Workshop on Elastic Waves
Col de Porte

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UNIVERSITY OF MANITOBA

ULTRASONICS RESEARCH LABORATORY

Anderson localization of ultrasonic waves in three dimensions

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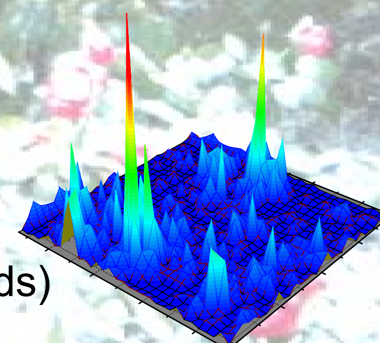
At Manitoba, we use ultrasound to study wave phenomena in mesostructured materials, and to probe the physical properties of mesoscopic materials.

- ballistic and diffusive wave transport in random media
- field fluctuation spectroscopy (DSS, DAWS…)
- wave transport & focusing in phononic crystals
- ultrasound in complex materials (e.g., soft matter, foods)



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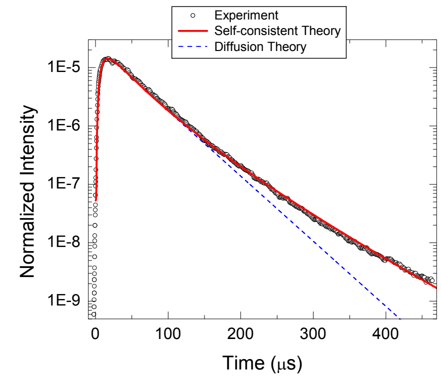
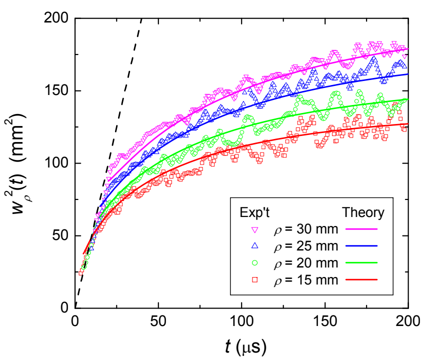
Outline: Localization of Elastic Waves

I. Introduction:
What is Anderson Localization?
Our samples & their basic (wave) properties

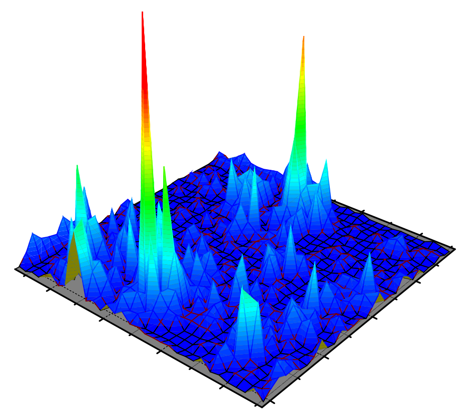
For a recent overview, see Physics Today, August 2009



II. Time-dependent transmission, $I(t)$



III. Transverse confinement of ultrasonic waves due to localization
("3D transverse localization")



IV. Statistical approach to localization
– non-Rayleigh statistics, variance, multifractality.

V. Conclusions

Hu *et al.*, *Nature Physics*, **4**, 945 (Dec, 2008) arXiv:0805.1502

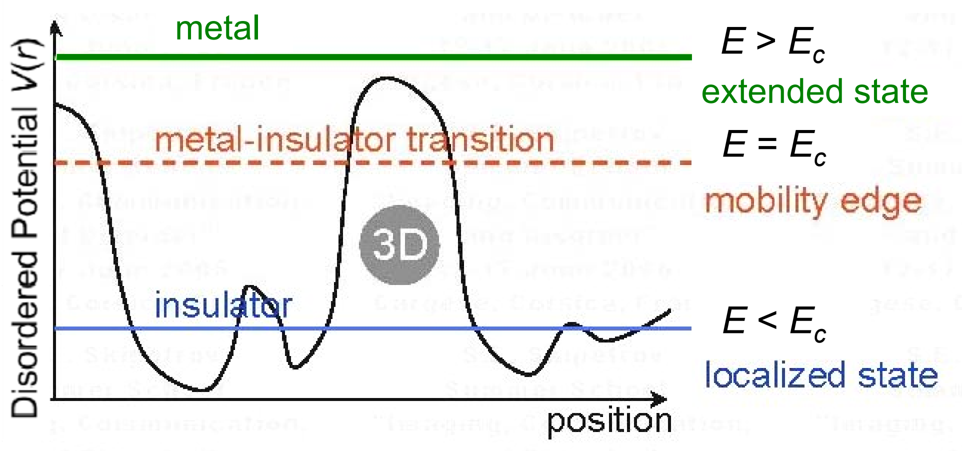
Introduction: Anderson localization of electrons (quantum particles)

P.W. Anderson
1958

(~50 years ago)

Schrodinger equation:
$V(\mathbf{r})$ varies randomly in space

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$



"Localization [..], very few believed it at the time, and even fewer saw its importance, among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it."
P.W. Anderson, Nobel Lecture, 1977

Many theoretical breakthroughs:

e.g. Scaling theory (1979) (~30 years ago)
Self consistent theory (1980)

Experiments:

Hampered by interactions and finite temperatures

Introduction: Anderson localization of electrons (quantum particles)

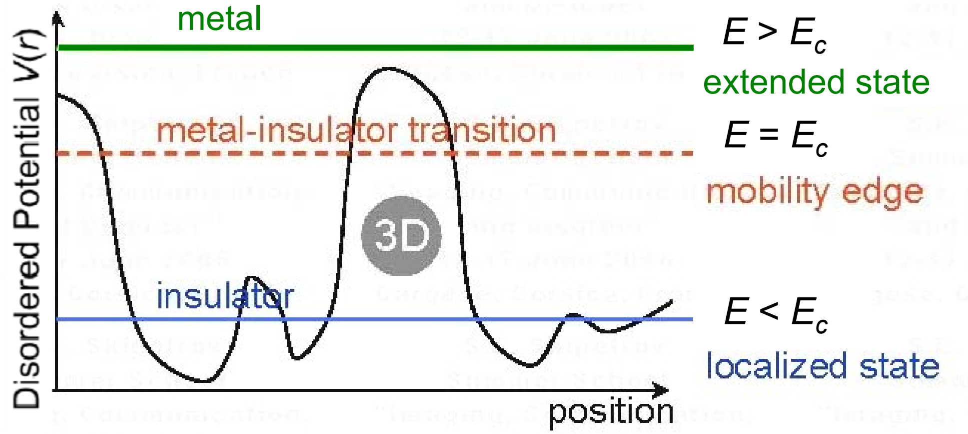
P.W. Anderson
1958

(~50 years ago)

Schrodinger equation:

V(**r**) varies randomly
in space

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) = E\psi(\mathbf{r})$$



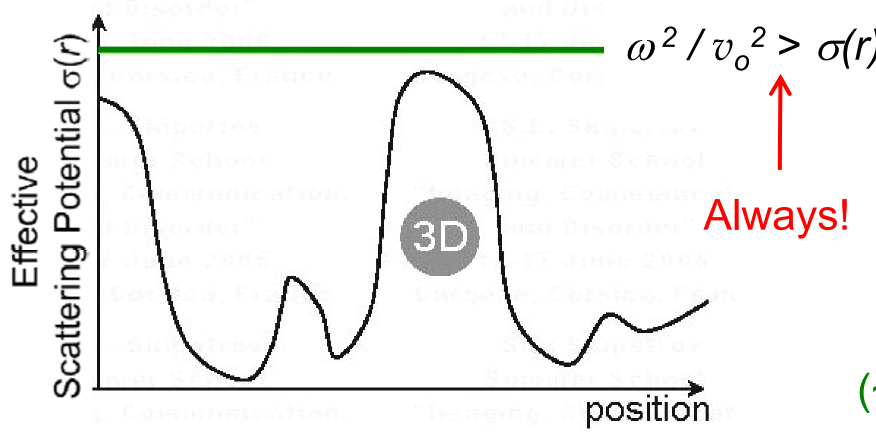
Localization of classical waves (sound or light)

e.g., scalar wave equation with disorder:

$$\left[-\nabla^2 + \sigma(r)\right]\psi(r) = \frac{\omega^2}{v_0^2}\psi(r)$$

$$\text{where} \quad \sigma(r) = \frac{\omega^2}{v_0^2} - \frac{\omega^2}{v^2(r)}$$

deviations from a uniform medium with velocity v_0

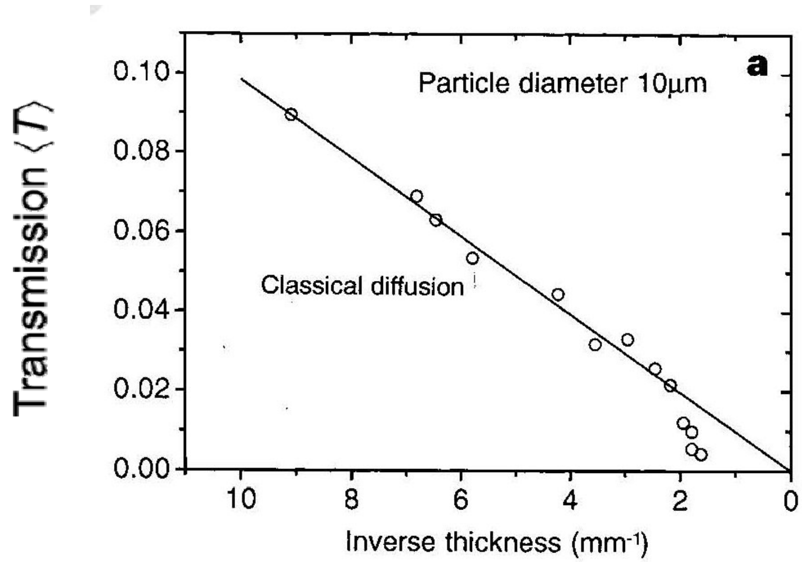


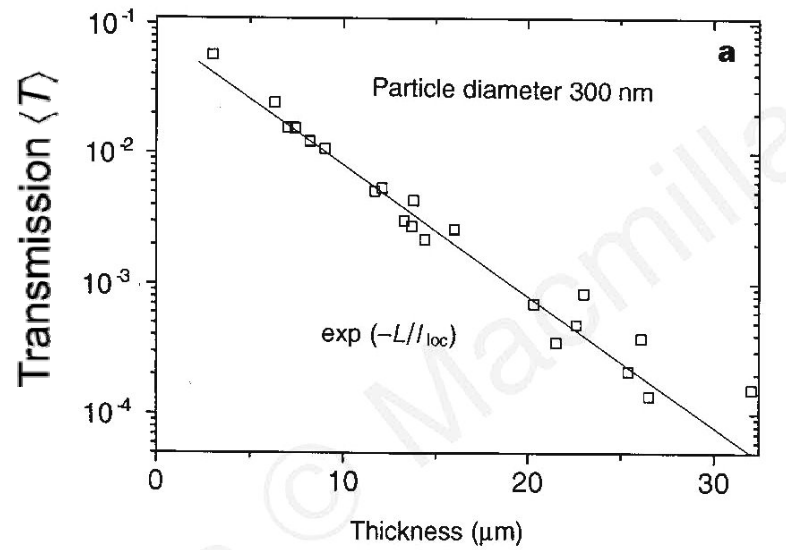
Sajeev John
1983

(~25 years ago)

Previous experiments with light in 3D:

Exponential scaling of the average transmission (for monochromatic waves) with thickness *L*. [Wiersma *et al*., *Nature* **390**, 671 (1997)]





Diffuse regime:

$$\langle T \rangle \propto \frac{\ell^*}{L}$$

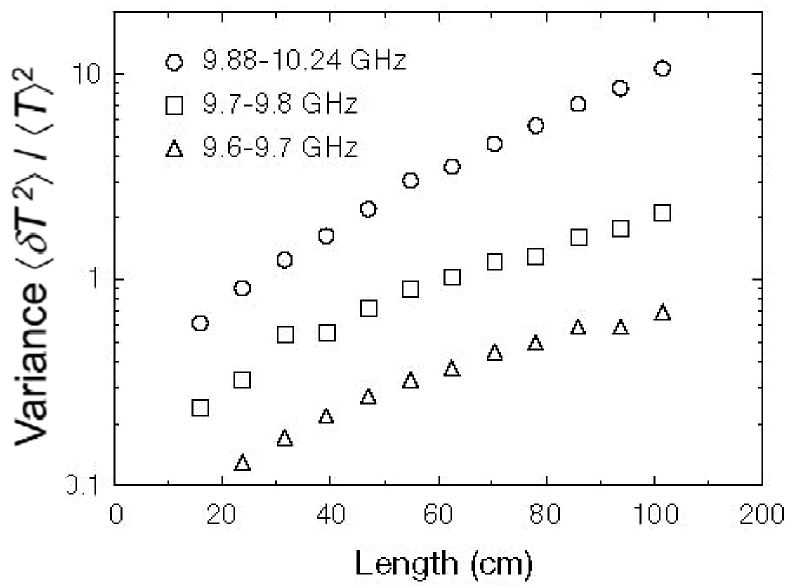
Localized regime

$$\langle T \rangle \propto \exp\left[-\frac{L}{\xi}\right]$$

- Difficult to distinguish from effects of absorption ($\propto \exp[-L/\ell_a]$)

Previous experiments with microwaves in quasi-1D:

Enhanced fluctuations of total transmission.
[Chabanov *et al*., *Nature* **404**, 850 (2000)]



Diffuse regime:

$$\frac{\langle \delta T^2 \rangle}{\langle T \rangle^2} \ll 1$$

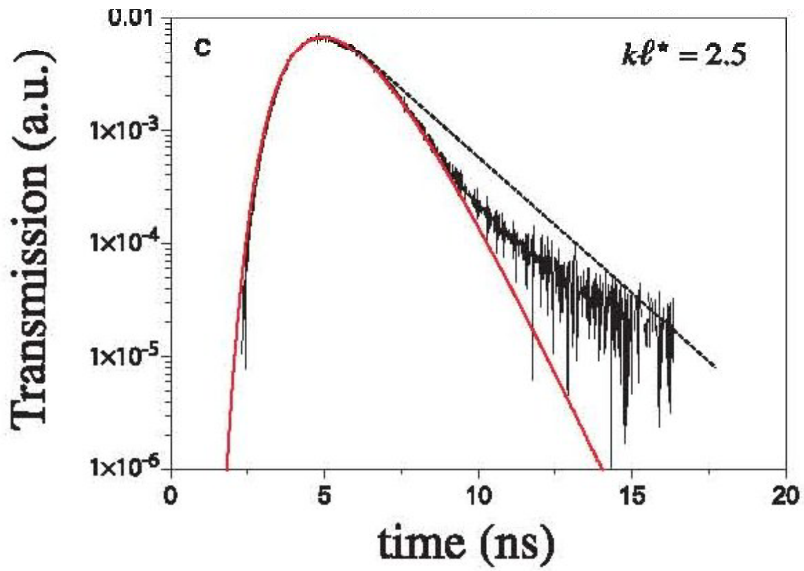
Localized regime

$$\frac{\langle \delta T^2 \rangle}{\langle T \rangle^2} > \text{const} \sim 1$$

• Chabanov *et al*. proposed that this criterion for localization is independent of absorption, but their experiments were limited to quasi-1-dimensional samples.

