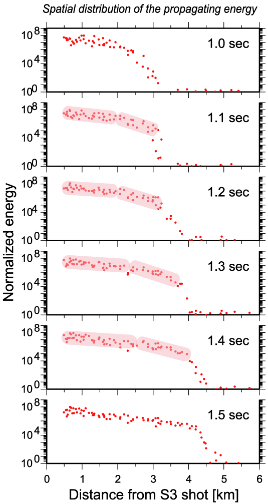
Experiment at Osama Volcano (Japan)




Courtesy of M. Yamamoto

- 5 Dynamite shots
- Vertical sensors 2Hz
- Station spacing: 50-150m
- Data normalized with late coda
- Shot 3 in the 8-16 Hz frequency band

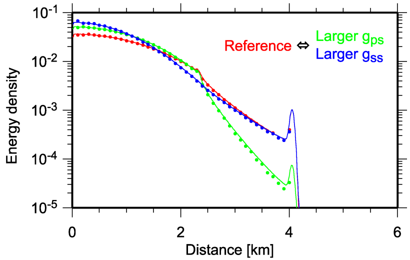
# Spatial Distribution of Energy




Courtesy of M. Yamamoto

- Observe the two slopes
- Shear waves are mode converted
- Spatial homogenization of energy at late time

# Role of the scattering parameters

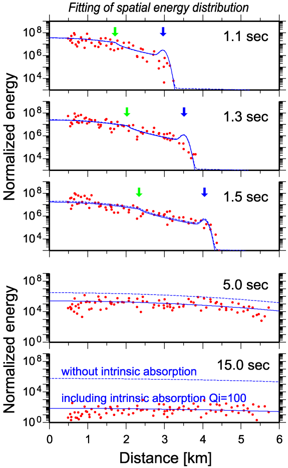



- Reference: $v^p = 2.7$km/s, $v^p/v^s = \sqrt{3}$, $l^{pp} = 3$ km, $l^{pp}/l^{ps} = 2$, $l^{pp}/l^{ss} = 3$
- Smaller $l^{ps}$: $l^{pp}/l^{ps} = 4$
- Smaller $l^{ss}$: $l^{pp}/l^{ss} = 6$

Courtesy of M. Yamamoto

- 3-D elastic radiative transfer
- Isotropic scattering
- Explosion: isotropic $P$ source
- Reciprocity: $l^{sp} = 2\left(\frac{v^p}{v^s}\right)^2 l^{ps}$
- Vary the ratios $l^{pp}/l^{ps}$ and $l^{pp}/l^{ss}$

# Constraint on medium heterogeneity




Impressive agreement!

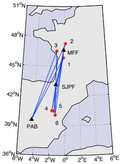
Yamamoto and Sato, to appear in J.G.R.

## Inversion results

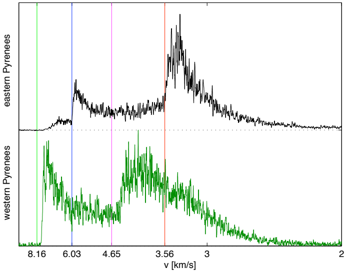
- $v^p = 2.7$ km/s
- $l^{pp} = 3.8$ km, $l^{pp} = 2.4 l^{ps}$, $l^{pp} = 3 l^{ss}$
- Absorption: $Q_i = 100$
- Extremely strong scattering: $l^p \approx l^s \approx 1$km
- Coupling parameters: rich information on the nature of heterogeneity

# Modeling of *Lg* blockage through the Pyrenees

Wavepaths through the Pyrenees

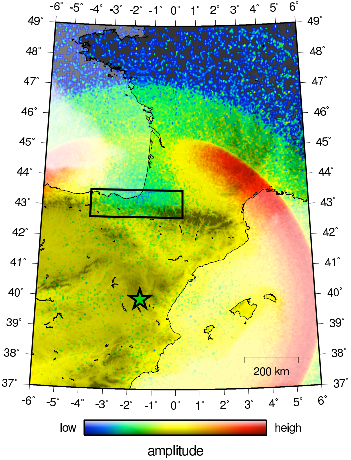



Extinction of Lg waves

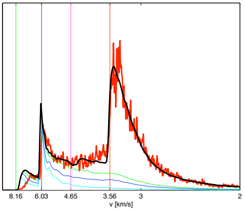



- Blockage in a localized zone
- Not explained by Moho jump or faults
- Monte Carlo simulations of RT
- 3-D anistropic scattering and mode conversion
- Depth varying velocity and scattering properties

