Polarization, multiple scattering and the Berry phase

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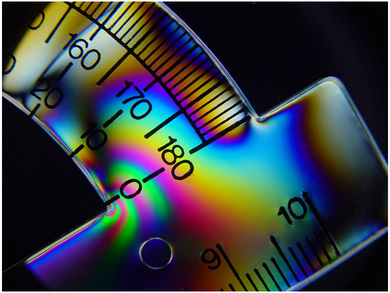
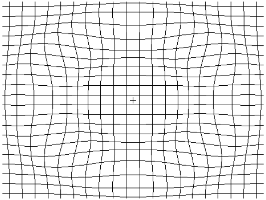
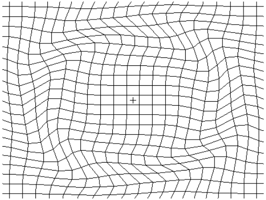
Col de Porte, January 14th 2010

Presentation overview

Polarization
for different waves



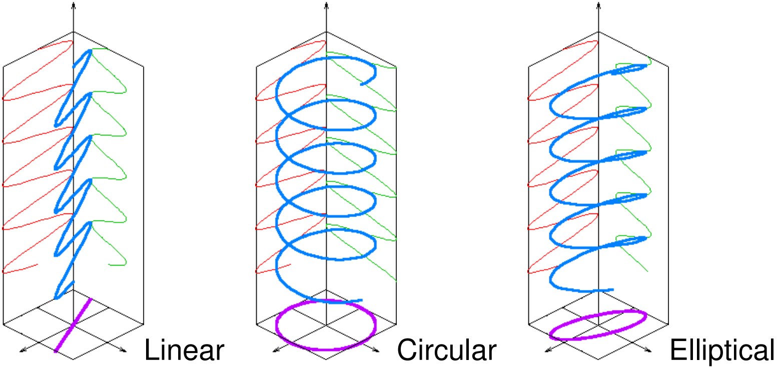




Polarization
generalities

Acoustic waves	1 d.o.f.	no polarization
Electromagnetic waves	2 d.o.f.	polarization
Elastic waves	3 d.o.f.	polarization

Polarization depends on *relative* phases and amplitudes



For the *field* : Jones representations

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

cartesian

For the *intensity* : Stokes representations

$$\begin{pmatrix} \mathrm{I} \\ \mathrm{I}_\perp \\ \mathrm{Q} \\ \mathrm{U} \\ \mathrm{V} \end{pmatrix} \text{ with } \begin{cases} \mathrm{I} = E^\dagger E \\ \mathrm{I}_\perp = E^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} E \\ \ldots \end{cases}$$

Polarization
representations

For the *field* : Jones representations

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad \begin{pmatrix} E_+ \\ E_- \\ E_0 \end{pmatrix}$$

cartesian — circular

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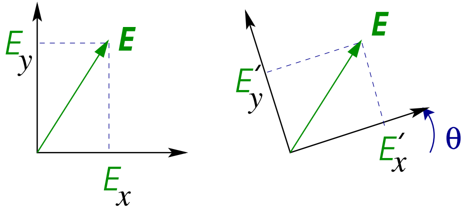
Polarization

frames



$$\begin{pmatrix} E'_x \\ E'_y \\ E'_z \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

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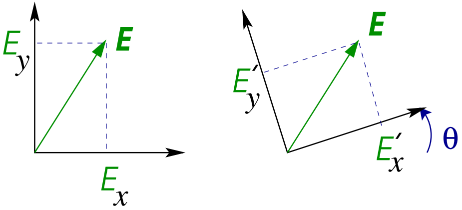

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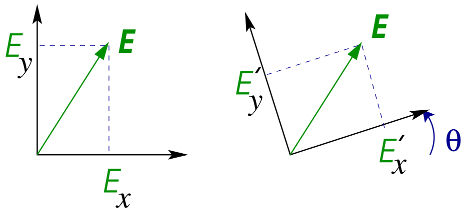

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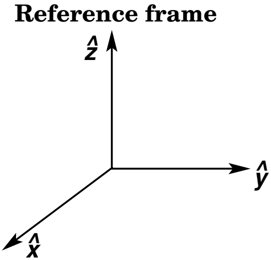
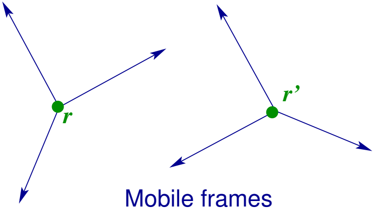
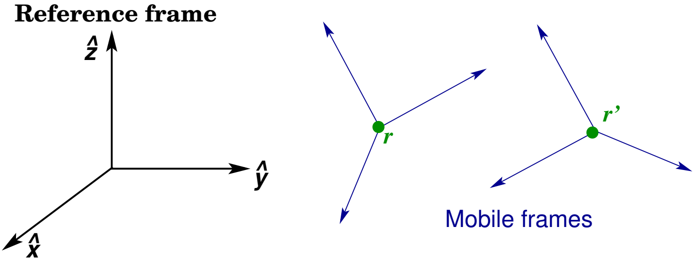
Polarization
mobile frames





$$\boldsymbol{E}(\boldsymbol{r},\ t,\ \mathrm{R})$$

$$E_{\mathrm{s}}(\boldsymbol{r},\ t,\ \mathrm{RZ}(\alpha)) = \mathrm{e}^{-\mathrm{i}s\alpha} E_{\mathrm{s}}(\boldsymbol{r},\ t,\ \mathrm{R})$$