More recent experiments with light in 3D:

Time-dependent transmission through thick samples of TiO_2 particles
[Störzer et al., *PRL* **96**, 063904 (2006)]



Non-exponential tail at long times:
interpreted as a slowing down of diffusion with propagation time due to localization.

Current status (~50 years after Anderson's discovery):

- The subject is more alive than ever!
- Growing activity in optics, microwaves, acoustics, seismic waves, and atomic matter waves.

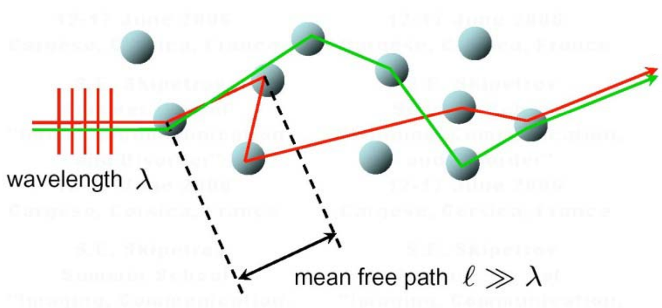
Question: Can we convincing observe the localization of ultrasound due to disorder in 3D, and, if so, can we learn something new?

NB Scaling theory ⇒ Only in 3D is there a real transition from extended to localized modes (*i.e.*, a mobility edge)

Weak disorder ($k\ell >> 1$):

Diffuse propagation

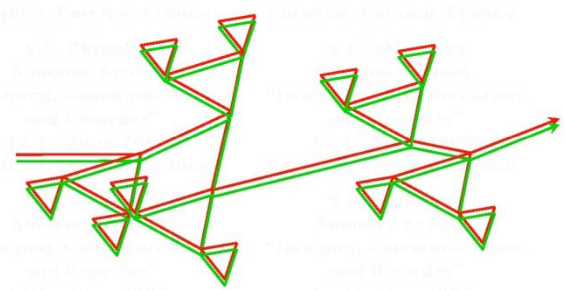
$D_B = ⅓\, v_E \ell_B^*$ (neglect interference)



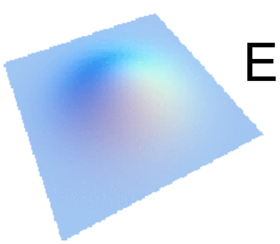
Strong disorder ($k\ell \sim 1$):

Anderson localization

(interference is important!)

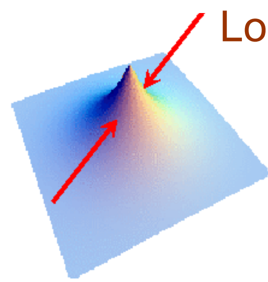


e.g., After a short pulse of ultrasound is incident on the medium…



Energy density spreads diffusively from the source

Localization length ξ



Energy remains localized the vicinity of the source

Our samples: "Mesoglasses" fabricated by sintering aluminum beads together to form a porous, solid 3D elastic network.

Aluminum volume fraction: $\phi = 0.55$

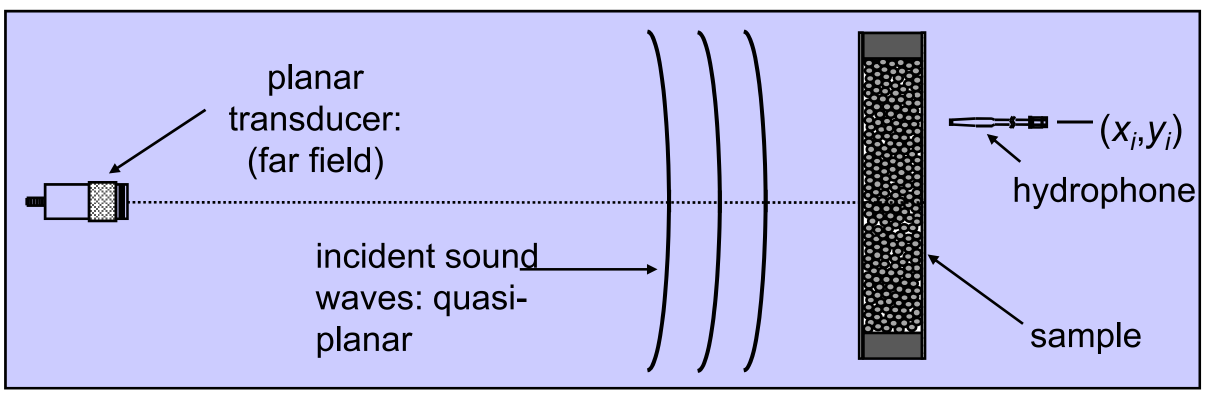
Monodisperse beads:

radius, $a_{bead} = 2.05$ mm

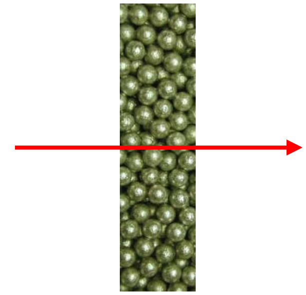
Sample width >> thickness (L: 8 to 23 mm)

Experiment: Pulsed ultrasonic transmission measurements (waterproofed samples, in a water tank)

Frequency range: 0.1 to 3 MHz ($6 \geq \lambda / a \geq 1$)



<u>Coherent transport in disordered Al mesostructures</u>:



Ballistic transport: Average the transmitted field to recover the weak coherent pulse and measure :

- phase velocity: $v_p = \omega / k$
- group velocity: $v_g = \mathrm{d}\omega / \mathrm{d}k$
- scattering mean free path, ℓ : $I = I_0 \exp\left[-L / \ell\right]$

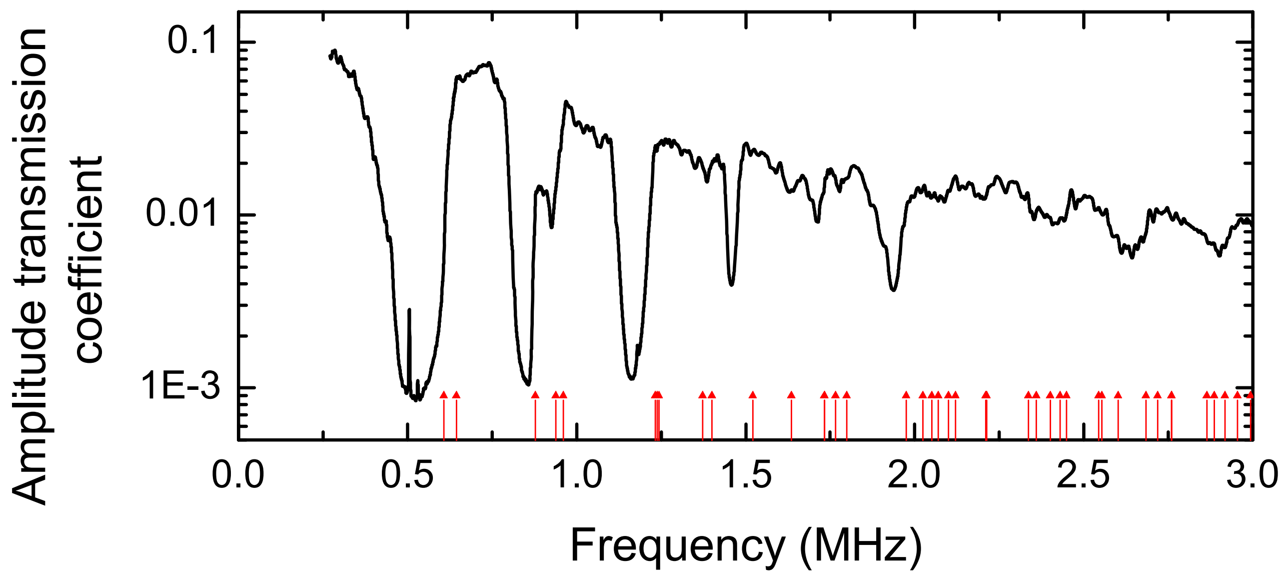
Very strong scattering in the intermediate frequency regime (0.2 – 3 MHz) :

$1 \leq k\ell \leq 2.5$

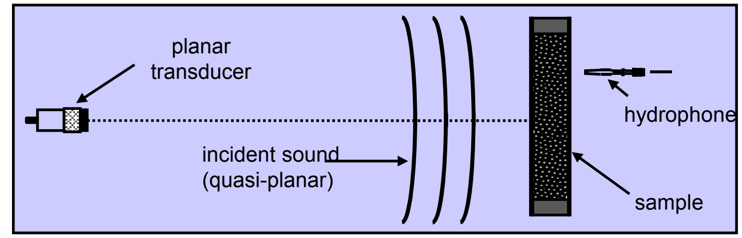
(outside the bandgaps)

Amplitude transmission coefficient:

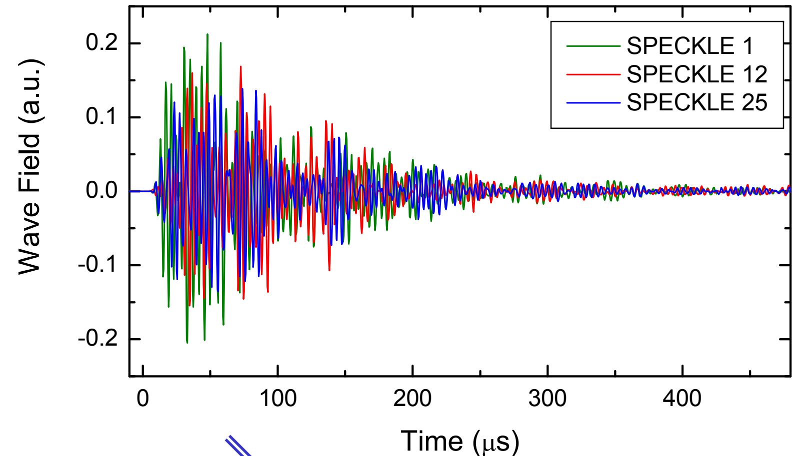
Bandgaps arise from weakly coupled resonances of the aluminum beads (Turner & Weaver, 1998)



II. Time-dependent transmission, $I(t)$.

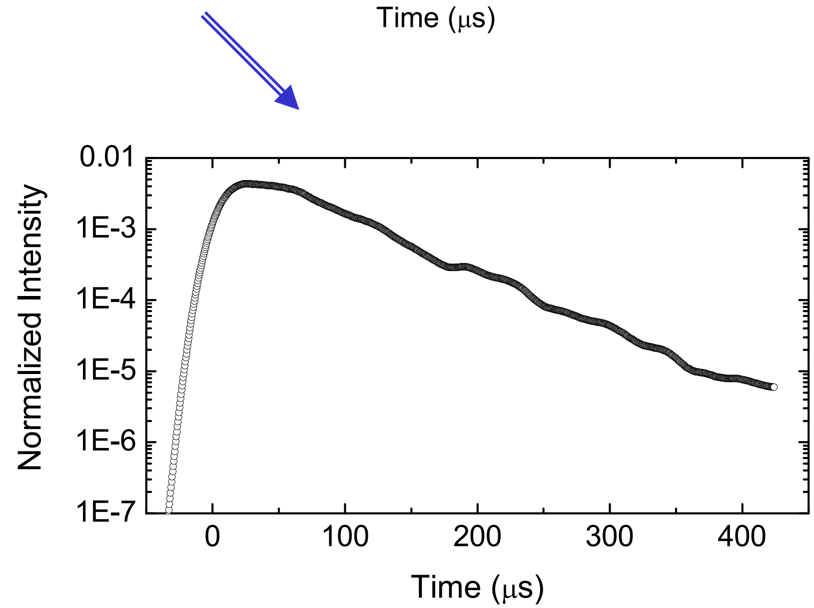


• Measure multiply scattered field in many independent speckles by scanning the hydrophone.



• Digitally filter the field to limit bandwidth (~5% usually)

• Determine $I(t)$ by averaging the squared transmitted pulse envelopes. (Normalize by the peak of the input pulse)



• First compare with the diffusion model, using realistic boundary conditions

(e.g. see Page *et al*., Phys. Rev. E **52**, 3106 (1995) for ultrasonic waves)

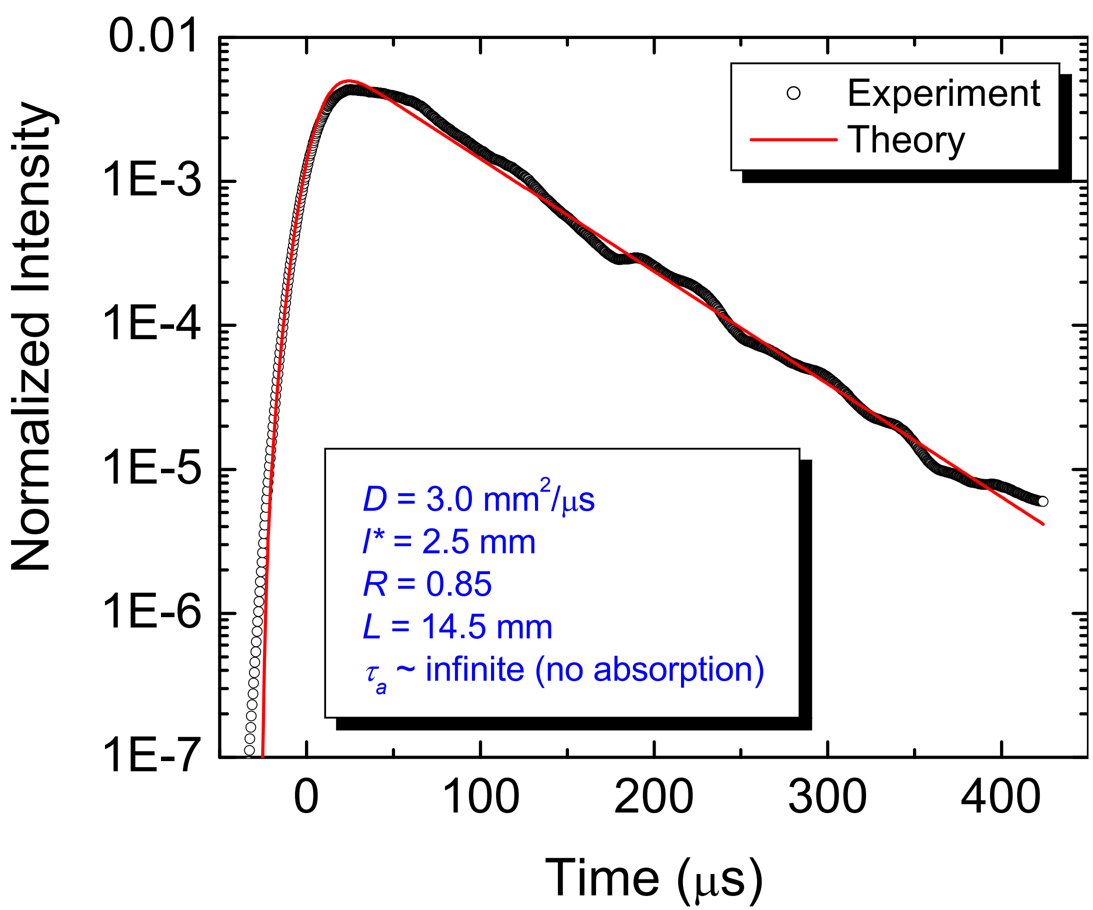
[z_0 - extrapolation length; z' - penetration depth; τ_a - absorption time]

• For elastic media, the diffusion coefficient $D_B = \frac{1}{3} v_E \ell^*$ is the energy-density weighted average of longitudinal and transverse waves.