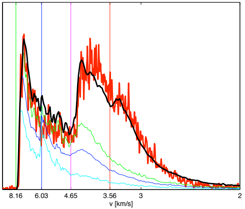
## Snapshot of numerical simulation




## Undisturbed zone



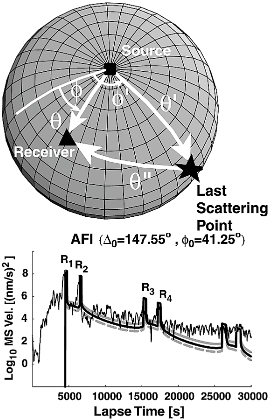

## Through anomaly




Sens-Schönfelder et al., J. Geophys. Res., 114, B07309, 2009

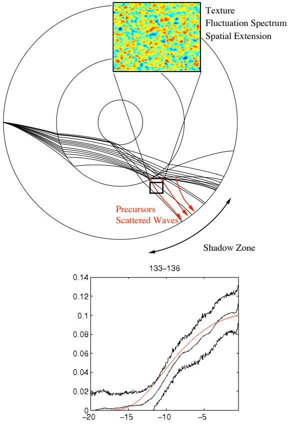
# Applications at the global scale

## Multiple scattering of Rayleigh waves




Sato and Nishino, J. Geophys. Res., 107, 2343, 2001

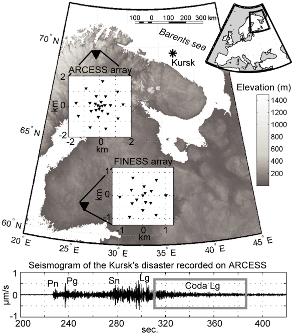
## Scattering in the lower mantle



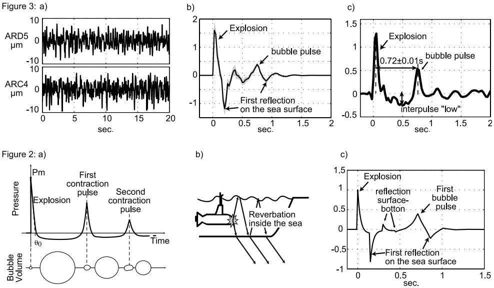

Margerin and Nolet, J. Geophys. Res., 108, 2514, 2003

# Application to source studies

## The Kursk Explosion




## Comparison between observed and "theoretical" explosion




Sèbe et al., Geophys. Res. Lett., 32, L14308, 2005

# More radiative transfer

## Asymptotic solution of Bethe-Salpeter equation

Wigner distribution of two Green functions:

$$\Gamma(\mathbf{x}, \mathbf{r}; t, \tau) = \langle G(\mathbf{x} + \mathbf{r}/2, t + \tau/2) G(\mathbf{x} - \mathbf{r}/2, t - \tau/2)^* \rangle$$

Fourier transform over $\tau$:

$$C(\mathbf{x}, \mathbf{r}; t, \omega) = \int \Gamma(\mathbf{x}, \mathbf{r}; t, \tau) e^{-i\omega\tau} d\omega$$

Asymptotic result $t \to \infty$

$$C(\mathbf{x}, \mathbf{r}; t, \omega) \sim \frac{\mathrm{e}^{-\omega t/Q_i}}{(Dt)^{3/2}} \operatorname{Im} \langle G(\mathbf{r}, \omega) \rangle$$

# Consequence 1: Equipartition

## Full-Space Elastic Green Function

Spectral domain:

$$\mathrm{Im}\,\hat{G}_{ij}(\mathbf{k},\omega) \sim (\delta_{ij} - \hat{k}_i\hat{k}_j)\delta(\omega^2 - v_s^2 k^2) + \hat{k}_i\hat{k}_j\delta(\omega^2 - v_p^2 k^2)$$