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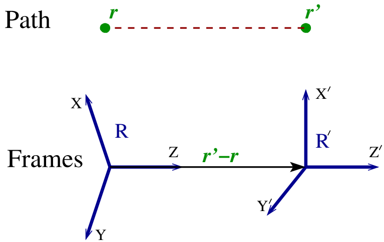
Green's function

in vacuum

G_0 relates amplitudes

- at positions $\boldsymbol{r}$ and $\boldsymbol{r}'$
- at times t and t'
- in frames R and R'



Euler angles $\mathrm{R} = \mathrm{Z}(\phi)\mathrm{Y}(\theta)\mathrm{Z}(\psi)$

$$\Delta(\mathrm{R},\mathrm{R}') = \delta(\phi' - \phi)\delta(\cos\theta' - \cos\theta)$$

$$\boldsymbol{r}' - \boldsymbol{r} = R\,\mathrm{D}\hat{\boldsymbol{z}}$$

$$G_0(\boldsymbol{r}, \boldsymbol{r}', t, t', \mathrm{R}, \mathrm{R}')\big|_{ss'} =$$

$$\delta_{ss'}\,\Delta(\mathrm{R},\mathrm{R}')\,\Delta(\mathrm{R},\mathrm{D})\,e^{i(s'\psi' - s\psi)}\,\mathcal{G}_0(\boldsymbol{r}' - \boldsymbol{r}, t' - t)$$

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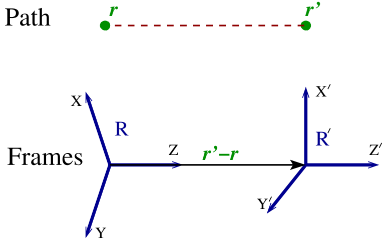

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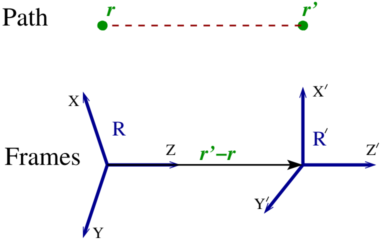

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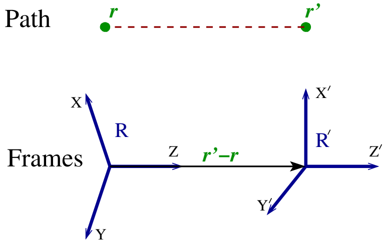

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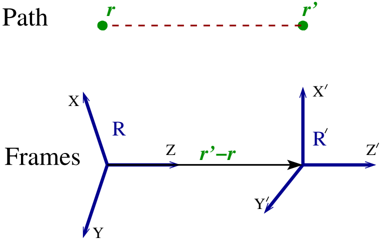

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Green's function
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$$\tilde{G}_0(\boldsymbol{q},\, \omega,\, R,\, R')\big|_{ss'} = \delta_{ss'} \frac{\Delta(R,\, R')}{\left(\frac{\omega}{c} - \boldsymbol{q} \cdot R\hat{\boldsymbol{z}}\right)^2} e^{i(s'\psi' - s\psi)}$$

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⇒ Directivity is essential to represent polarized waves in mobile frames.

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$\Rightarrow$ Directivity is essential to represent polarized waves in mobile frames.

Green's function
and medium anisotropies

- Absorption (dichroism)

$$\exp\left(-\kappa_s R\right)$$

- Birefringence

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- Spin flip (for photons)

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$$\boldsymbol{q} \to \boldsymbol{q} - sV\mathbf{B}$$

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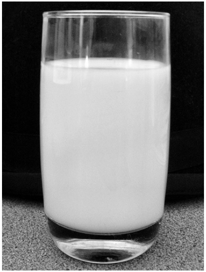
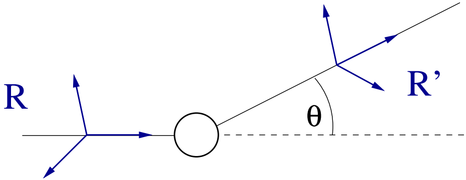
Multiple scattering

several systems, many scales



Single scattering

simpler things first



$$\mathrm{R} \underset{\tilde{\mathrm{R}}}{\longrightarrow} \mathrm{R}'$$

rotation $\quad \mathrm{R}^{-1}\mathrm{R}' = \tilde{\mathrm{R}} = \mathrm{Z}(\tilde{\phi})\mathrm{Y}(\tilde{\theta})\mathrm{Z}(\tilde{\psi})$

$$\mathrm{T}_{\mathsf{ss}'}(\omega, \mathrm{R}, \mathrm{R}') = e^{\mathrm{i}(\mathsf{s}\tilde{\phi}+\mathsf{s}'\tilde{\psi})} \; f_{\mathsf{ss}'}(\omega, \tilde{\theta})$$

Single scattering

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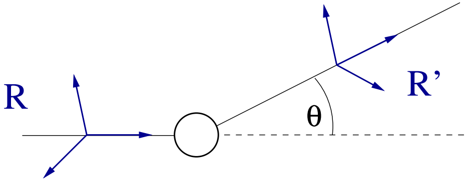

$$R \underset{\tilde{R}}{\longrightarrow} R'$$

rotation $\quad R^{-1}R' = \tilde{R} = Z(\tilde{\phi})Y(\tilde{\theta})Z(\tilde{\psi})$

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Single scattering

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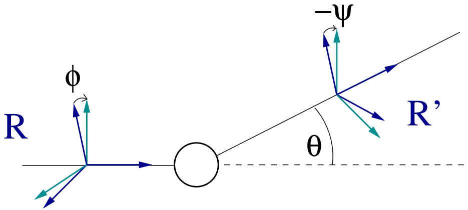

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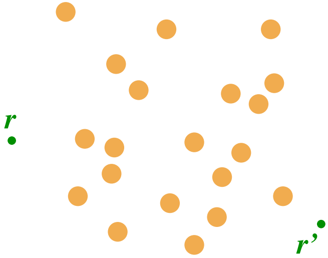
Find G the *effective* Green's function of a medium filled with scatterers.