Time-dependent transmission at low frequencies:
(below the lowest band gap)

Good fit to the predictions of the diffusion approximation for a plane wave source $\Rightarrow$ measure *D*. (Absorption is too small to measure.)

f = 0.2 MHz:



$I(t)$ decays exponentially at long times

$$I(t) \sim \exp\left[-t/\tau_D\right]$$

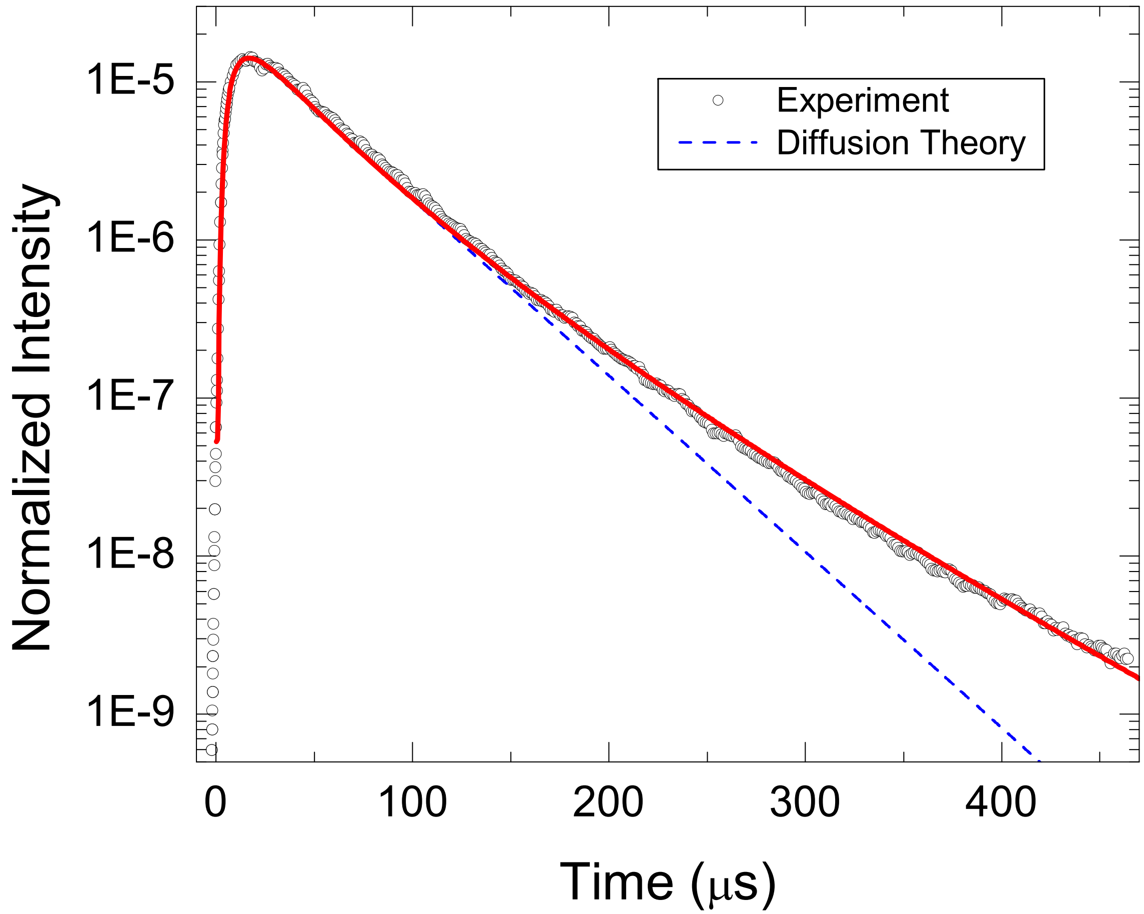
with

$$\tau_D = \left(L + z_0\right)^2 / \pi^2 D_B$$

Normal diffusive behaviour

$I(t)$ at higher frequencies (e.g. 2.4 MHz)

Find non-exponential decay of $I(t)$ at long times ($t \gg \tau_D$) $\Rightarrow$ Looks like a diffusion process with $D(t)$ decreasing with propagation time.



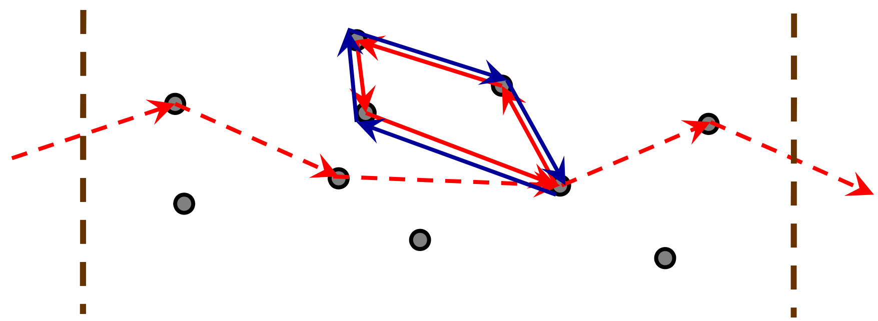
Suggests that sound may be localized

Quantitative analysis of $I(t)$ at high frequencies (2.4 MHz)

– fit the (plane wave) data directly with the recently improved self-consistent theory of localization [Skipetrov & van Tiggelen (2006)]

Basic idea:

The presence of loops increases the return probability as compared to 'normal' diffusion



⇩

Diffusion slows down

⇩

Diffusion constant should be renormalized $D_{\mathrm{B}} \rightarrow D < D_{\mathrm{B}}$

Generalization to Open Media:

Loops are less probable near the boundaries

⇩

Slowing down of diffusion is spatially heterogeneous

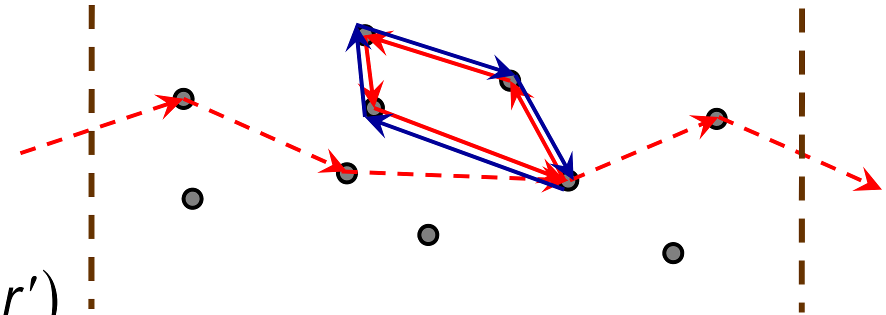
⇩

Diffusion constant becomes position-dependent $D_{\mathrm{B}} \rightarrow D(\mathbf{r}) < D_{\mathrm{B}}$

Quantitative analysis of $I(t)$ at high frequencies (2.4 MHz)

– fit the (plane wave) data directly with the recently improved self-consistent theory of localization [Skipetrov & van Tiggelen (2006)]

Mathematical formulation:



Diffusion equation

$$\left[-i\Omega - \nabla \cdot D(r,\Omega)\nabla\right] G(r,r',\Omega) = \delta(r-r')$$

($G(r,r',\Omega)$ – Intensity Green's function)

+

Self-consistent equation for the diffusion coefficient

$$\frac{1}{D(r,\Omega)} = \frac{1}{D_B} + \frac{3}{\pi\rho(\omega)D_B} G(r, r'=r, \Omega)$$

($\rho(\omega)$ – density of states)

+

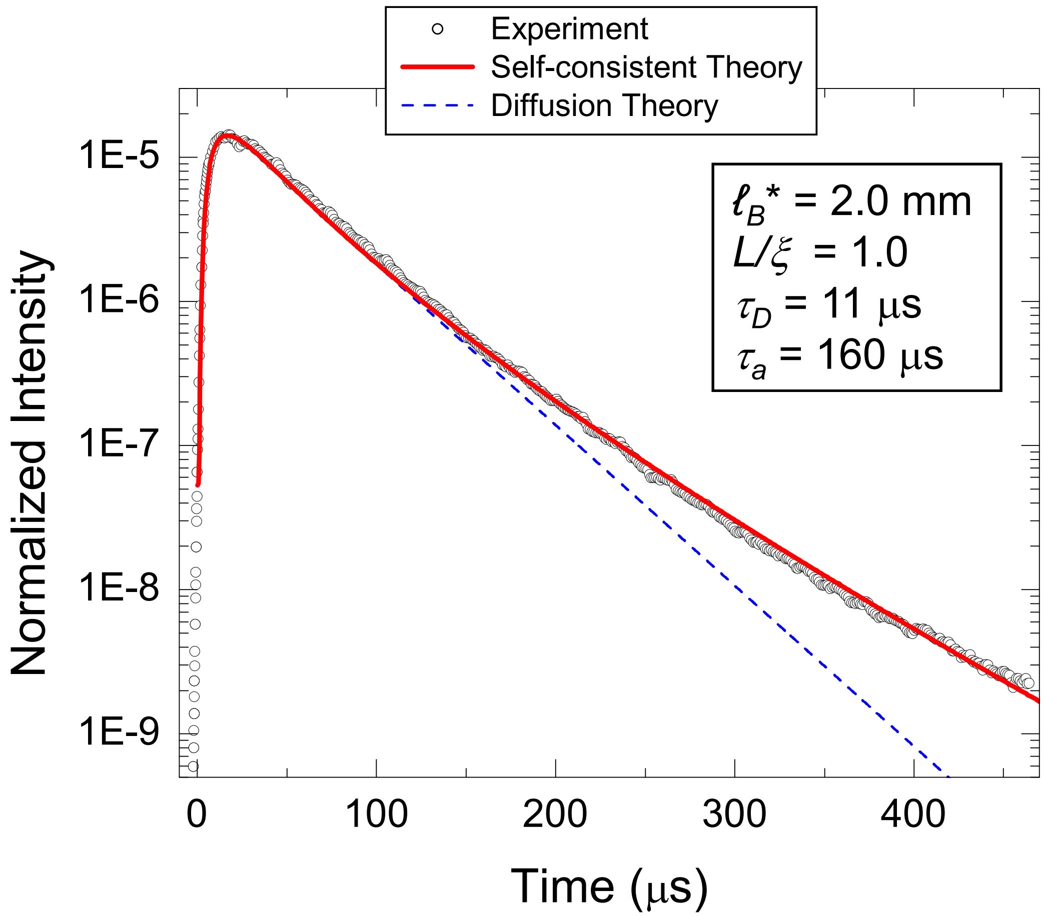
Boundary conditions

$$G(r,r',\Omega) - z_0 \frac{D(r,\Omega)}{D_B}\left(\mathbf{n}\cdot\nabla G(r,r',\Omega)\right) = 0$$

Diffusion coefficient depends on position **r** and frequency Ω

Quantitative analysis of $I(t)$ at high frequencies (2.4 MHz)

– fit the (plane wave) data directly with predictions of the self consistent theory of localization for $D(\mathbf{r},\Omega)$ [Skipetrov & van Tiggelen (2006)]



Input parameters:

$L = 14.5$ mm (sample thickness)

$\ell = 0.6$ mm (scattering mean free path)

$R = 0.82$ (internal reflection coeff.)

$z_0 = \ell_B^* \tfrac{2}{3}(1+R)/(1-R) = 6.7\ \ell_B^*$

$v_p = 5.0$ km/s (phase velocity)

$k\ell = 1.82$

Fitted parameters:

ℓ_B^* (“bare” transport mean free path)

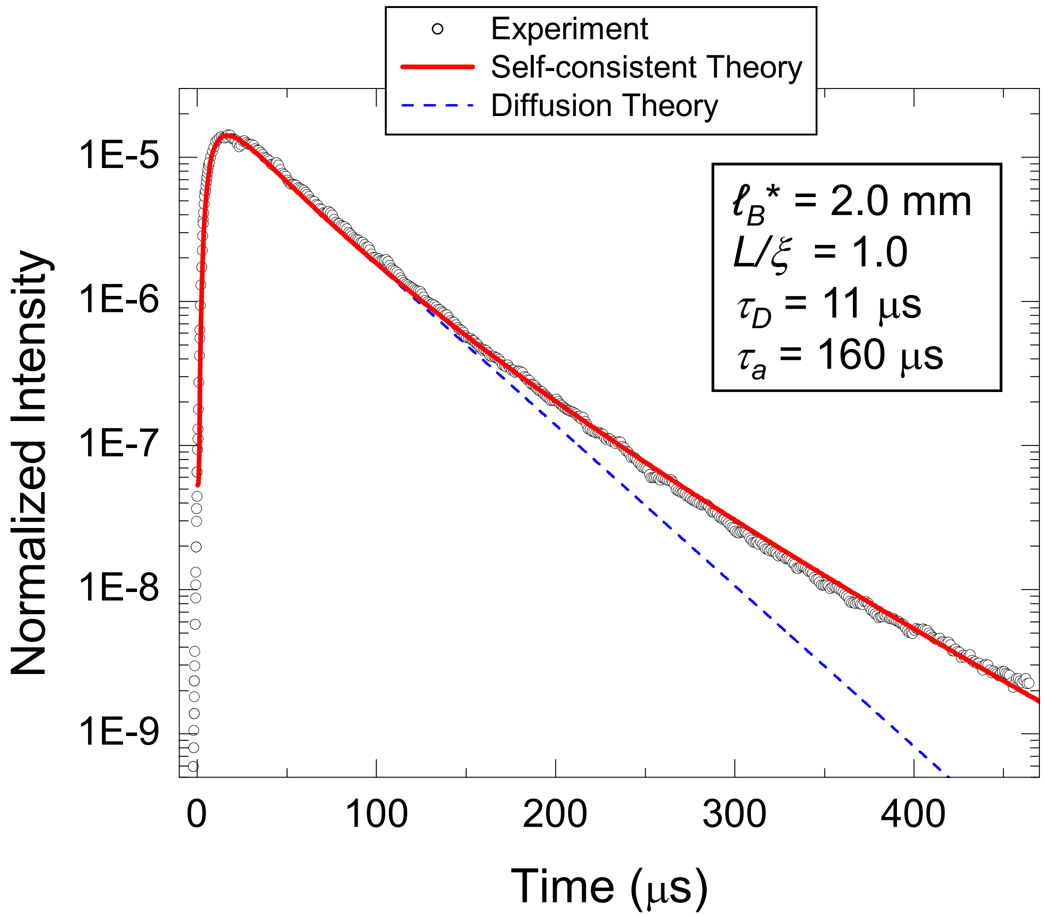
L/ξ (ξ is the localization length)

τ_D or D_B (bare diffusion coefficient)

τ_a (absorption time)

Excellent fit at all propagation times.

