Polarization, multiple scattering and the Berry phase

Vincent Rossetto
LPMMC Grenoble
Presentation overview

1. **Polarization**
   - Generalities
   - Green’s function

2. **Multiple scattering**
   - Single scattering
   - Born expansion
   - Dyson equation

3. **Berry phase**
   - Bringing the Berry phase to light
   - Geometry
   - Applications
Polarization

1. Polarization
   - Generalities
   - Green’s function

2. Multiple scattering
   - Single scattering
   - Born expansion
   - Dyson equation

3. Berry phase
   - Bringing the Berry phase to light
   - Geometry
   - Applications
Polarization
generalities

<table>
<thead>
<tr>
<th>Acoustic waves</th>
<th>Electromagnetic waves</th>
<th>Elastic waves</th>
</tr>
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<tbody>
<tr>
<td>1 d.o.f.</td>
<td>2 d.o.f.</td>
<td>3 d.o.f.</td>
</tr>
<tr>
<td>no polarization</td>
<td>polarization</td>
<td>polarization</td>
</tr>
</tbody>
</table>

Polarization depends on *relative* phases and amplitudes

Linear  
Circular  
Elliptical
For the *field*: Jones representations

\[
\begin{pmatrix}
  E_x \\
  E_y \\
  E_z \\
\end{pmatrix}
\]

*cartesian*

For the *intensity*: Stokes representations

\[
\begin{pmatrix}
  I \\
  I_\perp \\
  Q \\
  U \\
  V \\
\end{pmatrix}
\]

with \( I = E^\dagger E \)

\( I_\perp = E^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} E \)

\( \ldots \)
For the field: Jones representations

\[
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\]
cartesian

\[
\begin{pmatrix}
E_+
\\
E_-
\\
E_0
\end{pmatrix}
\]
circular

For the intensity: Stokes representations

\[
\begin{pmatrix}
I \\
I_\perp \\
Q \\
U \\
V
\end{pmatrix}
\]
with
\[
\begin{align*}
I &= E^\dagger E \\
I_\perp &= E^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} E \\
\ldots
\end{align*}
\]
For the *field* : Jones representations

\[
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\quad \text{cartesian}
\]

\[
\begin{pmatrix}
E_+ \\
E_-
\end{pmatrix}
\quad \text{circular}
\]

For the *intensity* : Stokes representations

\[
\begin{pmatrix}
I \\
I_\perp \\
Q \\
U \\
V
\end{pmatrix}
\quad \text{with} \quad \begin{cases}
I = E^\dagger E \\
I_\perp = E^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} E \\
\ldots
\end{cases}
\]
Polarization frames

\[
\begin{align*}
\begin{pmatrix}
E'_{x} \\
E'_{y} \\
E'_{z}
\end{pmatrix} &=
\begin{pmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
E_{x} \\
E_{y} \\
E_{z}
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix}
E'_{+} \\
E'_{-} \\
E'_{0}
\end{pmatrix} &=
\begin{pmatrix}
e^{-i\theta} & 0 & 0 \\
0 & e^{i\theta} & 0 \\
0 & 0 & e^{0}
\end{pmatrix}
\begin{pmatrix}
E_{+} \\
E_{-} \\
E_{0}
\end{pmatrix}
\end{align*}
\]
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\begin{pmatrix}
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E'_- \\
E'_0
\end{pmatrix} =
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0 & 0 & e^0
\end{pmatrix}
\begin{pmatrix}
E_+ \\
E_- \\
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\end{pmatrix}
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Polarization frames

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E_{+} \\
E_{-} \\
E_{0}
\end{pmatrix}
\]
$E(r, t, R)$

$E_s(r, t, RZ(\alpha)) = e^{-i\alpha} E_s(r, t, R)$

Reference frame

Mobile frames
$E(r, t, R)$

$E_s(r, t, RZ(\alpha)) = e^{-i\alpha} E_s(r, t, R)$
Green's function
in vacuum

\( \mathbf{G}_0 \) relates amplitudes
- at positions \( \mathbf{r} \) and \( \mathbf{r}' \)
- at times \( t \) and \( t' \)
- in frames \( \mathbf{R} \) and \( \mathbf{R}' \)