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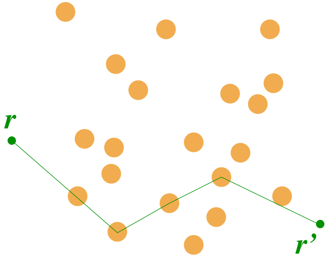

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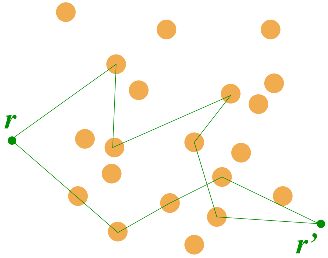

Multiple scattering

classical picture

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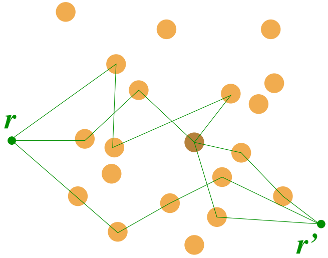

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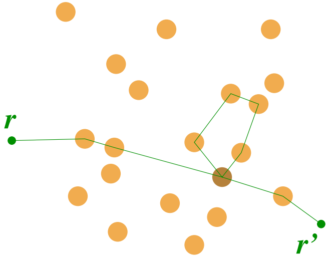

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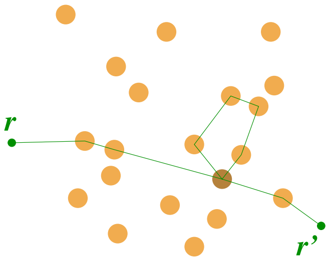
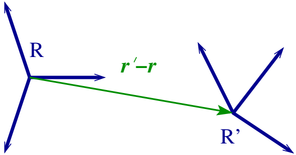
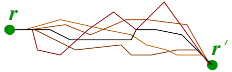




Paths



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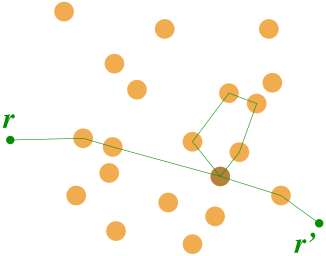

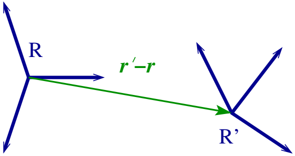

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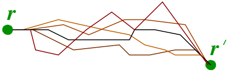

$$\mathrm{G}(\boldsymbol{q}, \omega, \mathrm{R}, \mathrm{R}')$$

Without correlations : independent scattering approximation

$$\overline{\mathcal{G}}(\boldsymbol{r}, \boldsymbol{r}') = \mathcal{G}_0(\boldsymbol{r}, \boldsymbol{r}') + \int \rho \mathrm{d}\boldsymbol{x}_1 \, \mathcal{G}_0(\boldsymbol{r}, \boldsymbol{x}_1) \mathrm{T} \mathcal{G}_0(\boldsymbol{x}_1, \boldsymbol{r}') + \cdots$$

Multiple scattering
Born expansion

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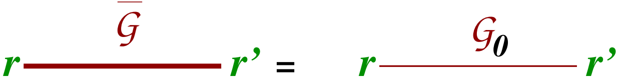

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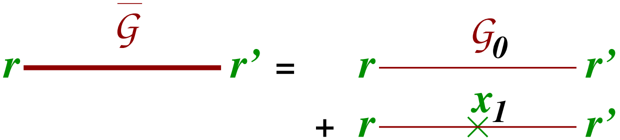

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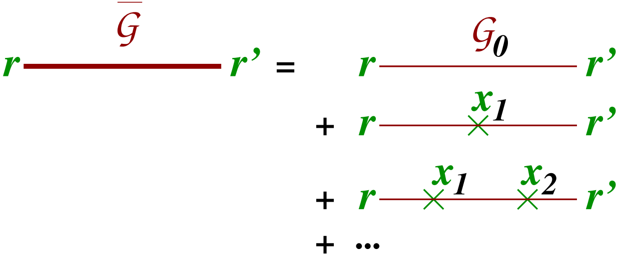
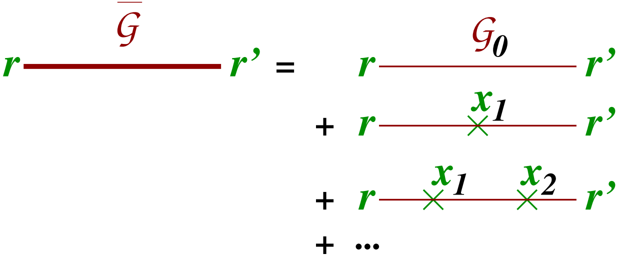
$$\overline{\mathcal{G}}(\boldsymbol{r}, \boldsymbol{r}') = \mathcal{G}_0(\boldsymbol{r}, \boldsymbol{r}') + \int \rho \mathrm{d}\boldsymbol{x}_1 \, \mathcal{G}_0(\boldsymbol{r}, \boldsymbol{x}_1) \mathrm{T} \mathcal{G}_0(\boldsymbol{x}_1, \boldsymbol{r}') + \cdots$$



Multiple scattering
Born expansion

Without correlations : independent scattering approximation

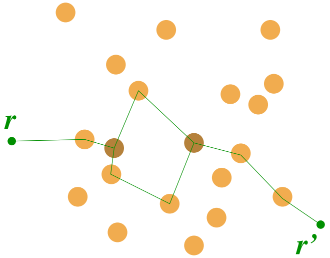
$$\overline{\mathcal{G}}(\boldsymbol{r},\,\boldsymbol{r}') = \mathcal{G}_0(\boldsymbol{r},\,\boldsymbol{r}') + \int \rho \mathrm{d}\boldsymbol{x}_1\;\mathcal{G}_0(\boldsymbol{r},\,\boldsymbol{x}_1)\mathrm{T}\mathcal{G}_0(\boldsymbol{x}_1,\,\boldsymbol{r}') + \cdots$$