Quantitative analysis of $I(t)$ at high frequencies (2.4 MHz)
– fit the (plane wave) data directly with predictions of the self consistent theory of localization for $D(\mathbf{r},\Omega)$ [Skipetrov & van Tiggelen (2006)]



Localization length ξ:

$$\frac{\xi}{\ell_B^*} = \left[\frac{6}{(k\,\ell_B^*)_c^2}\right]\frac{\chi^2}{1-\chi^4}$$

where $\chi = k\ell/(k\ell)_c$

Localization regime:

$\xi > 0\,, \quad k\ell < (k\ell)_c$

Diffuse regime:

$\xi < 0\,, \quad k\ell > (k\ell)_c$

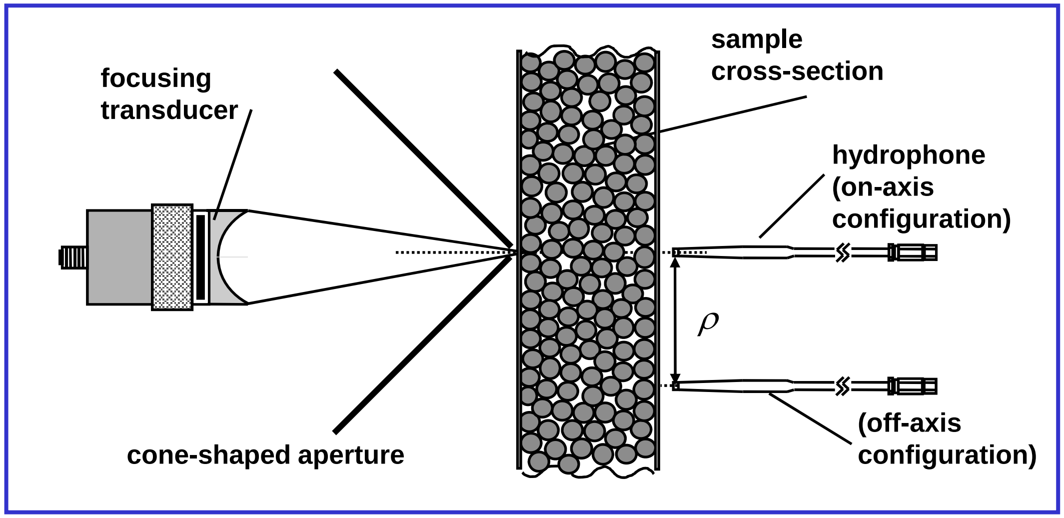
Excellent fit at all propagation times with $\xi > 0 \quad (L > \xi > L/4)$

$\Rightarrow$ Convincing evidence for the localization of sound

III. Transverse confinement ("transverse localization in 3D")

Experiment (displaced point source technique):

- Point source (focusing transducer + small aperture)
- Point detector, placed a transverse distance ρ away
- Scan *x*-*y* position of the sample to determine $I(\rho,t)$.



The ratio $I(\rho,t)/I(0,t)$ probes the transverse growth (dynamic spreading) of the intensity profile.

- Diffuse regime – measure the effective width of the "diffuse halo", which provides a method of measuring *D* independent of boundary conditions and absorption. [Page *et al.*, Phys. Rev. E **52**, 3106 (1995)]

$$\frac{I(\rho,t)}{I(0,t)} = \exp\left[-\rho^2/4Dt\right] \equiv \exp\left[-\rho^2/w^2(t)\right]$$

so the effective width *w(t)* is

$$w^2(t) = -\frac{\rho^2}{\ln\left[I(\rho,t)/I(0,t)\right]} = 4Dt$$

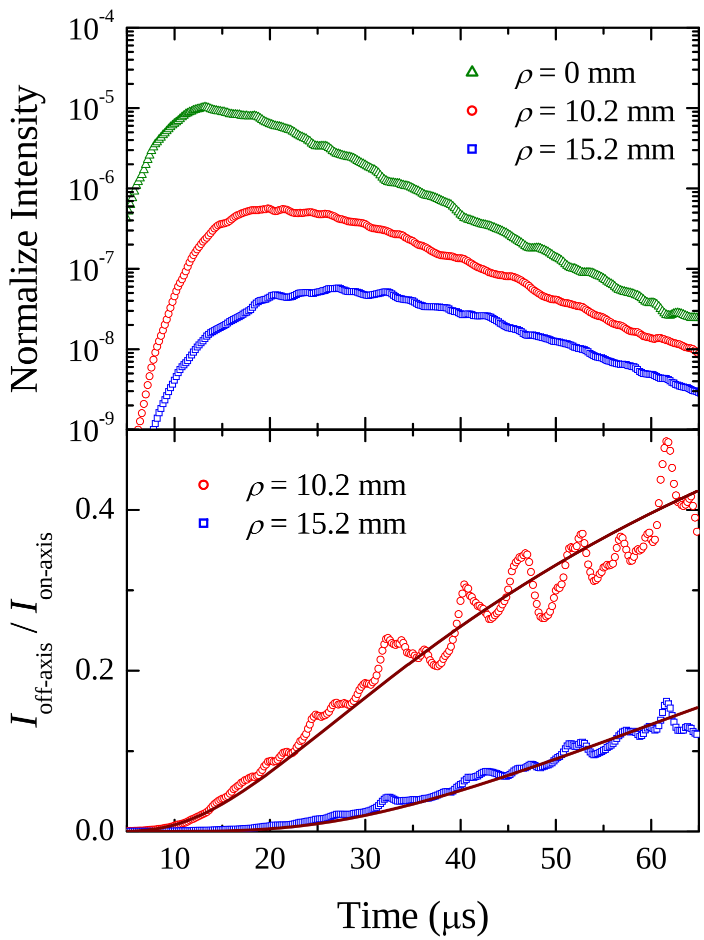
Diffuse regime –the effective width of the "diffuse halo" grows linearly in time

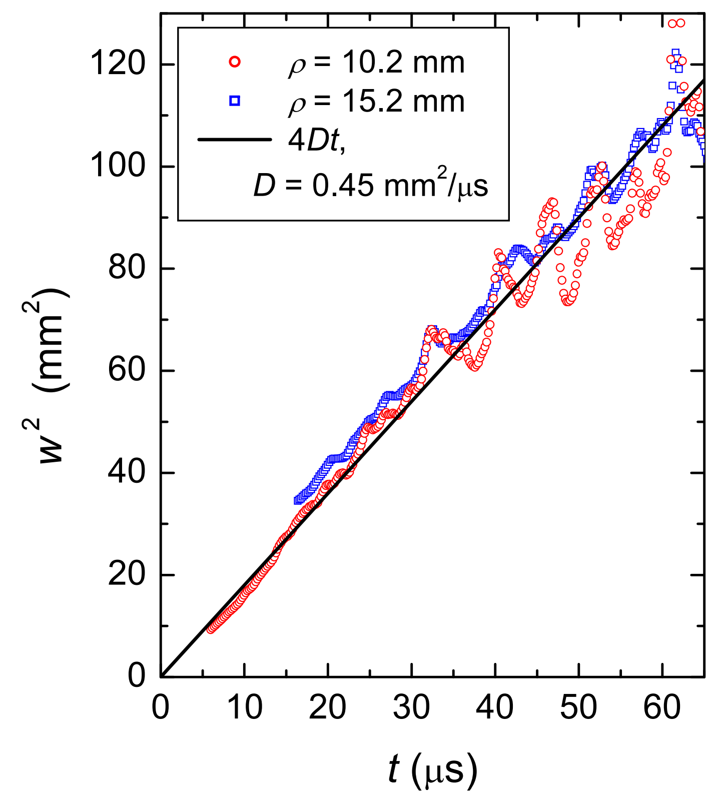
Data (from 1995) on a suspension of glass beads in water ($k\ell \sim 7$)

[Page *et al.*, Phys. Rev. E **52**, 3106 (1995)]

$$\frac{I(\rho,t)}{I(0,t)} = \exp\left[-\rho^2/4Dt\right] \equiv \exp\left[-\rho^2/w^2(t)\right]$$

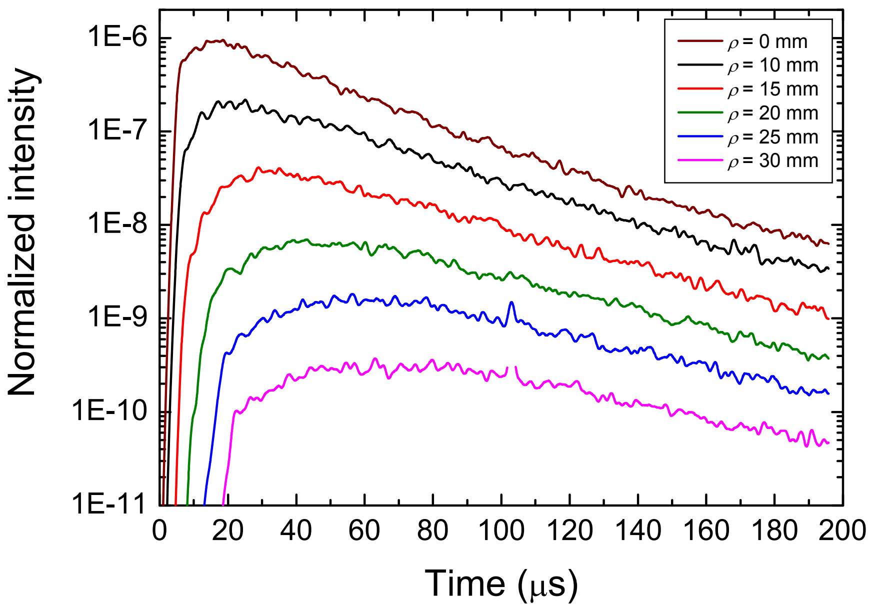
$$w^2(t) = -\frac{\rho^2}{\ln\left[I(\rho,t)/I(0,t)\right]} = 4Dt$$

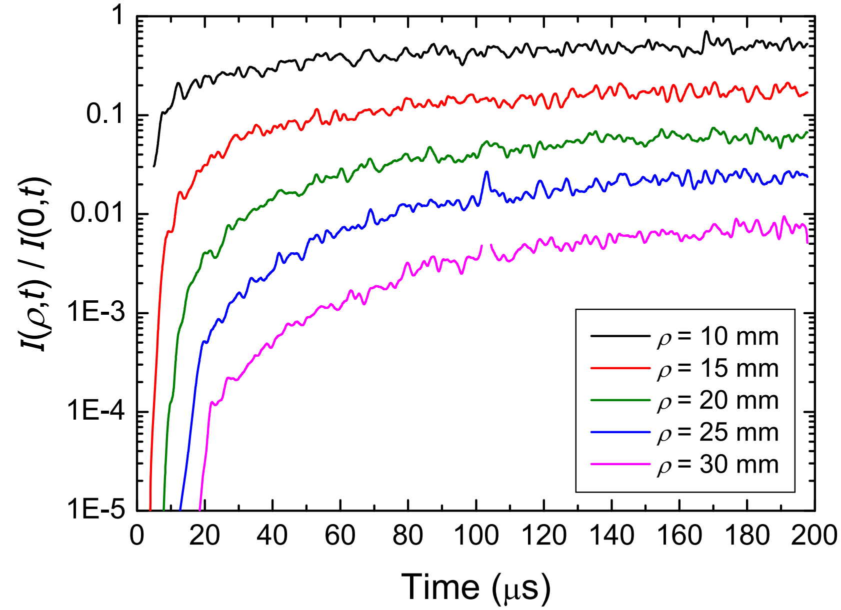




Measure D_B independent of boundary conditions and absorption.

Question: What happens to $I(\rho,t)$ & $w(t)$ in the localization regime?

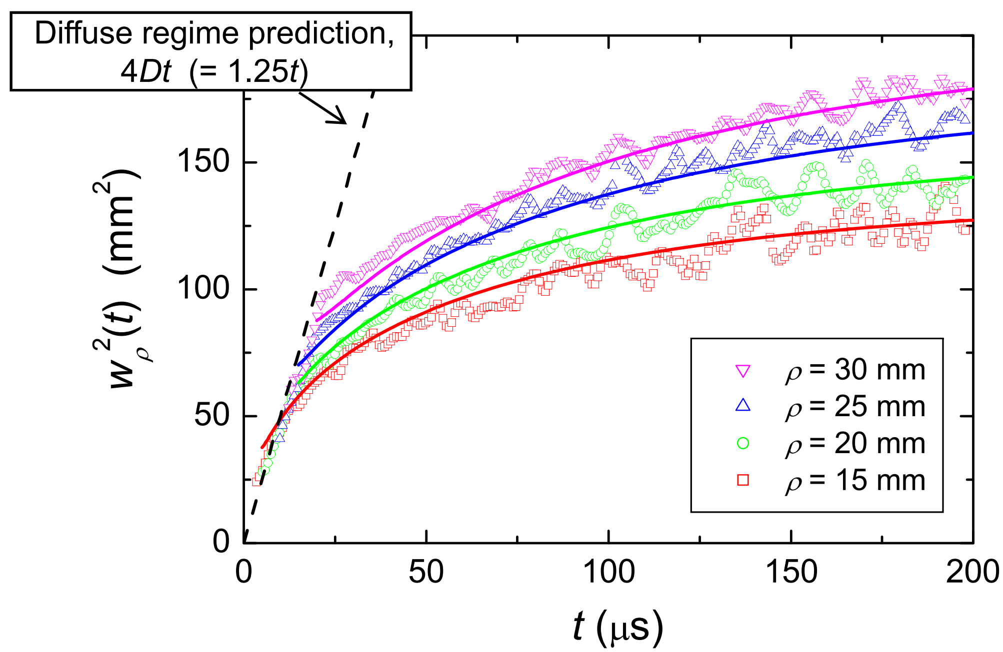




Dynamic transverse width at 2.4 MHz:

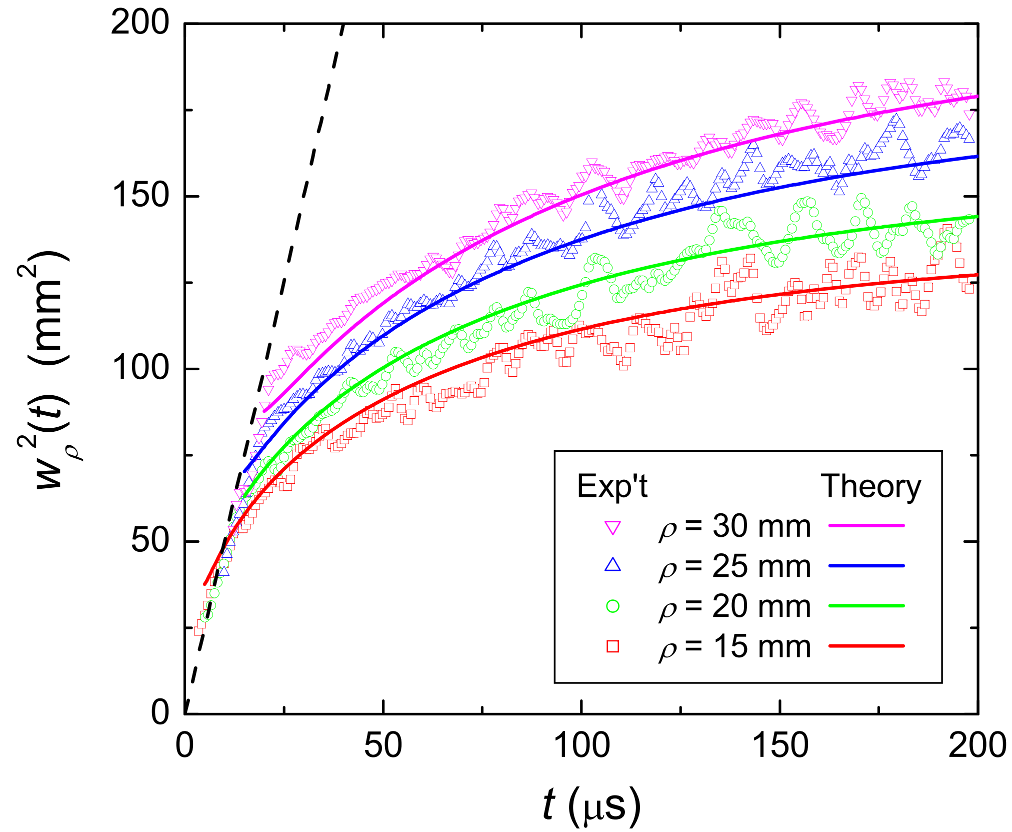
Localization dramatically inhibits the expansion of the intensity profile in the transverse direction.