Euler angles
\[
\mathbf{R} = \mathbf{Z}(\phi)\mathbf{Y}(\theta)\mathbf{Z}(\psi)
\]
\[
\Delta(\mathbf{R}, \mathbf{R}') = \delta(\phi' - \phi)\delta(\cos \theta' - \cos \theta)
\]
\[
\mathbf{r}' - \mathbf{r} = \mathbf{R} \hat{\mathbf{z}}
\]

\[
\mathbf{G}_0(\mathbf{r}, \mathbf{r}', t, t', \mathbf{R}, \mathbf{R}') \big|_{ss'} = \delta_{ss'} \Delta(\mathbf{R}, \mathbf{R}') \Delta(\mathbf{R}, \mathbf{D}) e^{i(s'\psi' - s\psi)} \mathbf{g}_0(\mathbf{r}' - \mathbf{r}, t' - t)
\]
Green’s function in vacuum

$G_0$ relates amplitudes
- at positions $r$ and $r'$
- at times $t$ and $t'$
- in frames $R$ and $R'$

Euler angles

$R = Z(\phi)Y(\theta)Z(\psi)$

$\Delta(R, R') = \delta(\phi' - \phi)\delta(\cos \theta' - \cos \theta)$

$r' - r = R \hat{D} \hat{z}$

$G_0(r, r', t, t', R, R')|_{ss'} = \delta_{ss'} \Delta(R, R') \Delta(R, D) e^{i(s'\psi' - s\psi)} g_0(r' - r, t' - t)$
Green’s function in vacuum

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Euler angles

$$R = Z(\phi)Y(\theta)Z(\psi)$$

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$$G_0(r, r', t, t', R, R')_{ss'} =$$

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Green’s function in vacuum

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Euler angles

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\mathbf{r}' - \mathbf{r} = \mathbf{R} \mathbf{D} \hat{\mathbf{z}}
\]

\[
G_0(\mathbf{r}, \mathbf{r}', t, t', \mathbf{R}, \mathbf{R}')_{ss'} =
\]
\[
\delta_{ss'} \Delta(\mathbf{R}, \mathbf{R}') \Delta(\mathbf{R}, \mathbf{D}) e^{i(s'\psi' - s\psi)} g_0(\mathbf{r}' - \mathbf{r}, t' - t)
\]
Green’s function in Fourier domain

\[ \tilde{G}_0(q, \omega, R, R') \bigg|_{ss'} = \delta_{ss'} \frac{\Delta(R, R')}{\left( \frac{\omega}{c} - \mathbf{q} \cdot \hat{z} \right)^2} e^{i(s'\psi' - s\psi)} \]

\[ \int dR \int dR' \tilde{G}_0(q, \omega, R, R') = \frac{1}{\left( \frac{\omega}{c} \right)^2 - q^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

⇒ Directivity is essential to represent polarized waves in mobile frames.
Green’s function
in Fourier domain

\[ \tilde{G}_0(q, \omega, R, R')|_{ss'} = \delta_{ss'} \frac{\Delta(R, R')}{\left(\frac{\omega}{c} - q \cdot \hat{z}\right)^2} e^{i(s'\psi' - s\psi)} \]

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⇒ Directivity is essential to represent polarized waves in mobile frames.
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\int dR \int dR' \tilde{G}_0(q, \omega, R, R') = \frac{1}{(\frac{\omega}{c})^2 - q^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

⇒ Directivity is essential to represent polarized waves in mobile frames.
Absorption (dichroism)

\[ \exp(-\kappa_s R) \]

Birefringence

\[ \exp \left( -i \frac{\omega}{c_s} R \right) \]

Spin flip (for photons)

\[ G_0 \bigg|_{s,-s} \neq 0 \]

Faraday effect (for photons)

\[ q \rightarrow q - s V_B \]
Absorption (dichroism)

\[ \exp \left( -\kappa_s R \right) \]

Birefringence

\[ \exp \left( -i \frac{\omega}{c_s} R \right) \]

Spin flip (for photons)

\[ G_0 \bigg|_{s,-s} \neq 0 \]

Faraday effect (for photons)

\[ q \rightarrow q - s \sqrt{B} \]
Green's function and medium anisotropies

- Absorption (dichroism)
  \[ \exp(-\kappa_s R) \]

- Birefringence
  \[ \exp\left(-i\frac{\omega}{c_s} R\right) \]

- Spin flip (for photons)
  \[ G_0|_{s,-s} \neq 0 \]

- Faraday effect (for photons)
  \[ q \rightarrow q - s V_B \]
Absorption (dichroism)
\[ \exp(-\kappa_s R) \]

Birefringence
\[ \exp\left(-i \frac{\omega}{c_s} R\right) \]

Spin flip (for photons)
\[ G_0 \big|_{s, -s} \neq 0 \]

Faraday effect (for photons)
\[ q \rightarrow q - s V_B \]
1. **Polarization**
   - Generalities
   - Green’s function

2. **Multiple scattering**
   - Single scattering
   - Born expansion
   - Dyson equation