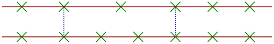


Self-consistent equation

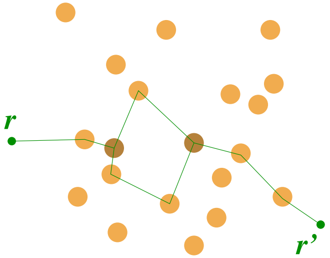
Multiple scattering
self-energy

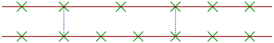
Polarization
V. Rossetto
Polarization
Multiple scattering
Single scattering
Born expansion
Dyson equation
Berry phase
Conclusion





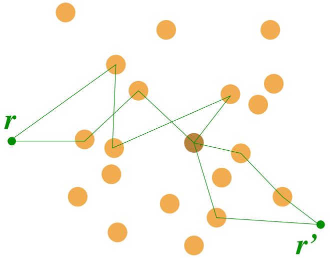
Multiple scattering
self-energy

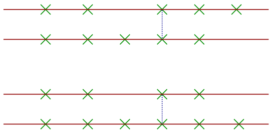


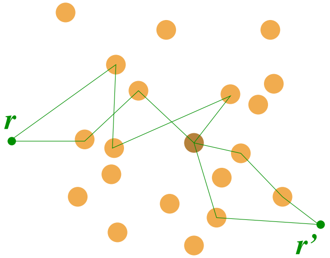


= 0

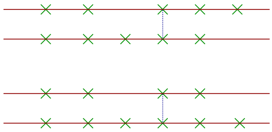
Multiple scattering
self-energy





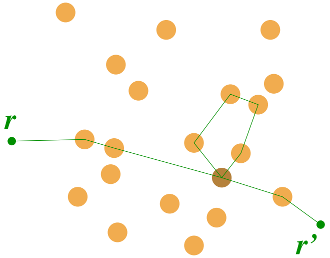


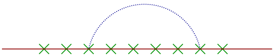
r
r’

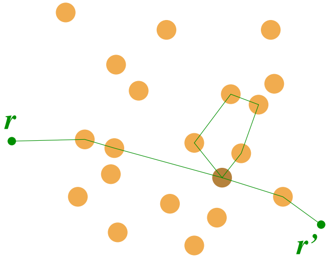


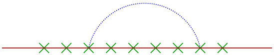
= 0
= 0

Multiple scattering
self-energy

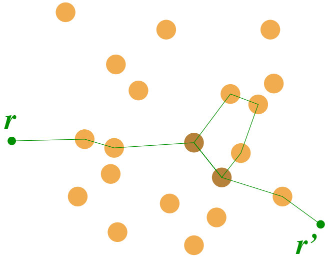








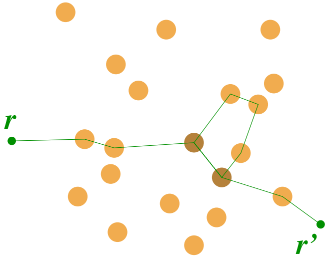
$\neq 0$

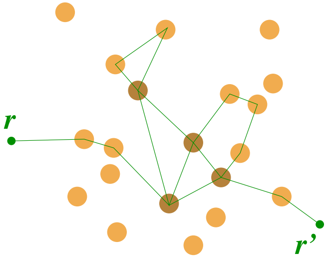


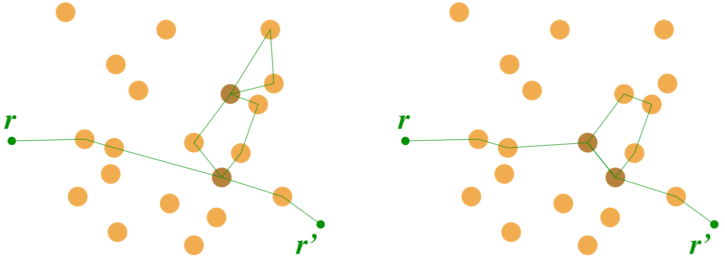
r
r'

Multiple scattering

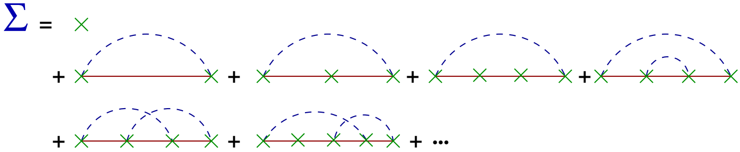
self-energy







r
r'
r
r'



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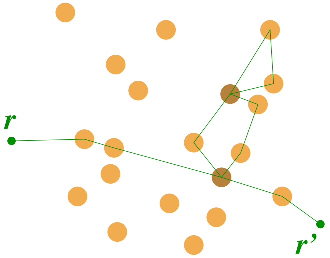
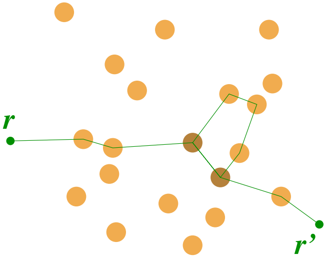
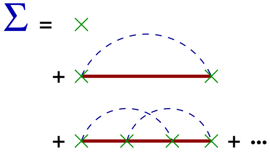