$$\frac{I(\rho,t)}{I(0,t)} = \exp\left[-\rho^2/w^2(t)\right]$$



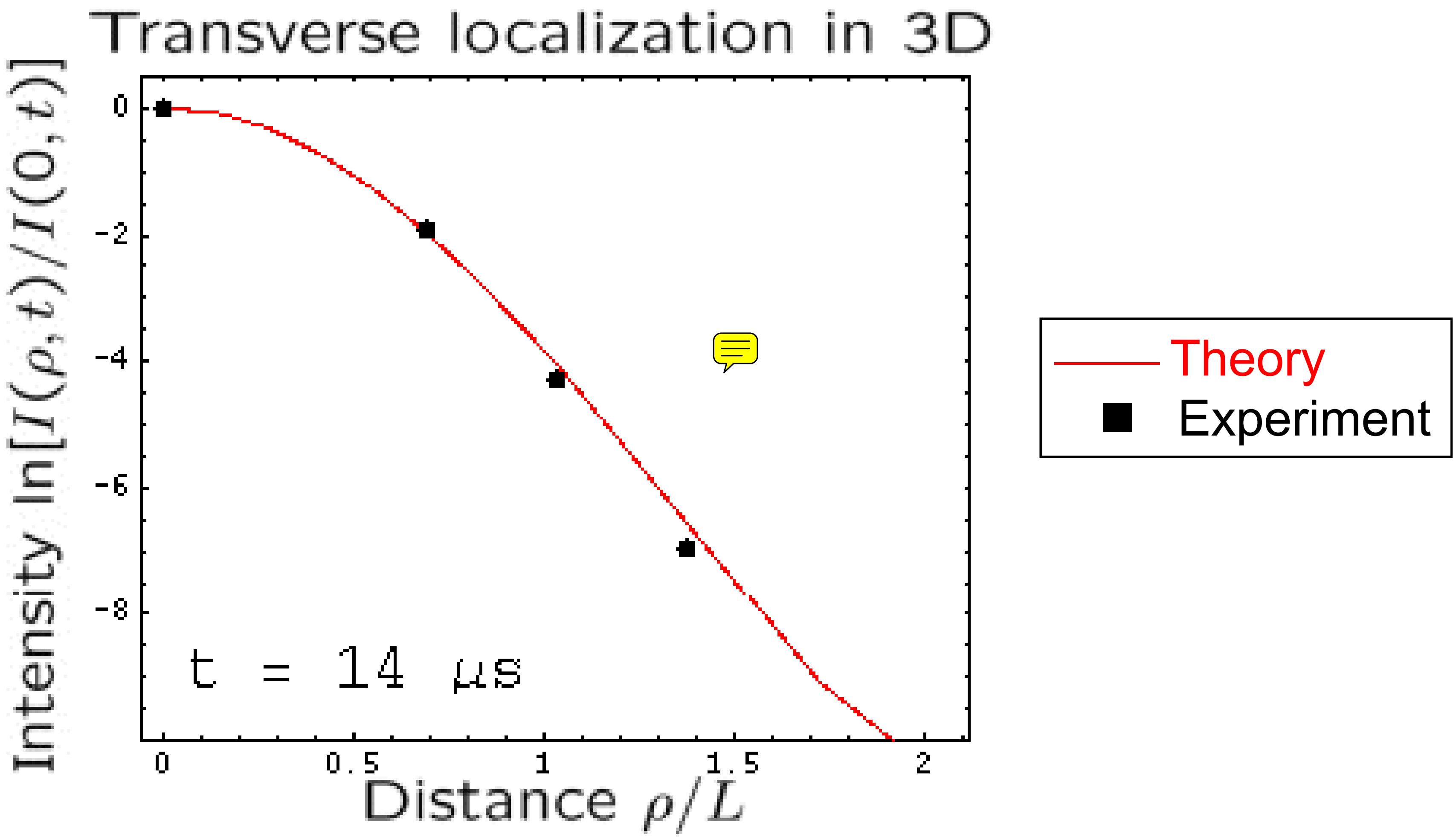
Quantitative analysis of the dynamic transverse width, $w(t)$:

- Fit the data using the new self consistent theory that allows for the position dependence of the renormalized diffusion coefficient in 3D.



- Excellent fit for all four ρ with:
  $\ell_B^* = 2.0$ mm
  $L/\xi = 1.0$
  $\tau_D = 17$ μs
  (τ_a cancels in ratio)
- Fit is more sensitive to ξ than plane wave $I(t)$
- Again, find $\xi > 0 \Rightarrow$ classical wave localization is convincingly demonstrated in this 3D "phononic" mesoglass.
- First direct measurement and theory for the transverse structure of localized waves in 3D. Find $w \sim$ 12-14 mm $\sim \xi$ for this sample

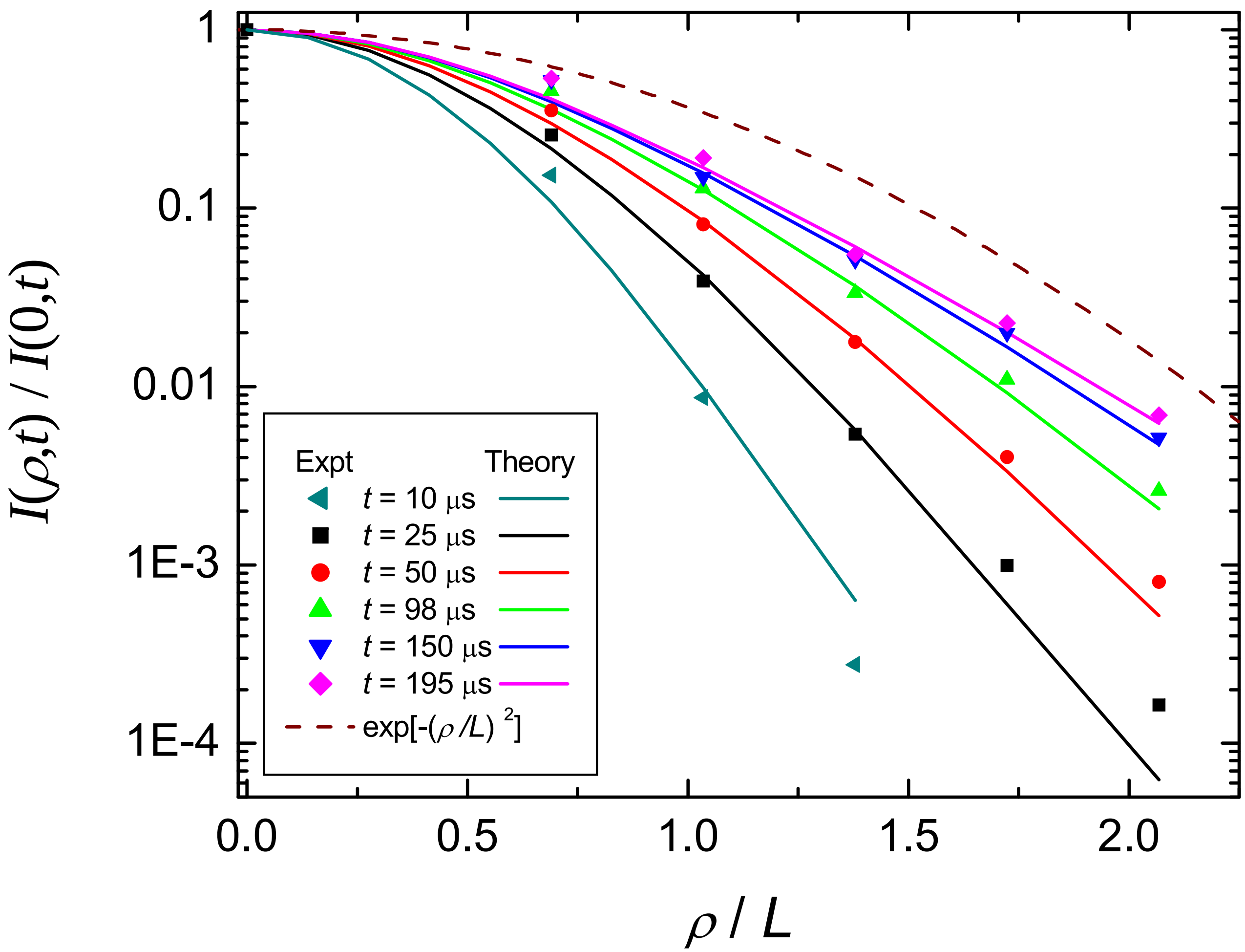
3D Transverse Localization: this animation (prepared by Sergey Skipetrov) shows the “freezing” of the transverse profile at long times (saturation of $I(\rho,t)/I(\rho,0)$ occurs for $t > t_{loc} \sim 100\ \mu s$ in this case.)



Decrease of $I(\rho,t)$ with transverse distance ρ is not Gaussian

⇒ Near the mobility edge ($k\ell/(k\ell)_c$ = 0.99 for this sample at this frequency), w varies somewhat with transverse displacement ρ.

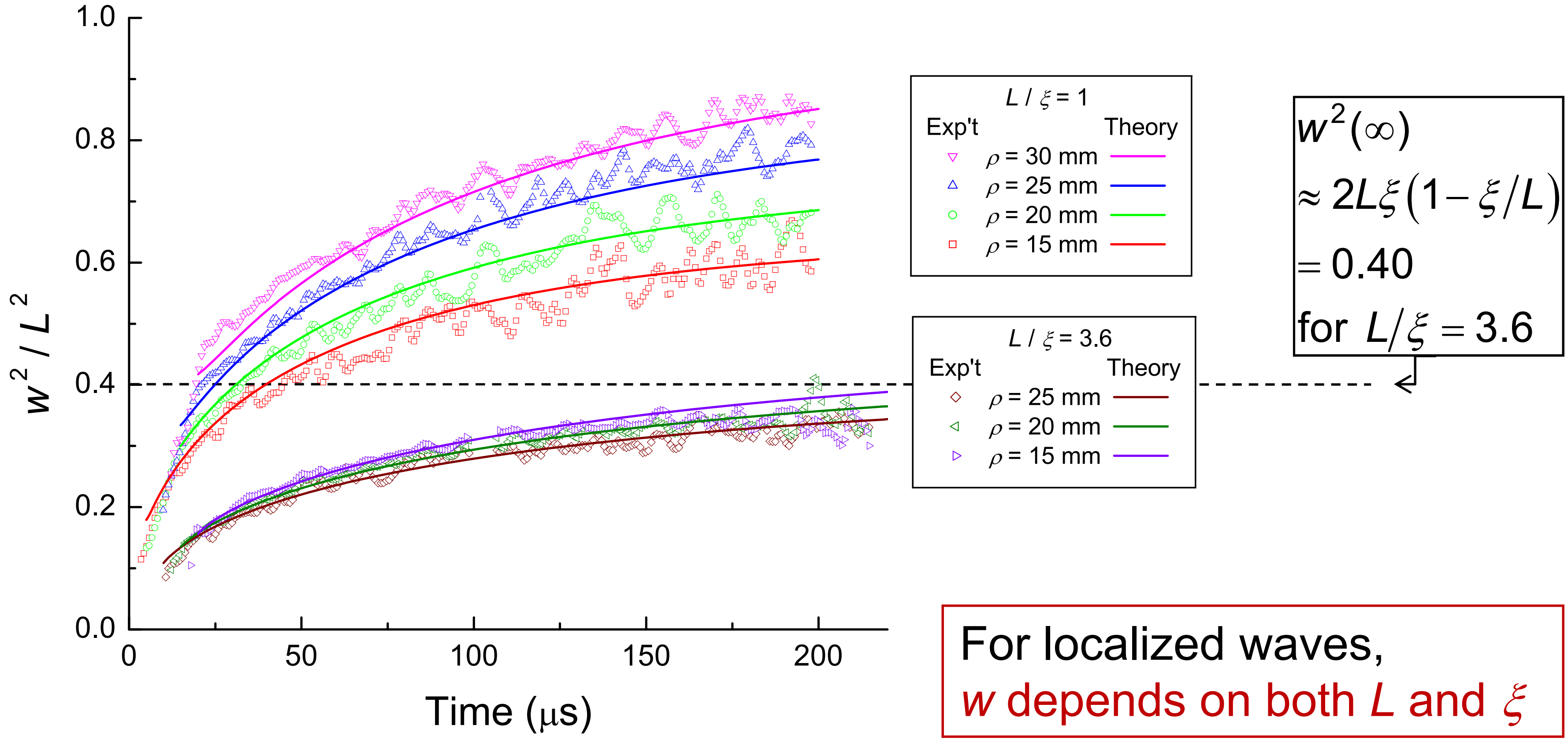
The self-consistent theory (solid curves) captures the experimentally observed dependence of $w(t)$ on ρ very well.



Question: What determines the magnitude of the dynamic transverse width $w_\rho(t)$?

- For thick samples, w becomes independent of ρ.
- Behaviour at long times: SC theory predictions for the saturated width when $L >> \xi$: $$w^2(t \to \infty) \approx 2L\xi\left(1 - \xi / L\right)$$

[Cherroret, Skipetrov and van Tiggelen, aiXiv:0810.0767v1]



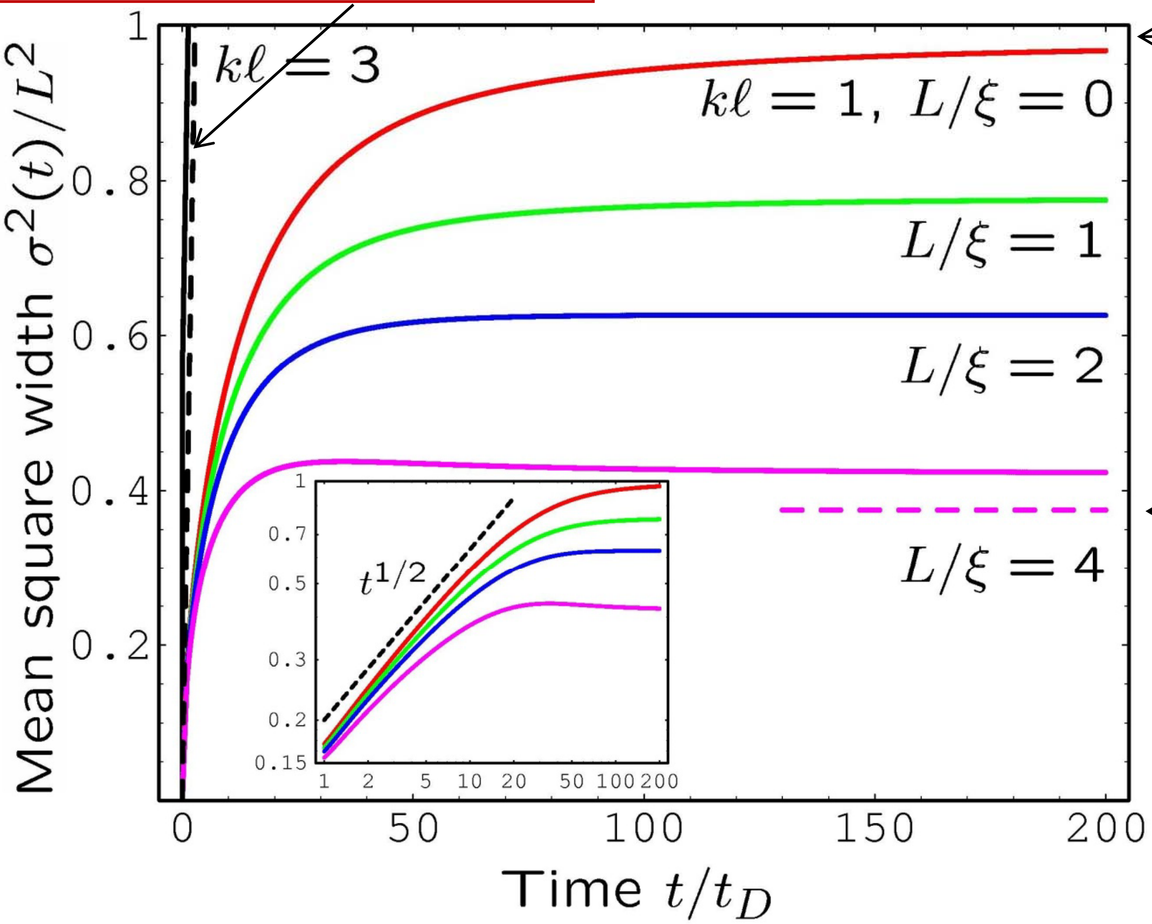
For localized waves,
w depends on both L and ξ

The saturation of $w(t)$ at long times is predicted *even* at the mobility edge [Cherroret, Skipetrov and van Tiggelen, arXiv:0810.0767v1].