3. **Berry phase**
   - Bringing the Berry phase to light
   - Geometry
   - Applications
Multiple scattering
several systems, many scales
Single scattering
simpler things first

\[ R \rightarrow R' \]

\[ \tilde{R} = Z(\tilde{\phi})Y(\tilde{\theta})Z(\tilde{\psi}) \]

\[ T_{ss'}(\omega, R, R') = e^{i(s\tilde{\phi} + s'\tilde{\psi})} f_{ss'}(\omega, \tilde{\theta}) \]
Single scattering
simpler things first

rotation \quad R^{-1}R' = \tilde{R} = Z(\tilde{\phi})Y(\tilde{\theta})Z(\tilde{\psi})

T_{ss'}(\omega, R, R') = e^{i(s\tilde{\phi} + s'\tilde{\psi})} f_{ss'}(\omega, \tilde{\theta})
Single scattering
simpler things first

rotation \[ R^{-1}R' = \tilde{R} = Z(\tilde{\phi})Y(\tilde{\theta})Z(\tilde{\psi}) \]

\[ T_{ss'}(\omega, R, R') = e^{i(s\tilde{\phi} + s'\tilde{\psi})} f_{ss'}(\omega, \tilde{\theta}) \]
Find \( G \) the *effective* Green’s function of a medium filled with scatterers.
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Find $G$ the *effective* Green’s function of a medium filled with scatterers.
Without correlations: independent scattering approximation

\[ \overline{G}(r, r') = G_0(r, r') + \int \rho dx_1 \, G_0(r, x_1) T G_0(x_1, r') + \cdots \]
Without correlations: independent scattering approximation

\[ \overline{G}(r, r') = G_0(r, r') + \int \rho d\mathbf{x}_1 \, G_0(r, \mathbf{x}_1) T G_0(\mathbf{x}_1, r') + \cdots \]

\[ r \overline{G} r' = \overline{G_0} r' \]
Without correlations: independent scattering approximation

\[ \overline{G}(r, r') = G_0(r, r') + \int \rho d\mathbf{x}_1 \ G_0(r, \mathbf{x}_1)T G_0(\mathbf{x}_1, r') + \cdots \]

\[ \overline{G} \]

\[ \frac{\mathbf{r}}{r} \rightarrow \frac{\mathbf{r}}{r'} = \frac{\mathbf{G}_0}{r'} \]

\[ + \frac{\mathbf{r}}{r} \times \frac{\mathbf{x}_1}{\mathbf{r}} \]
Without correlations: independent scattering approximation

\[ \overline{g}(r, r') = g_0(r, r') + \int \rho d\mathbf{x}_1 \ g_0(r, \mathbf{x}_1) T g_0(\mathbf{x}_1, r') + \cdots \]

\[ \overline{g} \]

\[ r \quad \overline{g} \quad r' = r \quad g_0 \quad r' \]

+ \[ r \quad \mathbf{x}_1 \quad r' \]

+ \[ r \quad \mathbf{x}_1 \quad \mathbf{x}_2 \quad r' \]

+ \[ \cdots \]
Without correlations: independent scattering approximation

\[ \overline{G}(r, r') = G_0(r, r') + \int \rho \, dx_1 \, G_0(r, x_1) T G_0(x_1, r') + \cdots \]

\[ \overline{G} \]
\[ r \quad r' \]
\[ = \]
\[ r \quad G_0 \quad r' \]
\[ + r \quad x_1 \quad r' \]
\[ + r \quad x_1 \quad x_2 \quad r' \]
\[ + \cdots \]

Self-consistent equation

\[ = \quad + \quad \times \]
Multiple scattering

self-energy

Polarization

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Multiple scattering

Single scattering

Born expansion

Dyson equation

Berry phase

Conclusion
Multiple scattering

self-energy

\[ r = 0 \]

\[ r' = 0 \]
Multiple scattering
self-energy
Multiple scattering
self-energy

\[ \sum_{k=1}^{\infty} \frac{1}{k!} \left( \frac{\partial}{\partial r} \right)^k \left[ \frac{1}{r} \right] = 0 \]

\[ = 0 \]
Multiple scattering

Self-energy

Polarization
V. Rossetto

Multiple scattering
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Multiple scattering
self-energy

\[ r \neq 0 \]
Multiple scattering
self-energy

Polarization
V. Rossetto

Polarization

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Multiple scattering
self-energy
Multiple scattering
self-energy

\[ \sum \]
Green–Dyson equation
Change scattering definition:

\[ = \int dR_1 \int dR_2 \, G_0(q, \omega, R_2, R') T(\omega, R_1, R_2) G_0(q, \omega, R, R_1) \]