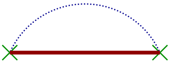


Green–Dyson equation

Change scattering definition :

$$= \int \mathrm{dR}_1 \int \mathrm{dR}_2 \, \mathrm{G}_0(\boldsymbol{q}, \omega, \mathrm{R}_2, \mathrm{R}')\mathrm{T}(\omega, \mathrm{R}_1, \mathrm{R}_2)\mathrm{G}_0(\boldsymbol{q}, \omega, \mathrm{R}, \mathrm{R}_1)$$

Self energy



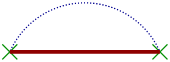
$$= \lim_{|\boldsymbol{q}|\to\infty} \int \mathrm{dR}_1 \int \mathrm{dR}_2 \, \mathrm{T}(\omega, \mathrm{R}_2, \mathrm{R}')\overline{\mathrm{G}}(, \omega, \mathrm{R}_1, \mathrm{R}_2)\mathrm{T}(\omega, \mathrm{R}, \mathrm{R}_1)$$

$$\int \mathrm{dR} \quad \Longrightarrow \quad \text{matrix product}$$

Change scattering definition :

$$= \int \mathrm{d}R_1 \int \mathrm{d}R_2\, G_0(\boldsymbol{q}, \omega, R_2, R')T(\omega, R_1, R_2)G_0(\boldsymbol{q}, \omega, R, R_1)$$

Self energy



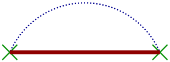
$$= \lim_{|\boldsymbol{q}|\to\infty} \int \mathrm{d}R_1 \int \mathrm{d}R_2\, T(\omega, R_2, R')\overline{G}(\boldsymbol{q}, \omega, R_1, R_2)T(\omega, R, R_1)$$

$$\int \mathrm{d}R \quad \Longrightarrow \quad \text{matrix product}$$

Change scattering definition :

$$= \int dR_1 \int dR_2 \, G_0(\boldsymbol{q}, \omega, R_2, R') T(\omega, R_1, R_2) G_0(\boldsymbol{q}, \omega, R, R_1)$$

Self energy



$$= \lim_{|\boldsymbol{q}|\to\infty} \int dR_1 \int dR_2 \, T(\omega, R_2, R') \overline{G}(\boldsymbol{q}, \omega, R_1, R_2) T(\omega, R, R_1)$$

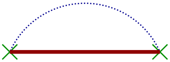
$\uparrow$has a direction

$$\int dR \quad \Longrightarrow \quad \text{matrix product}$$

Change scattering definition :

$$= \int \mathrm{dR}_1 \int \mathrm{dR}_2 \, \mathrm{G}_0(\boldsymbol{q}, \omega, \mathrm{R}_2, \mathrm{R}')\mathrm{T}(\omega, \mathrm{R}_1, \mathrm{R}_2)\mathrm{G}_0(\boldsymbol{q}, \omega, \mathrm{R}, \mathrm{R}_1)$$

Self energy $\Sigma(\hat{\boldsymbol{q}}, \omega, \mathrm{R}, \mathrm{R}')$



$$= \lim_{|\boldsymbol{q}|\to\infty} \int \mathrm{dR}_1 \int \mathrm{dR}_2 \, \mathrm{T}(\omega, \mathrm{R}_2, \mathrm{R}')\overline{\mathrm{G}}(\boldsymbol{q}, \omega, \mathrm{R}_1, \mathrm{R}_2)\mathrm{T}(\omega, \mathrm{R}, \mathrm{R}_1)$$

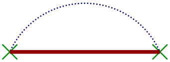
↑has a direction

$\int \mathrm{dR} \quad \Longrightarrow \quad$ matrix product

Change scattering definition :

$$= \int dR_1 \int dR_2 \, G_0(\boldsymbol{q}, \omega, R_2, R')T(\omega, R_1, R_2)G_0(\boldsymbol{q}, \omega, R, R_1)$$

Self energy $\Sigma(\hat{\boldsymbol{q}}, \omega, R, R')$



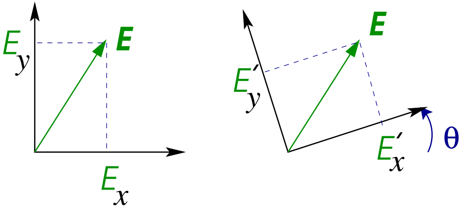
$$= \lim_{|\boldsymbol{q}|\to\infty} \int dR_1 \int dR_2 \, T(\omega, R_2, R')\overline{G}(\boldsymbol{q}, \omega, R_1, R_2)T(\omega, R, R_1)$$

↑has a direction

$$\int dR \quad \Longrightarrow \quad \text{matrix product}$$

The Berry phase

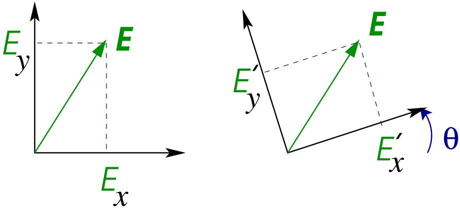
phase and rotation



$$E_{\mathsf{s}}(\boldsymbol{r},\, t,\, \mathrm{RZ}(\alpha)) = \mathrm{e}^{-\mathrm{i}\mathsf{s}\alpha} E_{\mathsf{s}}(\boldsymbol{r},\, t,\, \mathrm{R})$$

$$\mathrm{T}_{\mathsf{ss'}}(\omega,\, \mathrm{R},\, \mathrm{R'}) = \mathrm{e}^{\mathrm{i}(\mathsf{s}\tilde{\phi}+\mathsf{s'}\tilde{\psi})}\; f_{\mathsf{ss'}}(\omega,\, \tilde{\theta})$$