Numerical calculations using the dynamic self-consistent theory:

In the diffuse regime:
$w^2(t \to \infty) = 4D\,[1-(k\ell)^{-2}]\,t$

At the mobility edge:
$w(t \to \infty) \approx L$

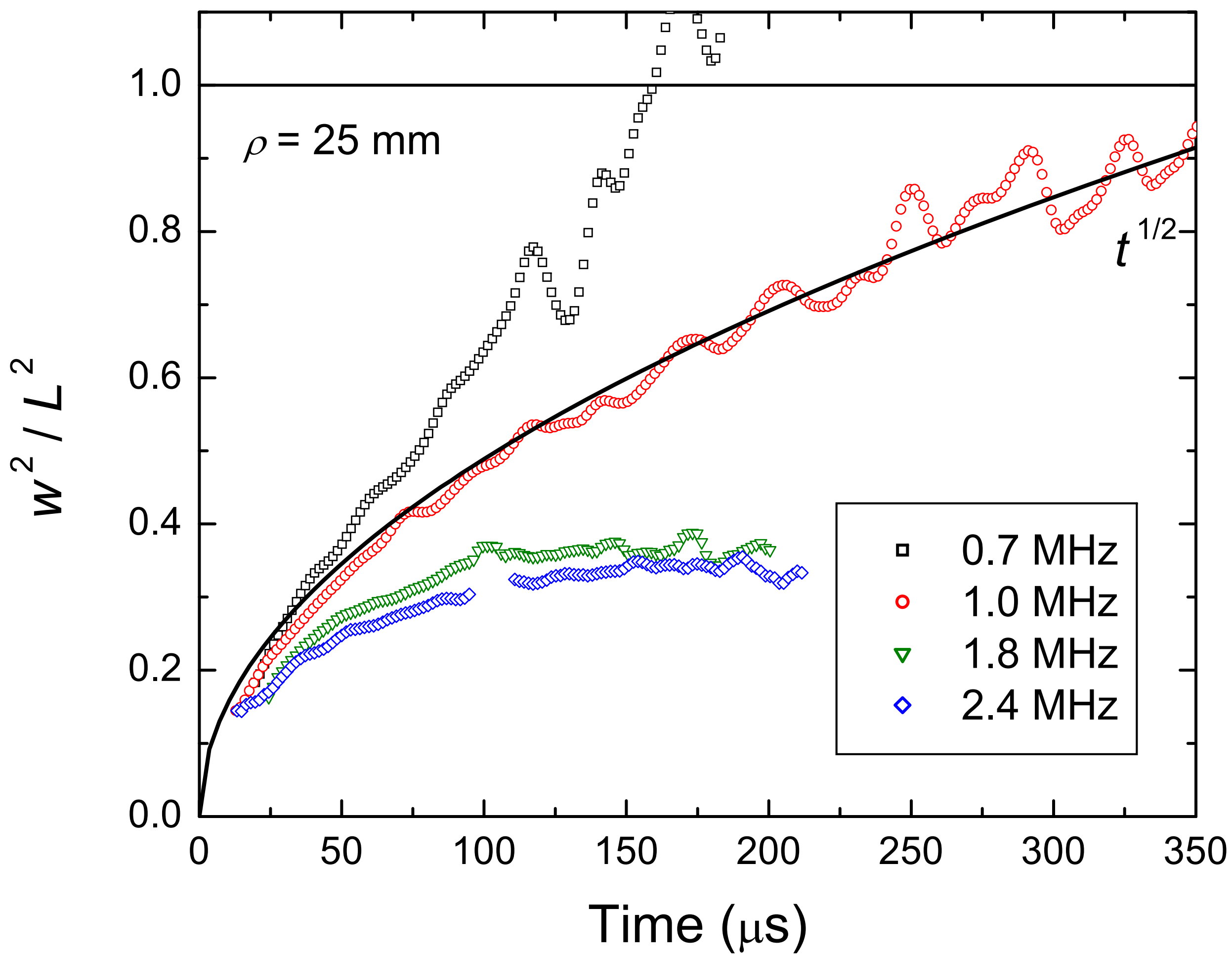


($L = 100\,\ell$)

Deep in the localization regime:

$$w^2(t \to \infty) \approx 2L\xi\left(1 - \xi / L\right)$$

What happens when we vary the frequency?

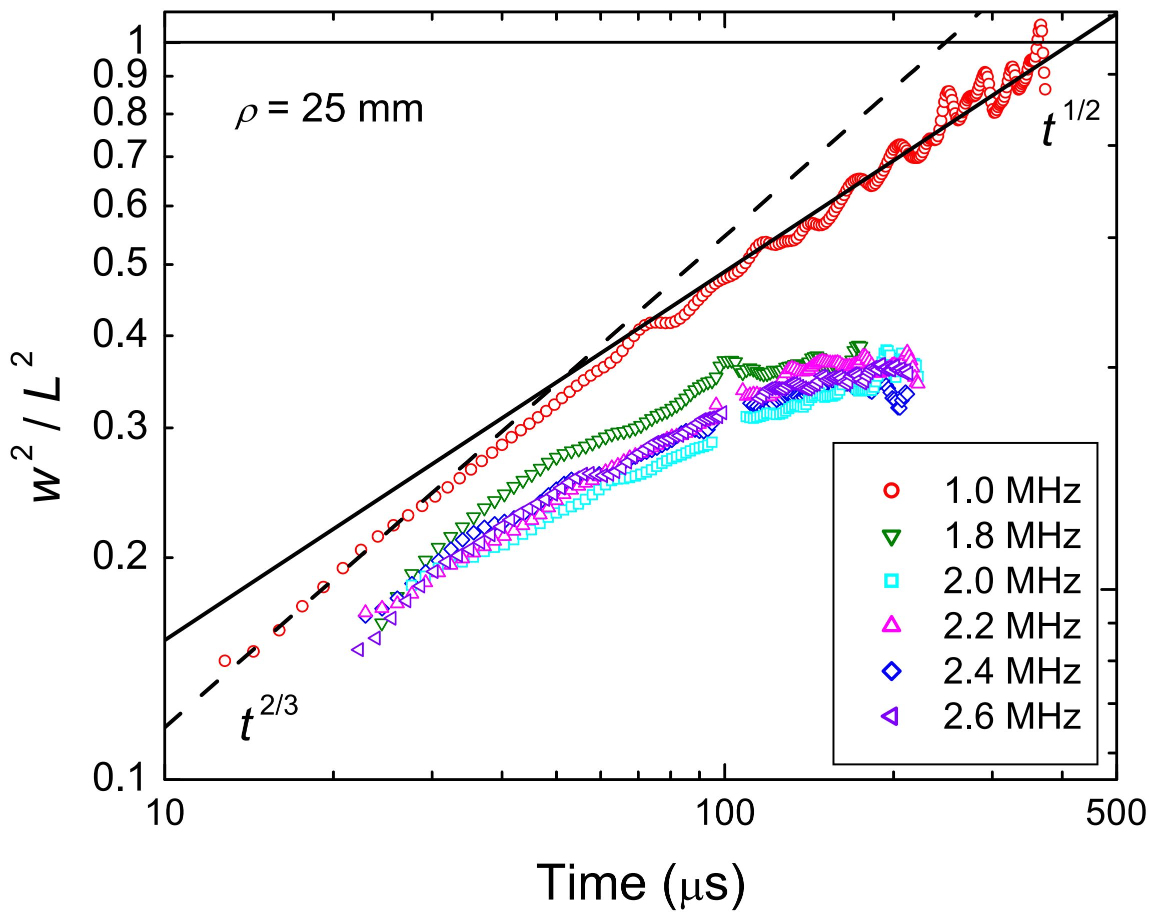


At 0.7 and 1.0 MHz, $w^2(t)$ does not saturate $\Rightarrow$ above the mobility edge.
(at 0.7 MHz, the time dependence is almost linear)

Should be feasible to measure ξ as the mobility edge is approached

What happens when we vary the frequency?

Plot on log scales to show the time dependence



Near the mobility edge, we see

$w^2(t) \propto t^{2/3}$ for $t < \tau_D$ &

$w^2(t) \propto t^{1/2}$ for a limited range of $t > \tau_D$

Agrees with predictions of the self-consistent theory.

Summary: Transverse confinement (3D transverse localization)

• The dynamic transverse width $w^2(t)$ has completely different properties for diffuse and localized modes

Diffuse: $w^2(t) \propto t$ and increases without bound.

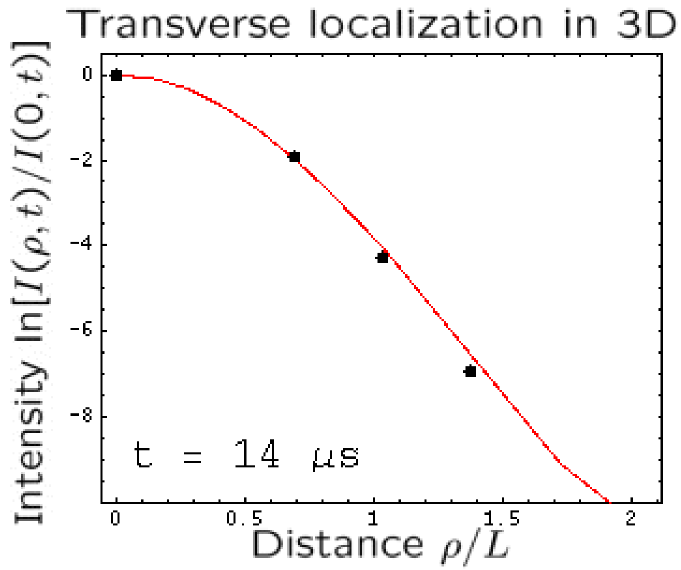
Localized: $w^2(t)$ saturates at long times.

At the mobility edge: $w(t \to \infty) \approx L$

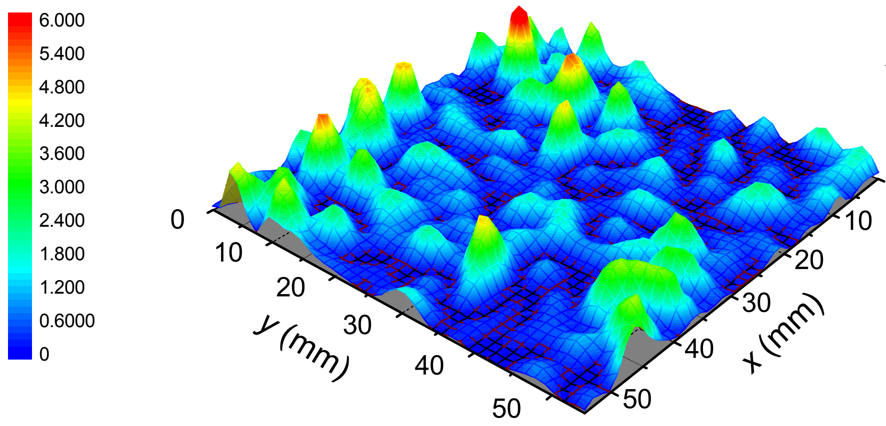
Deep in the localization regime: $w^2(t \to \infty) \approx 2L\xi\left(1 - \xi / L\right)$

• $w^2(t)$ is independent of absorption $\rightarrow$ its measurement (*for any kind of wave*) provides a valuable method for assessing whether or not the waves are localized. (No risk of confusing absorption with localization.)

• $w^2(t)$ can be used to measure the localization length ξ.



IV. Statistical approach to the localization of sound:



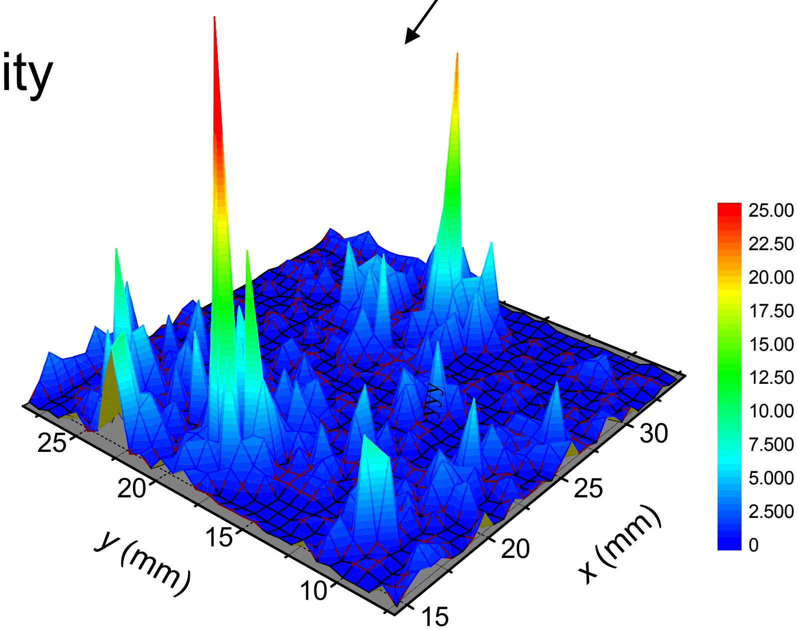
Diffuse ultrasound
(speckle pattern for our mesoglass at 0.20 MHz)

Localized ultrasound
(speckle pattern for our mesoglass at 2.4 MHz)

Large fluctuations in the transmitted intensity are characteristic of localized waves.

Signatures of these fluctuations are seen in:

- Near field speckle pattern
- Intensity distribution $P(I/\langle I \rangle)$
- Variance
- Multifractality



Transmitted intensity distributions for our mesoglass:

Measure the intensity I at each point in the near field speckle pattern when the sample is illuminated on the opposite side with a broad beam. When I is normalized by its average value to get $\hat{I} = I / \langle I \rangle$, its distribution is universal.