Total Energy:

$$\mathrm{Trace}\left[\mathrm{Im}\,G_{ik}(0,\omega)\right] = \mathrm{Trace}\left[\iiint \mathrm{Im}\,\hat{G}_{ij}(\mathbf{k},\omega) d^3k\right]$$

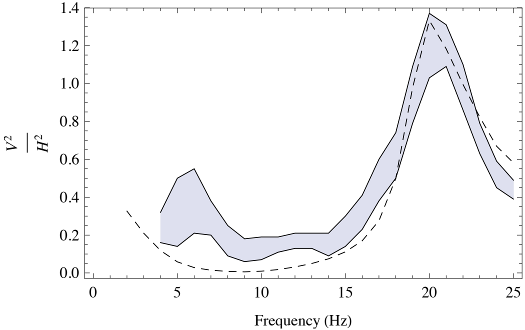
$$\sim \text{Shear} + \text{Long}$$

$$\sim \overbrace{2\frac{\omega^2}{v_s^3}} + \overbrace{\frac{\omega^2}{v_p^3}}$$

# Application of Equipartition to Site Effect Studies

| Velocity Model |
| --- |
| $h_1 = 4\text{m}$<br>$\alpha_1 = 300\text{m/s}$<br>$\beta_1 = 150\text{m/s}$<br>$\rho_1 = 2200\text{kg/m}^3$ |
| $h_2 = 11\text{m}$<br>$\alpha_2 = 900\text{m/s}$<br>$\beta_2 = 500\text{m/s}$<br>$\rho_2 = 2200\text{kg/m}^3$ |
| $h_3 = 50\text{m}$<br>$\alpha_3 = 3100\text{m/s}$<br>$\beta_3 = 1600\ \text{m/s}$<br>$\rho_3 = 2700\text{kg/m}^3$ |
| $\alpha_\infty = 5400\text{m/s}$<br>$\beta_\infty = 3000\text{m/s}$<br>$\rho_\infty = 2700\text{kg/m}^3$ |

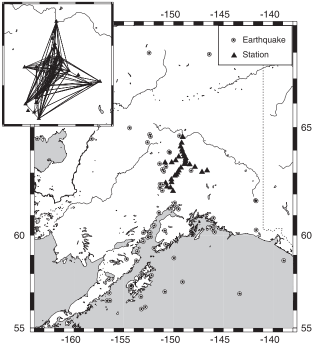
Energy partition in the coda



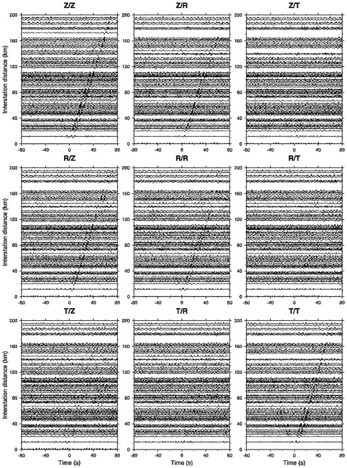

Margerin et al., Geophys. J. Int., 177, 571-585, 2009

## Consequence 2: Green function reconstruction

Temporary experiment in Alaska



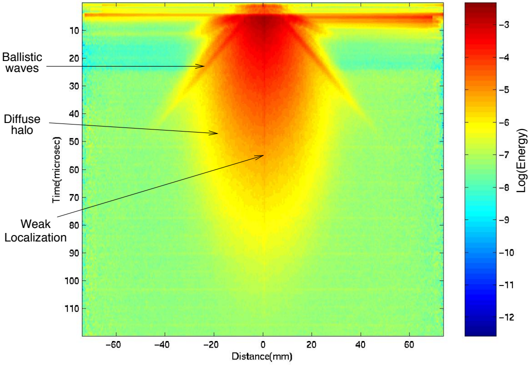

Correlation of Coda waves




Paul et al., J. Geophys. Res., 110, B08302, 2005

# Beyond radiative transfer: Weak localization

## Ultrasound propagation in sand




Courtesy A. Derode, L.O.A.

## Interference of reciprocal paths