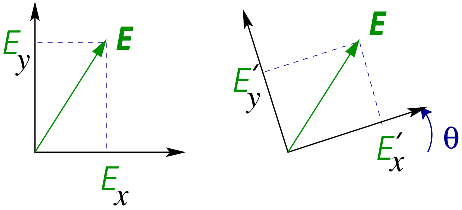
Total phase for a path ?



$$E_s(\boldsymbol{r},\, t,\, \mathrm{R}\mathrm{Z}(\alpha)) = \mathrm{e}^{-\mathrm{i}s\alpha} E_s(\boldsymbol{r},\, t,\, \mathrm{R})$$

$$\mathrm{T}_{ss'}(\omega,\, \mathrm{R},\, \mathrm{R}') = \mathrm{e}^{\mathrm{i}(s\tilde{\phi}+s'\tilde{\psi})} \quad f_{ss'}(\omega,\, \tilde{\theta})$$

Total phase for a path ?



$$E_s(\boldsymbol{r},\, t,\, \mathrm{RZ}(\alpha)) = \mathrm{e}^{-\mathrm{i}s\alpha} E_s(\boldsymbol{r},\, t,\, \mathrm{R})$$

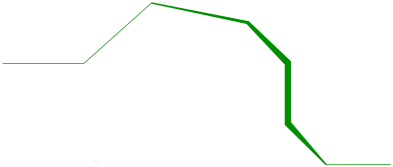
$$\mathrm{T}_{ss'}(\omega,\, \mathrm{R},\, \mathrm{R}') = \mathrm{e}^{\mathrm{i}(s\tilde{\phi}+s'\tilde{\psi})} \;\; f_{ss'}(\omega,\, \tilde{\theta})$$

Total phase for a path ?

The Berry phase
bringing it to light

Consider a path $(\boldsymbol{x}_1, \mathrm{R}_1) \ldots (\boldsymbol{x}_n, \mathrm{R}_n)$ such that $\mathrm{R}_n = \mathrm{R}_1$ and

$$\theta_{i+1} - \theta_i \ll 1 \qquad \phi_{i+1} - \phi_i \ll 1$$



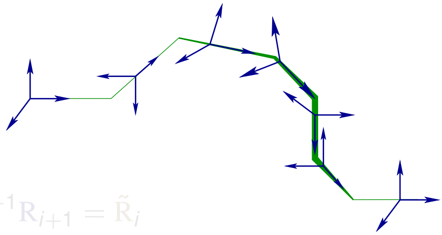
Rotations $\mathrm{R}_i^{-1}\mathrm{R}_{i+1} = \tilde{\mathrm{R}}_i$

$$\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i)\cos\theta_i + \psi_{i+1} - \psi_i$$

$$s\sum_{i=0}^{n-1} \tilde{\phi}_i + \tilde{\psi}_i \simeq -s\sum_{i=0}^{n-1}(1 - \cos\theta_i)(\phi_{i+1} - \phi_i) \mod 2\pi$$

Consider a path $(\boldsymbol{x}_1, \mathrm{R}_1) \ldots (\boldsymbol{x}_n, \mathrm{R}_n)$ such that $\mathrm{R}_n = \mathrm{R}_1$ and

$$\theta_{i+1} - \theta_i \ll 1 \qquad \phi_{i+1} - \phi_i \ll 1$$



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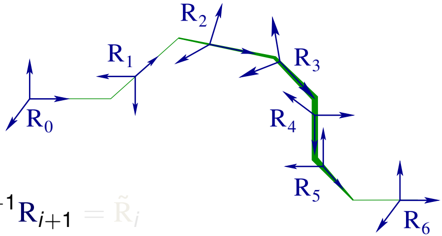
Consider a path $(\boldsymbol{x}_1, \mathrm{R}_1)\ldots(\boldsymbol{x}_n, \mathrm{R}_n)$ such that $\mathrm{R}_n = \mathrm{R}_1$ and

$$\theta_{i+1} - \theta_i \ll 1 \qquad \phi_{i+1} - \phi_i \ll 1$$



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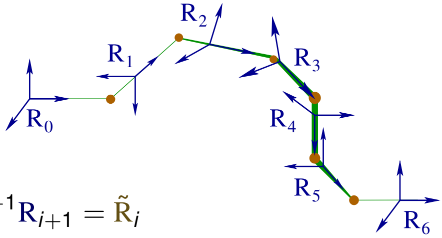
$$\theta_{i+1} - \theta_i \ll 1 \qquad \phi_{i+1} - \phi_i \ll 1$$



Rotations $\mathrm{R}_i^{-1}\mathrm{R}_{i+1} = \tilde{\mathrm{R}}_i$

$$\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i)\cos\theta_i + \psi_{i+1} - \psi_i$$

$$s\sum_{i=0}^{n-1}\tilde{\phi}_i + \tilde{\psi}_i \simeq -s\sum_{i=0}^{n-1}(1-\cos\theta_i)(\phi_{i+1}-\phi_i) \mod 2\pi$$

Consider a path $(\boldsymbol{x}_1, \mathrm{R}_1)\dots(\boldsymbol{x}_n, \mathrm{R}_n)$ such that $\mathrm{R}_n = \mathrm{R}_1$ and

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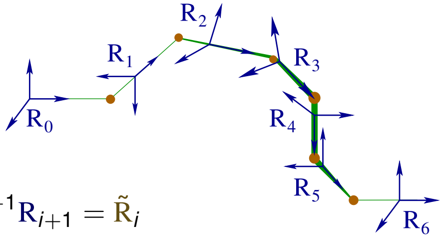

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$$\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i)\cos\theta_i + \psi_{i+1} - \psi_i$$

$$s\sum_{i=0}^{n-1}\tilde{\phi}_i + \tilde{\psi}_i \simeq -s\sum_{i=0}^{n-1}(1-\cos\theta_i)(\phi_{i+1}-\phi_i) \mod 2\pi$$