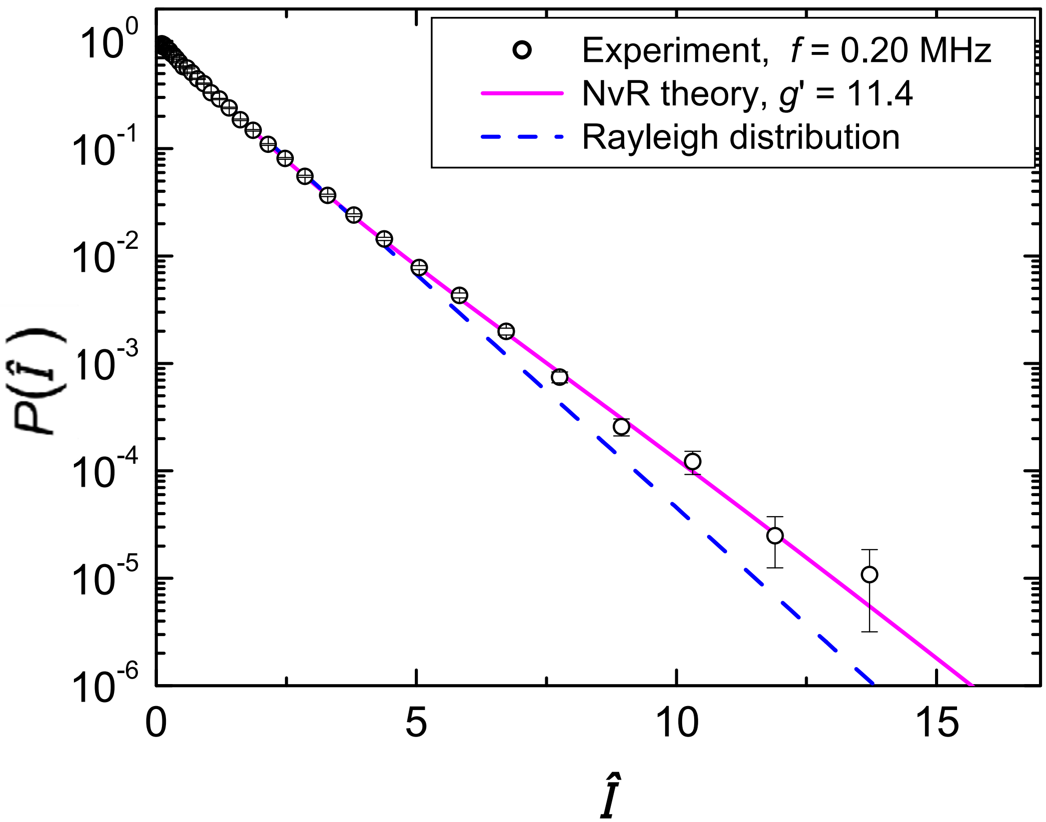
(a) Data at 0.20 MHz

Rayleigh distribution:
(random wave fields described by circular Gaussian statistics)

$$P(\hat{I}) = \exp(-\hat{I})$$

Leading order correction to Rayleigh statistics due to interference (no absorption) [Nieuwenhuizen & van Rossum, PRL **74**, 2674 (1995)] (g' = dimensionless conductance):

$$P(\hat{I}) = \exp(-\hat{I})\left[1 + \frac{1}{3g'}\left(\hat{I}^2 - 4\hat{I} + 2\right)\right]$$



Find g' = 11.4 >> 1
⇒ modes are extended

Transmitted intensity distributions for our mesoglass:

(b) Near 2.4 MHz (upper part of intermediate frequency regime), find very large departures from Rayleigh Statistics

Fit the entire distribution to predictions by van Rossum and Nieuewenhuizen [Rev. Mod. Phys. **71**, 313] for a slab geometry in 3D (red curve).

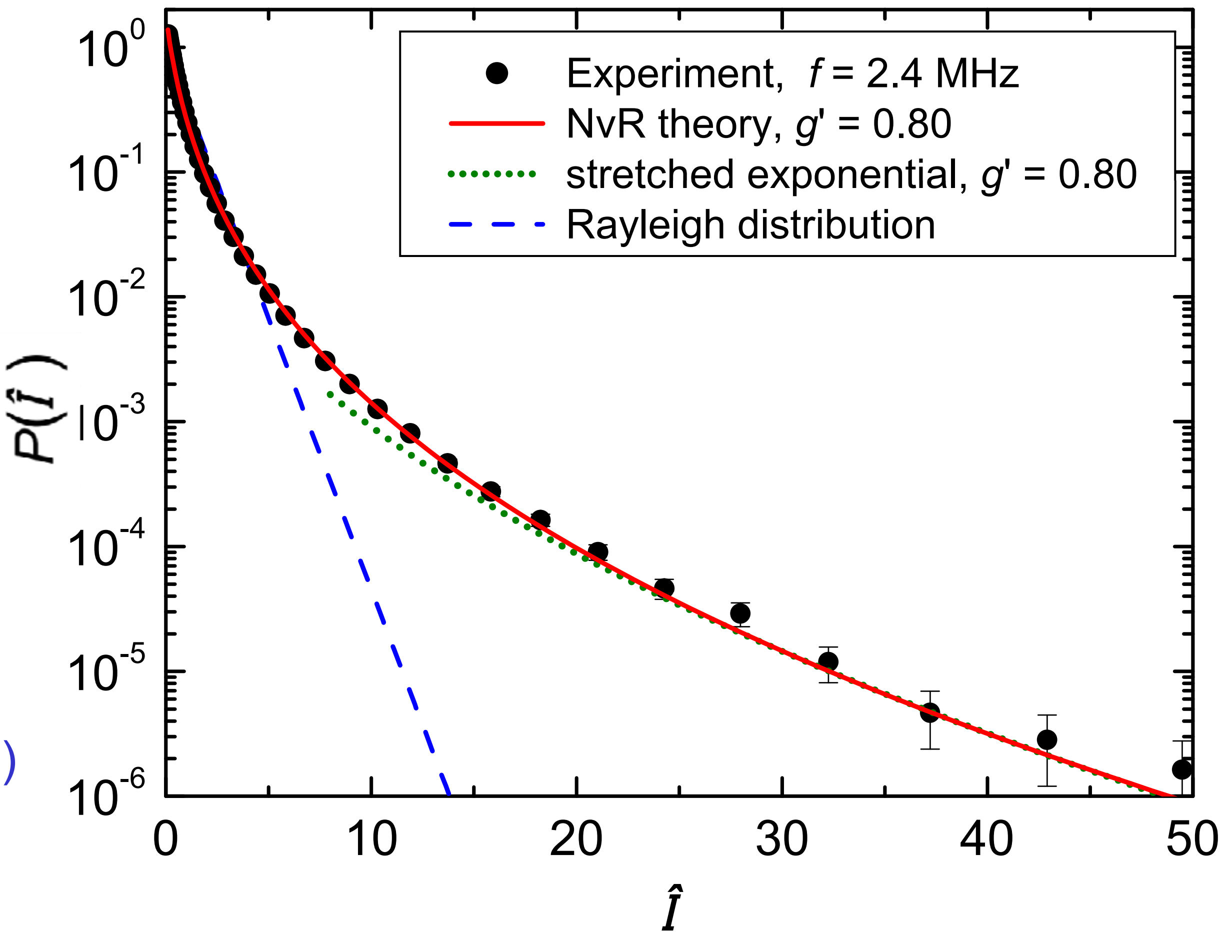
Remarkable agreement with experiment.

The tail of intensity distribution obeys a stretched exponential distribution

$$P\left(\hat{I}\right) \sim \exp\left(-2\sqrt{g'\hat{I}}\right)$$

(g' is the effective dimensionless conductance.)

Find $g' = 0.80 < 1$, indicating localization.



Variance of the transmitted intensity – another way to measure the dimensionless conductance g′:

Chabanov *et al.* [*Nature* **404**, 850 (2000)] have proposed that localization is achieved when the variance of the normalized *total* transmitted intensity , $\hat{T} = T/\langle T \rangle$ satisfies

$$\mathrm{var}(\hat{T}) \equiv \frac{\langle \delta T^2 \rangle}{\langle T \rangle^2} = \frac{2}{3g'} \geq \frac{2}{3}$$

whether absorption is present or not. This corresponds to the localization condition $g' \leq 1$.

But $\mathrm{var}(\hat{T})$ and $\mathrm{var}(\hat{I})$ are related: $\mathrm{var}(\hat{I}) = 2\,\mathrm{var}(\hat{T}) + 1$

Then, the Chabanov-Genack localization criterion gives $\mathrm{var}(\hat{I}) \geq 7/3$

e.g., for our data at 2.4 MHz:

Measure $\mathrm{var}(\hat{I}) = 2.74 \pm 0.09$ $\Rightarrow$ $g' = \dfrac{4}{3\left[\mathrm{var}(\hat{I}) - 1\right]} = 0.77 \pm 0.4$

Excellent agreement with $g' = 0.80 \pm 0.08$ measured from $P(\hat{I})$

Additional evidence that the modes are localized above ~ 2 MHz.

Multifractality (MF) of the wavefunction (with Sanli Faez, Ad Lagendijk):

[Faez et al., *PRL* **103**, 155703 (2009)]

Key idea: Large fluctuations ⇒ the moments of the wave function intensity

$$I(\mathbf{r}) = |\psi^2(\mathbf{r})| / \int |\psi^2(\mathbf{r})| d^d r$$

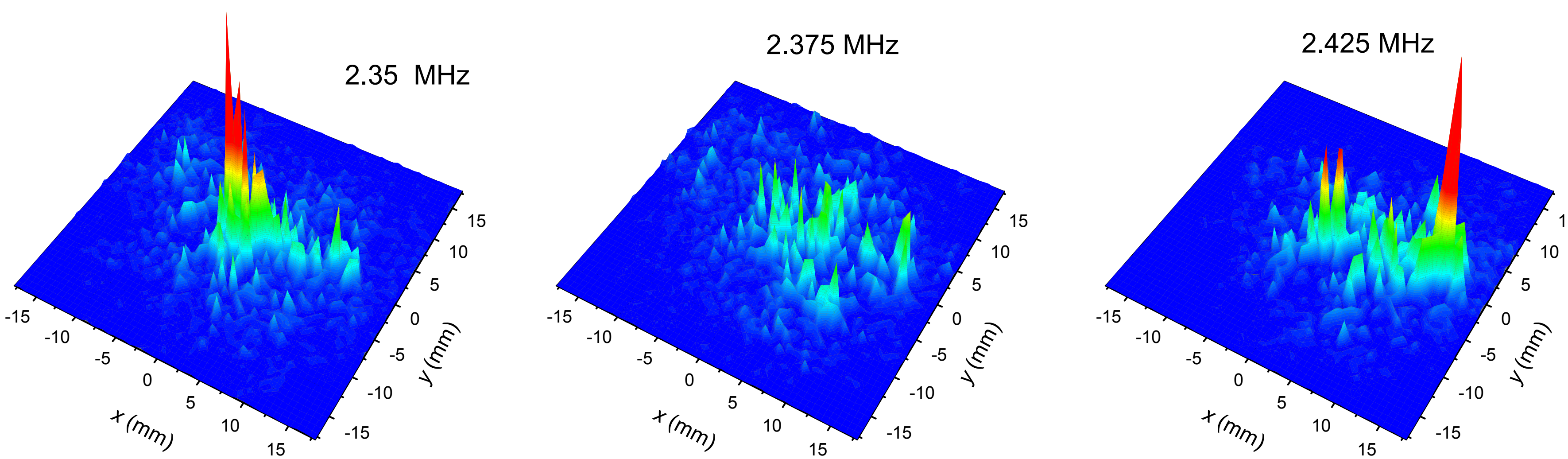
may depend anomalously on length scale at the Anderson transition, exhibiting multifractal behaviour

(MF ⇒ each moment scales with a different power- law exponent).

- Many theoretical predictions, but almost no experimental evidence

Question: Do the ultrasonic wavefunctions exhibit MF in our samples?

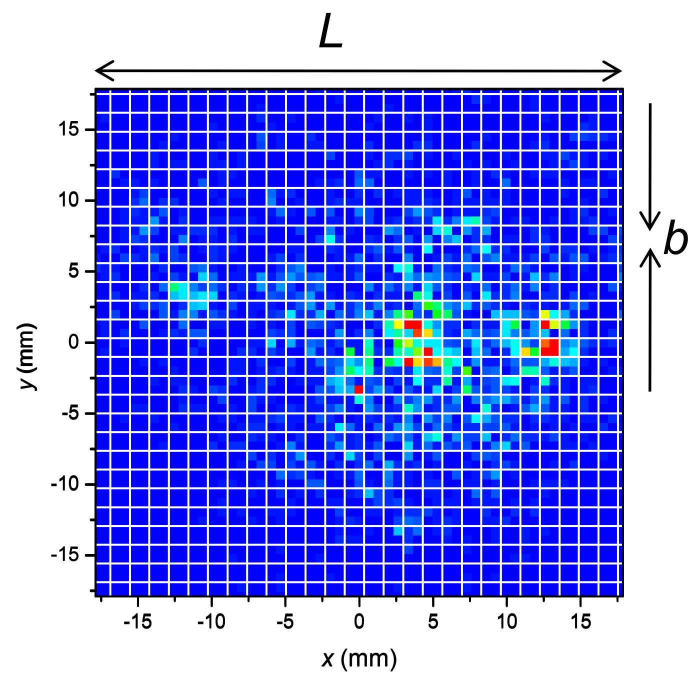
Transmitted speckle patterns $I(\mathbf{r})$ for a fixed point source (at $x = y = 0$).
Excite a single wave function at each frequency.



Multifractality (MF):

Characterizing the length scale dependence:

- Vary system size L, or
- Divide system into boxes of size b, and vary b with L fixed. ($\lambda < b < L$, L/b is the scaling length)



Generalized Inverse Participation Ratios (gIPR):

The gIPR quantify the non-trivial length scale dependence of the moments of the intensity.

$$P_q = \sum_{i=1}^{n} \left(I_{B_i} \right)^q = \sum_{i=1}^{n} \left[\int_{B_i} I(\mathbf{r})\, d^d\mathbf{r} \right]^q$$

$I(\mathbf{r}) = |\psi^2(\mathbf{r})| / \int |\psi^2(\mathbf{r})|\, d^d r$ (normalized intensity)

I_{Bi} is the integrated probability inside a box B_i of linear size b

$n = (L/b)^d$ is the number of boxes.

At criticality

$$\langle P_q \rangle \sim (L/b)^{-\tau(q)} \qquad \text{with} \qquad \tau(q) = d(q-1) + \Delta_q$$

normal dimension ($d(q-1)$) — anomalous dimension (Δ_q)

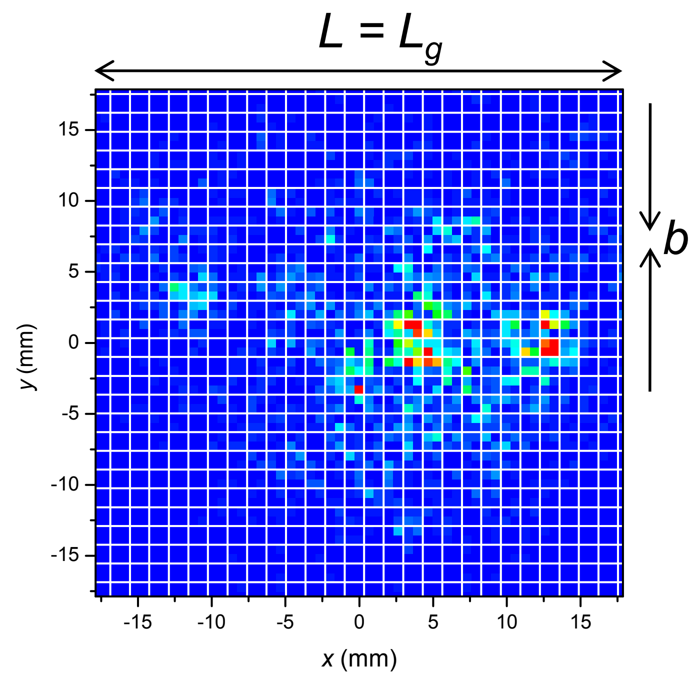
MF behaviour: τ is a continuous function of q (critical states).