Self energy

\[ = \lim_{|q| \to \infty} \int dR_1 \int dR_2 \, T(\omega, R_2, R') \overline{G}(\omega, R_1, R_2) T(\omega, R, R_1) \]

\[ \int dR \quad \Rightarrow \quad \text{matrix product} \]
Polarization

V. Rossetto

Multiple scattering

Single scattering
Born expansion
Dyson equation

Berry phase

Conclusion

Multiple scattering
of polarized light

Change scattering definition:

\[
\int d\mathbf{R}_1 \int d\mathbf{R}_2 \ G_0(\mathbf{q}, \omega, \mathbf{R}_2, \mathbf{R}') T(\omega, \mathbf{R}_1, \mathbf{R}_2) G_0(\mathbf{q}, \omega, \mathbf{R}, \mathbf{R}_1)
\]

Self energy

\[
= \lim_{|\mathbf{q}| \to \infty} \int d\mathbf{R}_1 \int d\mathbf{R}_2 \ T(\omega, \mathbf{R}_2, \mathbf{R}') \overline{G}(\mathbf{q}, \omega, \mathbf{R}_1, \mathbf{R}_2) T(\omega, \mathbf{R}, \mathbf{R}_1)
\]

\[
\int d\mathbf{R} \quad \implies \quad \text{matrix product}
\]
Multiple scattering
of polarized light

Change scattering definition:

\[
\begin{align*}
\ &= \int dR_1 \int dR_2 \, G_0(q, \omega, R_2, R')T(\omega, R_1, R_2)G_0(q, \omega, R, R_1)
\end{align*}
\]

Self energy

\[
\begin{align*}
\ &= \lim_{|q| \to \infty} \int dR_1 \int dR_2 \, T(\omega, R_2, R')\overline{G}(q, \omega, R_1, R_2)T(\omega, R, R_1) \\
\ &\text{has a direction}
\end{align*}
\]

\[
\int dR \quad \Rightarrow \quad \text{matrix product}
\]
Multiple scattering of polarized light

Change scattering definition:

\[
= \int dR_1 \int dR_2 \, G_0(q, \omega, R_2, R')T(\omega, R_1, R_2)G_0(q, \omega, R, R_1)
\]

Self energy \( \Sigma(\hat{q}, \omega, R, R') \)

\[
= \lim_{|q| \to \infty} \int dR_1 \int dR_2 \ T(\omega, R_2, R')\overline{G}(q, \omega, R_1, R_2)T(\omega, R, R_1)
\]

\[\int dR \quad \Longrightarrow \quad \text{matrix product}\]
Multiple scattering of polarized light

Change scattering definition:

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\]

Self energy \( \Sigma(\hat{q}, \omega, R, R') \)

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\]

\( \hat{q} \) has a direction

\[
\int dR \quad \Rightarrow \quad \text{matrix product}
\]
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The Berry phase
phase and rotation

\[ E_s(r, t, RZ(\alpha)) = e^{-is\alpha} E_s(r, t, R) \]

\[ T_{ss'}(\omega, R, R') = e^{i(s\bar{\phi} + s'\bar{\psi})} f_{ss'}(\omega, \bar{\theta}) \]

Total phase for a path?
The Berry phase
phase and rotation

\[ E_s(r, t, RZ(\alpha)) = e^{-i\alpha} E_s(r, t, R) \]

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Total phase for a path?
The Berry phase
phase and rotation

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\[ T_{ss'}(\omega, R, R') = e^{i(s\tilde{\phi} + s'\tilde{\psi})} f_{ss'}(\omega, \tilde{\theta}) \]

Total phase for a path?
Consider a path \((x_1, R_1) \ldots (x_n, R_n)\) such that \(R_n = R_1\) and
\[
\theta_{i+1} - \theta_i \ll 1 \quad \text{and} \quad \phi_{i+1} - \phi_i \ll 1
\]

Rotations \(R_i^{-1}R_{i+1} = \tilde{R}_i\)

\[
\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i) \cos \theta_i + \psi_{i+1} - \psi_i
\]

\[
s \sum_{i=0}^{n-1} \tilde{\phi}_i + \tilde{\psi}_i \simeq -s \sum_{i=0}^{n-1} (1 - \cos \theta_i)(\phi_{i+1} - \phi_i) \mod 2\pi
\]
Consider a path \((x_1, R_1) \ldots (x_n, R_n)\) such that \(R_n = R_1\) and \(\theta_{i+1} - \theta_i \ll 1\) \(\phi_{i+1} - \phi_i \ll 1\).