Consider a path $(\boldsymbol{x}_1, \mathrm{R}_1) \ldots (\boldsymbol{x}_n, \mathrm{R}_n)$ such that $\mathrm{R}_n = \mathrm{R}_1$ and

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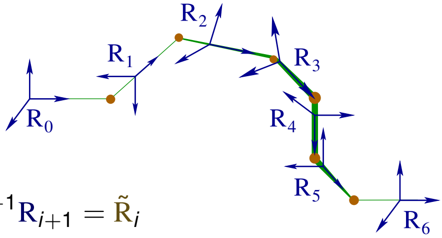

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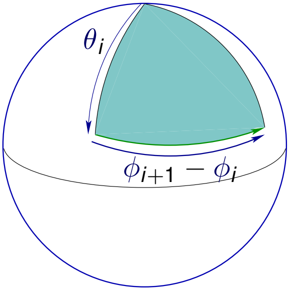
$$\tilde{\phi}_i + \tilde{\psi}_i \simeq (\phi_{i+1} - \phi_i)\cos\theta_i + \psi_{i+1} - \psi_i$$

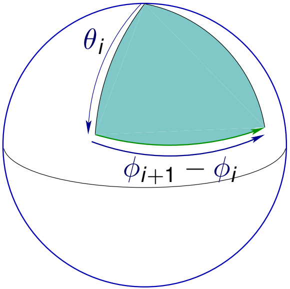
$$\mathsf{s}\sum_{i=0}^{n-1} \tilde{\phi}_i + \tilde{\psi}_i \simeq -\mathsf{s}\sum_{i=0}^{n-1}(1-\cos\theta_i)(\phi_{i+1}-\phi_i) \mod 2\pi$$

$$(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$

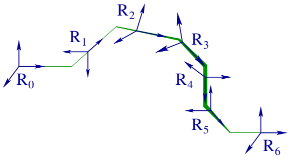
$$\sum_{i=0}^{n-1}(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$



$$(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$



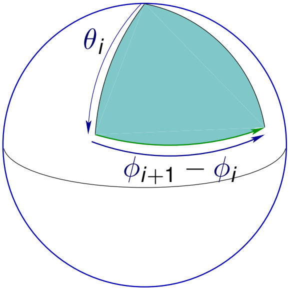
$$\sum_{i=0}^{n-1}(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$



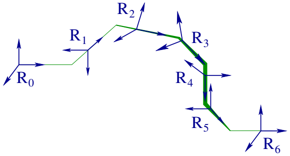
The Berry phase

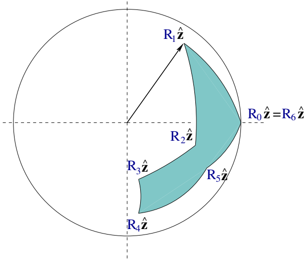
geometric interpretation

$$(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$

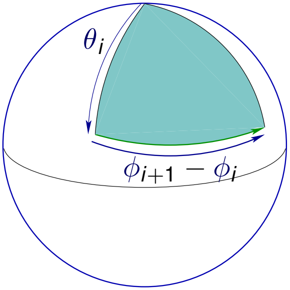


$$\sum_{i=0}^{n-1}(1 - \cos\theta_i)(\phi_{i+1} - \phi_i)$$

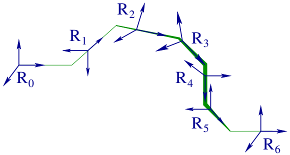




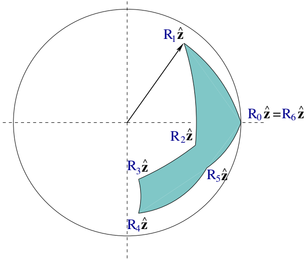
$$(1-\cos\theta_i)(\phi_{i+1}-\phi_i)$$



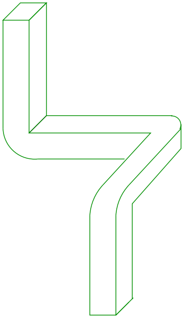
$$\sum_{i=0}^{n-1}(1-\cos\theta_i)(\phi_{i+1}-\phi_i)$$



$$\Omega_B = -s \left(\begin{array}{l} \text{Area enclosed} \\ \text{on the sphere} \\ \text{by the direction} \\ \text{of propagation} \end{array} \right)$$



“Parallel transport” of transverse polarization



The Berry phase

example

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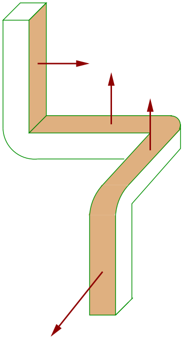

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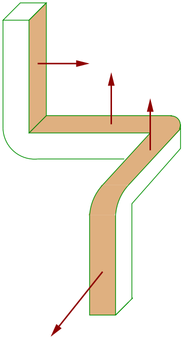

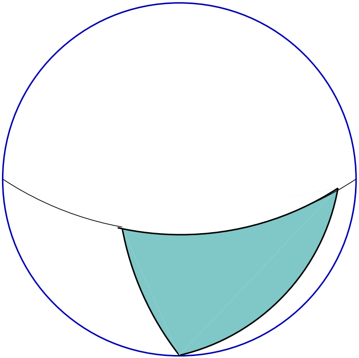

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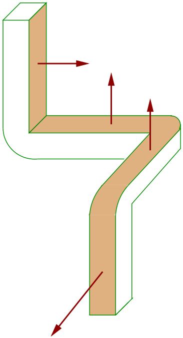

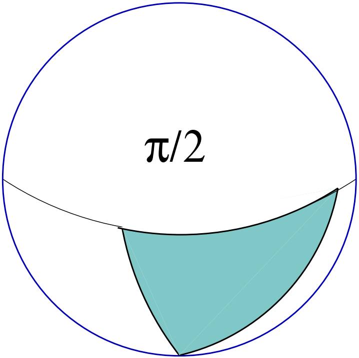

In a system with forward scattering, $\mathrm{G}(\boldsymbol{r}, \boldsymbol{r}', t, t', \mathrm{R}, \mathrm{R}')$ contains the statistics of the Berry phase for paths

- starting at $\boldsymbol{r}$ and ending at $\boldsymbol{r}'$
- of length $c(t' - t)$
- with initial and final directions $\mathrm{R}\hat{\boldsymbol{z}}$ and $\mathrm{R}'\hat{\boldsymbol{z}}$

Properties of the Berry phase statistics ?