Multifractality (MF):

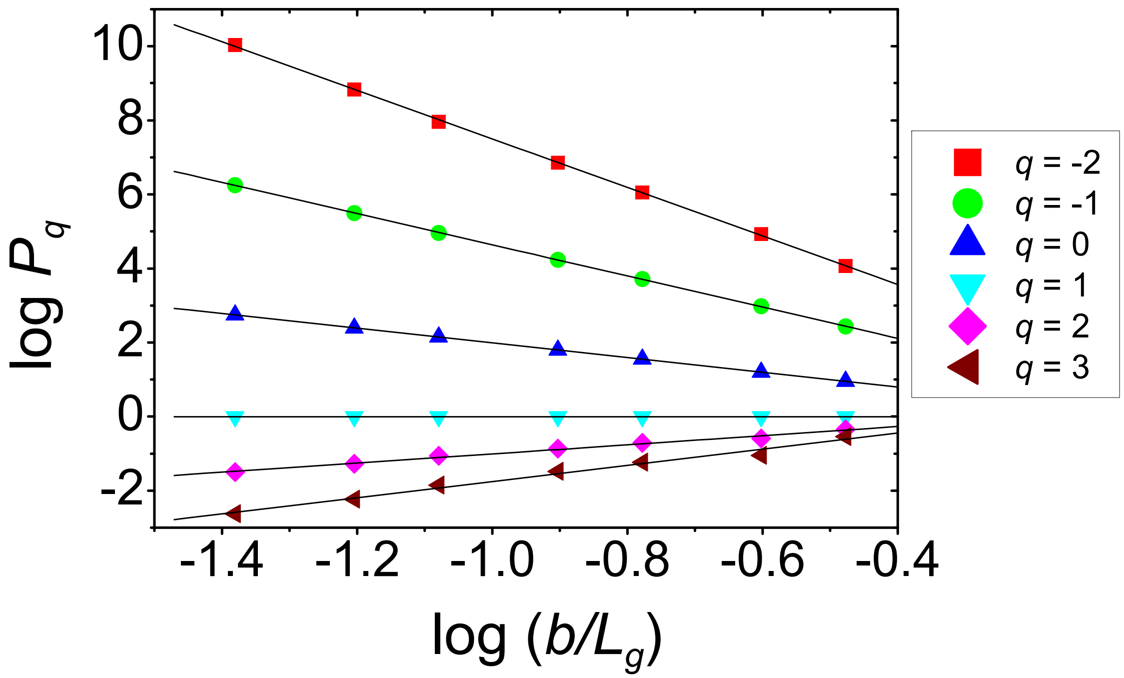
Generalized Inverse Participation Ratios (gIPR):

Find the "typically averaged" gIPR by box-sampling the wavefunctions (many frequencies) near the surface (d_{sampling} = 2, but sample is 3D) for a single realization of disorder.

$$\langle P_q \rangle_{typ} \sim \left(L_g / b \right)^{-2(q-1)-\Delta_q} \equiv \left(L_g / b \right)^{-\tau(q)}$$



Representative results at f = 2.40 MHz:



Extended states:
$\tau(q) = d(q-1)$ [i.e., $\Delta_q = 0$]

Near criticality:
$\tau(q)$, Δ_q, both continuous functions of q (MF)

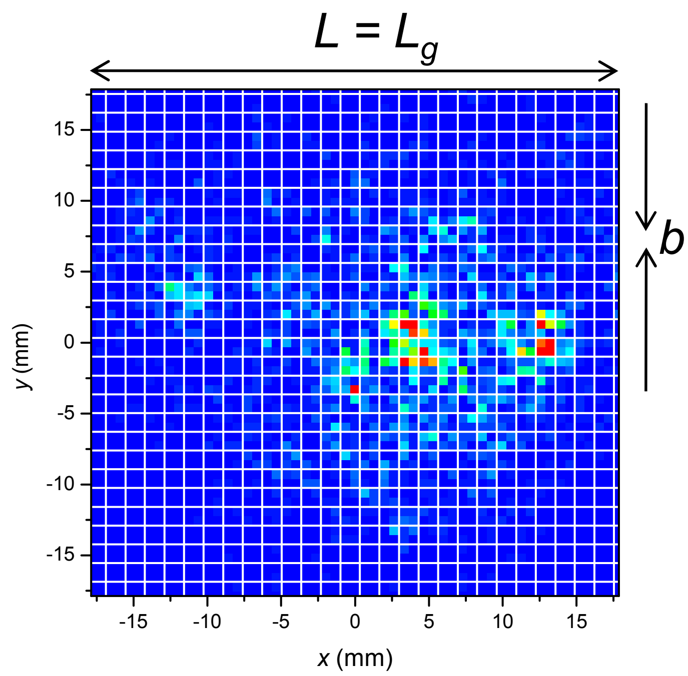
Deep in the localization regime: $\tau(q) = 0$

Multifractality (MF):

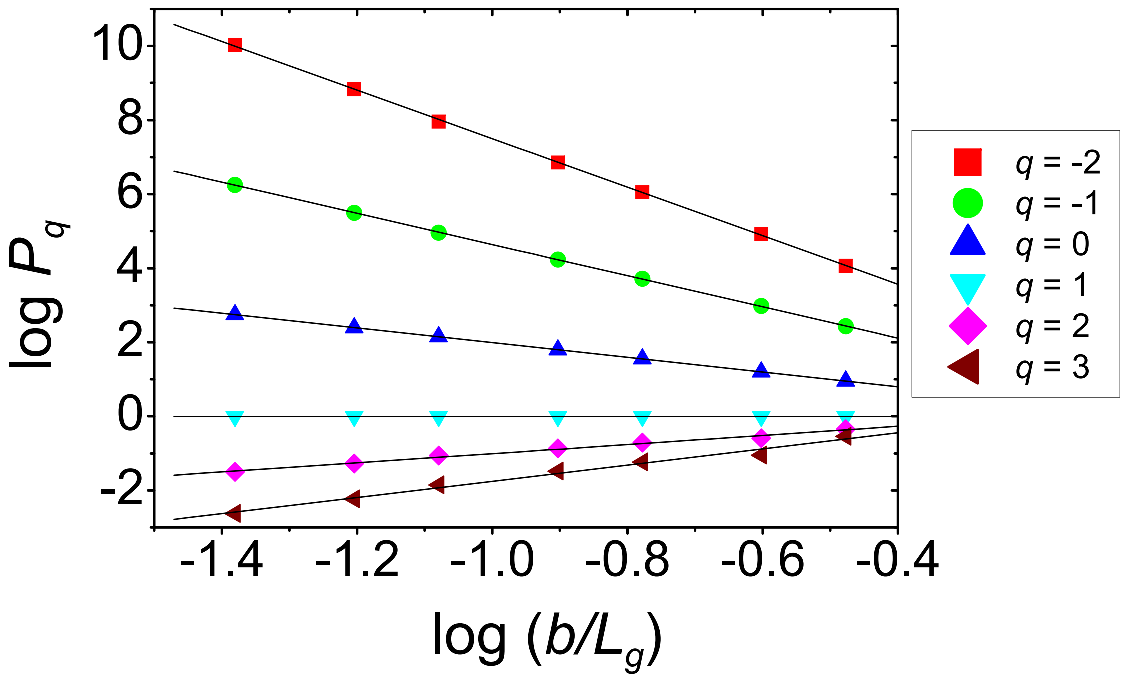
Generalized Inverse Participation Ratios (gIPR):

Find the "typically averaged" gIPR by box-sampling the wavefunctions (many frequencies) near the surface ($d_{\text{sampling space}}$ = 2, but sample is 3D) for a single realization of disorder.

$$\left\langle P_q \right\rangle_{typ} \sim \left(L_g / b \right)^{-2(q-1)-\Delta_q} \equiv \left(L_g / b \right)^{-\tau(q)}$$



Representative results at f = 2.40 MHz:



- Determine $\tau(q)$ from the slopes
- Subtract off the normal part of $\tau(q)$, $d(q-1)$, to determine Δ_q

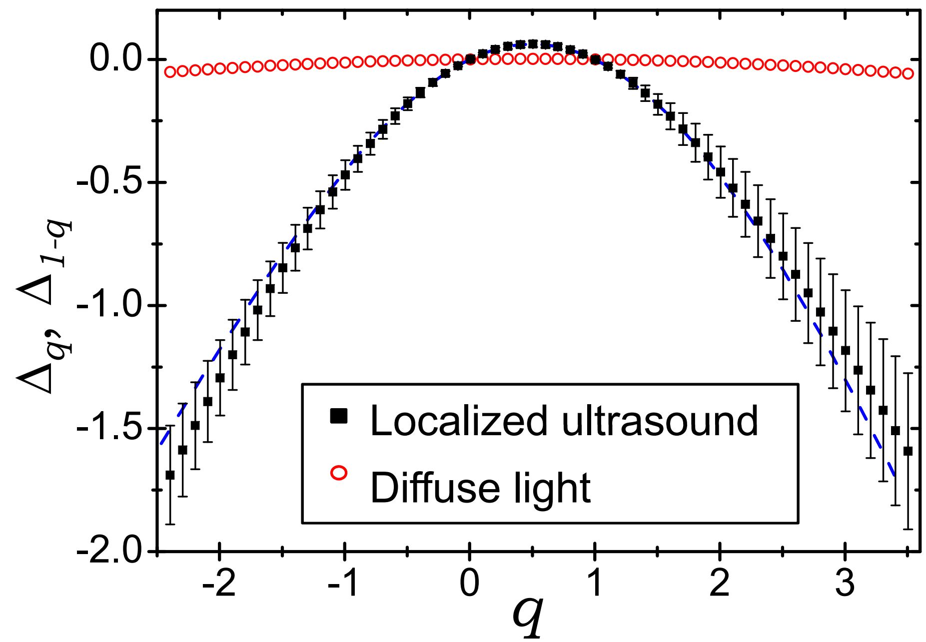
Multifractality (MF): the anomalous exponents (from the gIPR)

Anomalous exponents Δ_q

- The variation of Δ_q with q gives unambiguous evidence of MF for the localized ultrasonic wave functions

- Our data are consistent with an exact symmetry relation predicted by Mirlin *et al*. (PRL **9**7, 046803, 2006)

$$\Delta_q = \Delta_{1-q}$$

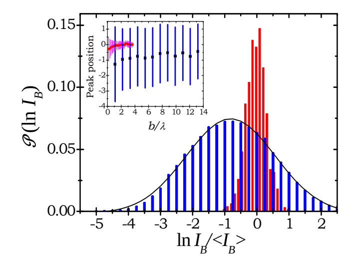


Additional evidence of wave function multifractality is given by

Probability density function (PDF)

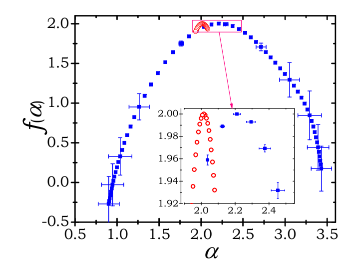
- exhibits log normal behaviour

$$\mathcal{P}(I_B) \sim \frac{1}{I_B}\left(\frac{L}{b}\right)^{-d+f(\alpha)}$$



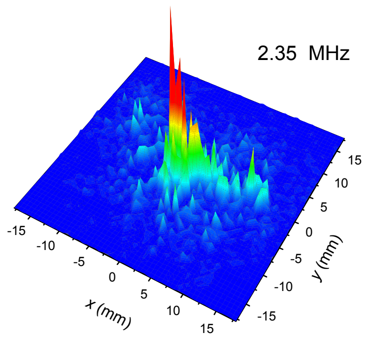
Singularity spectrum $f(\alpha)$ (related to $\tau(q)$ by a Legendre transform)

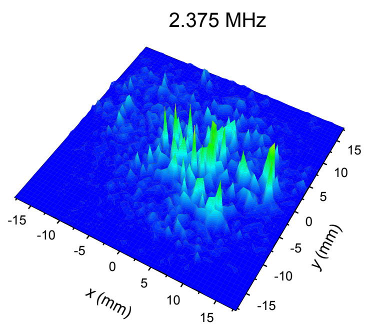
- peak is shifted from the Euclidean dimension d.

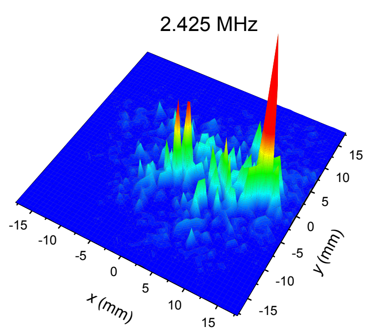


See Faez et al., *PRL* **103**, 155703 (2009)

Statistics - Summary







- Large fluctuations in the transmitted intensity for localized modes:

  non-Rayleigh statistics

  large variance, var($\hat{I}$) $\rightarrow g < 1$

- First experimental observations of wavefunction multifractality near the Anderson transition:

  scaling of the gIPR, $\langle P_q \rangle \sim (L/b)^{-\tau(q)}$

  probability density function (PDF is log normal)

  singularity spectrum, $f(\alpha)$ (α_{peak} > d)

Conclusions

We have used ultrasonic experiments and predictions of the self-consistent theory of dynamics of localization to demonstrate the localization of elastic waves in a 3D disordered mesoglass.

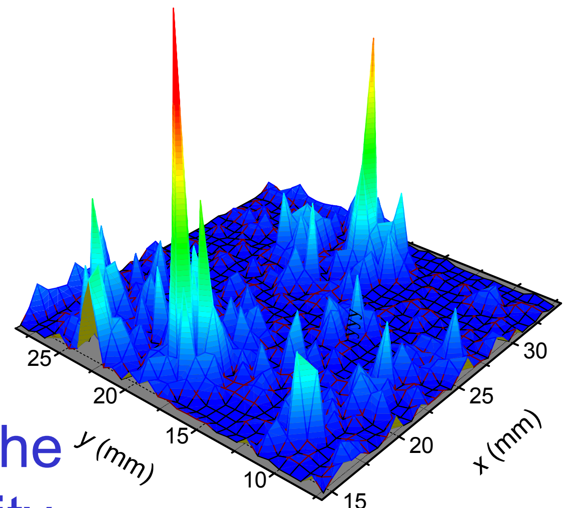
Localization signatures

- Time dependent transmitted intensity $I(t)$

$\rightarrow$ non-exponential decay of $I(t)$ at long times.

- Transverse confinement $\rightarrow$ first direct measurements and theory for $I(\rho,t)$, showing how localization cuts off the transverse spreading of the multiple scattering halo.

  $w^2(t)$ is independent of absorption and depends on the localization length ξ (and L)



- non-Rayleigh statistics and large variance of the transmitted intensity $\hat{I}$; wavefunction multifractality.

  dimensionless conductance $g' = 0.8 < 1$ (2.4 MHz)

Transverse confinement is a powerful new approach for guiding investigations of 3D Anderson localization for any type of wave.