Rotations \(R_i^{-1} R_{i+1} = \tilde{R}_i\)

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The Berry phase
bringing it to light

Consider a path \((x_1, R_1) \ldots (x_n, R_n)\) such that \(R_n = R_1\) and 
\[\theta_{i+1} - \theta_i \ll 1 \quad \text{and} \quad \phi_{i+1} - \phi_i \ll 1\]

Rotations \(R_i^{-1}R_{i+1} = \tilde{R}_i\)

\[\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i) \cos \theta_i + \psi_{i+1} - \psi_i\]

\[s \sum_{i=0}^{n-1} \tilde{\phi}_i + \tilde{\psi}_i \simeq -s \sum_{i=0}^{n-1} (1 - \cos \theta_i)(\phi_{i+1} - \phi_i) \mod 2\pi\]
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Rotations \(R_i R_{i+1}^{-1} = \tilde{R}_i\)

\[
\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i) \cos \theta_i + \psi_{i+1} - \psi_i
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\[
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\]
Consider a path \((x_1, R_1) \ldots (x_n, R_n)\) such that \(R_n = R_1\) and 
\[
\theta_{i+1} - \theta_i \ll 1 \quad \text{and} \quad \phi_{i+1} - \phi_i \ll 1
\]

Rotations \(R_{i+1}^{-1}R_i = \tilde{R}_i\)

\[
\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i) \cos \theta_i + \psi_{i+1} - \psi_i
\]

\[
s \sum_{i=0}^{n-1} \tilde{\phi}_i + \tilde{\psi}_i \simeq -s \sum_{i=0}^{n-1} (1 - \cos \theta_i)(\phi_{i+1} - \phi_i) \mod 2\pi
\]
The Berry phase
geometric interpretation

\[ \Omega = \sum_{i=0}^{n-1} (1 - \cos \theta_i)(\phi_{i+1} - \phi_i) \]

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\[ \Omega_B = -S \begin{pmatrix} \text{Area enclosed on the sphere by the direction of propagation} \end{pmatrix} \]
“Parallel transport” of transverse polarization
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In a system with forward scattering, $G(r, r', t, t', R, R')$ contains the statistics of the Berry phase for paths:

- starting at $r$ and ending at $r'$
- of length $c(t' - t)$
- with initial and final directions $\hat{R}z$ and $\hat{R}'z$

Properties of the Berry phase statistics:

- depends on the heterogeneity of the medium
- depends on transport properties (anisotropies)
- difficult to measure ( ?)
- interpretation of experiment data
- theory (?)
In a system with forward scattering, \( G(\mathbf{r}, \mathbf{r}', t, t', \mathbf{R}, \mathbf{R}') \) contains the statistics of the Berry phase for paths

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Forward scattering: the direction of propagation remains near \( \hat{z} \).

The trajectory on the sphere is almost flat.
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\[ L \sim l^* \]
Forward scattering: the direction of propagation remains near $\hat{z}$

The trajectory on the sphere is almost flat

The distribution of Berry phase is (Lévy):

$$p(\Omega_B) = \frac{\pi \ell^*}{L} \frac{1}{\cosh^2 \left(2\pi \frac{\Omega_B \ell^*}{L} \right)}$$
The Berry phase depolarization

Forward scattering: the direction of propagation remains near \( \hat{z} \)

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\[
p(\Omega_B) = \frac{\pi \ell^*}{L} \frac{1}{\cosh^2 \left( \frac{2\pi \ell^* \Omega_B}{L} \right)}
\]

The outgoing polarization is:

\[
\langle \cos 2\Omega_B \rangle = \frac{L}{2\ell^*} \frac{1}{\sinh \left( \frac{L}{2\ell^*} \right)}
\]
The Berry phase
numerical simulations

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Conclusion
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numerical simulations
The Berry phase
numerical simulations

Berry phase fluctuations

- $R=20$
- $R=100$
- $R=1000$
- $R=5000$
- $R=10000$
The Berry phase
backscattering experiment
The Berry phase backscattering experiment

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V. Rossetto

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The Berry phase
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The Berry phase backscattering experiment

\[ \frac{d\theta}{2} \]

\[ \frac{d\theta}{1} \]

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Multiple scattering theory for polarized waves
- takes into account several kinds of anisotropies
- and the Berry phase

Elastic waves should have a Berry phase
- but it was never observed! → experimental challenge?

The Berry phase contains informations on the paths statistics...
- ... therefore on the properties of the medium
- → investigate how to extract relevant informations

Use seismology techniques (stacking, correlations) to retrieve data
Conclusion and outlooks

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