- depends on the heterogeneity of the medium
- depends on transport properties (anisotropies)
- difficult to measure (?)
- interpretation of experiment data
- theory ?

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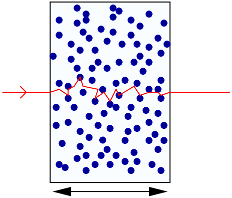
The Berry phase

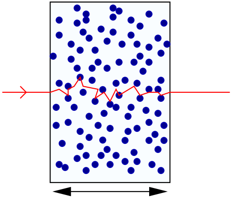
depolarization



$L \sim \ell^*$

Forward scattering : the direction of propagation remains near $\hat{\boldsymbol{z}}$

The trajectory on the sphere is almost flat



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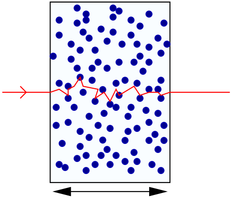

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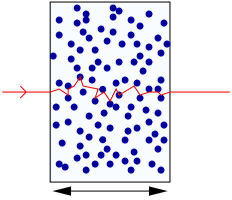
$$p(\Omega_B) = \frac{\pi \ell^*}{L} \frac{1}{\cosh^2\left(2\pi\Omega_B \frac{\ell^*}{L}\right)}$$



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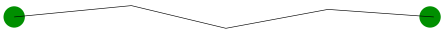
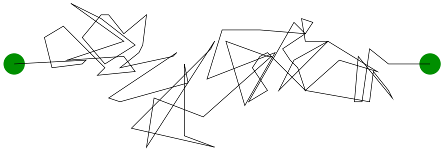
The outgoing polarisation is :

$$\langle \cos 2\Omega_B \rangle = \frac{L}{2\ell^*} \frac{1}{\sinh \frac{L}{2\ell^*}}$$

The Berry phase
numerical simulations





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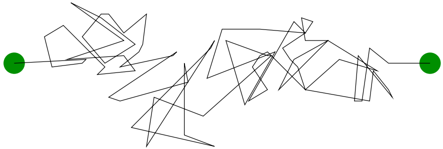

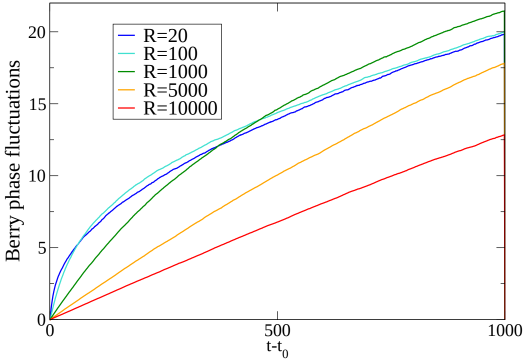

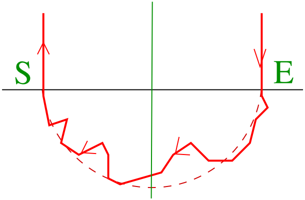
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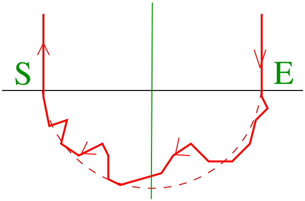
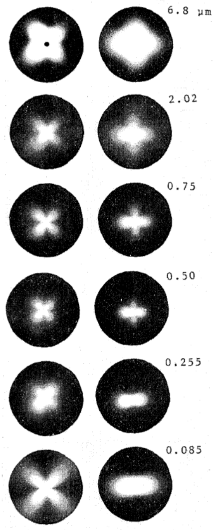
The Berry phase
backscattering experiment





The Berry phase

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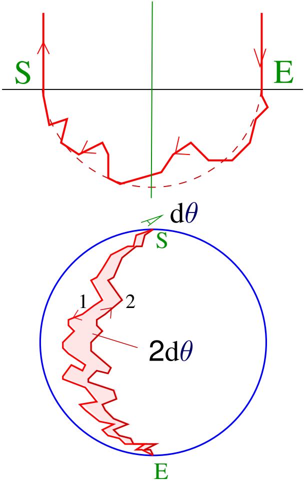

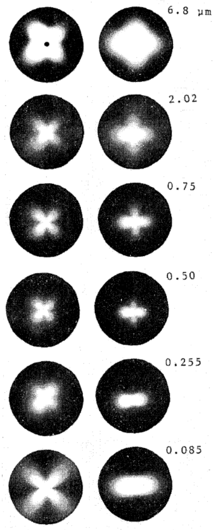

Conclusion and outlooks

- Multiple scattering theory for polarized waves
- takes into account several kinds of anisotropies
- and the Berry phase

- Elastic waves should have a Berry phase
- but it was never observed ! → experimental challenge ?
- The Berry phase contains informations on the paths statistics...
- ... therefore on the properties of the medium
- → investigate how to extract relevant informations
- Use seismology techniques (stacking, correlations) to retrieve data

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