Effect of particle losses on superpositions of phase states in Bose Josephson junctions

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PEPS-PTI

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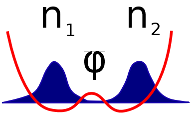
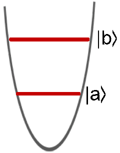
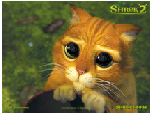
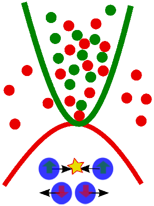
Outline

- Motivation
- N particles in two modes
- 'Typical' evolution - creation of entanglement
- Effect of particle losses









Motivation

'better' statistics = with uncertainties smaller than shot noise

NEEDED:
entangled states
limits due to decoherence and dephasing
here: effect of particle losses

Second quantization, N bosons in two modes

$|n, N-n\rangle$ - symmetric state with n atoms in mode 1, the rest in mode 2

Example: $\langle \boldsymbol{r}_1, \boldsymbol{r}_2 | 1, 1\rangle = \frac{1}{\sqrt{2}}\left(\phi_1(\boldsymbol{r}_1)\phi_2(\boldsymbol{r}_2) + \phi_1(\boldsymbol{r}_2)\phi_2(\boldsymbol{r}_1)\right)$
Thus $|N, N\rangle$ - very entangled state

$\hat{a}, \hat{b}$ - annihilation operators in modes 1 and 2
$[\hat{a}, \hat{a}^\dagger] = 1 \qquad [\hat{a}, \hat{b}] = 0 \quad [\hat{b}, \hat{b}^\dagger] = 1$

Coherent state:
$|\theta, \phi\rangle_N = \frac{1}{\sqrt{N!}}\left(\cos\theta e^{i\phi}\hat{a}^\dagger + \sin\theta e^{-i\phi}\hat{b}^\dagger\right)^N |0\rangle$
$\hat{a}|\theta, \phi\rangle_N = \sqrt{N}\cos\theta e^{i\phi}|\theta, \phi\rangle_{N-1}$

Initial states

$$|\phi = 0\rangle = \left(\frac{\hat{a}^\dagger + \hat{b}^\dagger}{\sqrt{2}}\right)^N |0\rangle = \sum_{n=0}^{N} \sqrt{\frac{1}{2^N}\binom{N}{n}}\, |n, N-n\rangle$$

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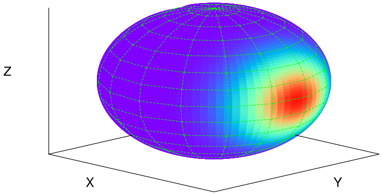
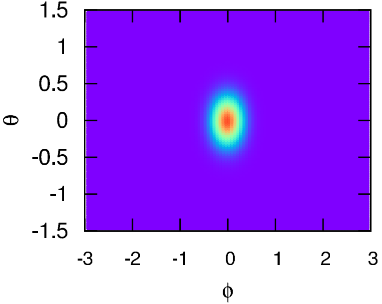




$$\hat{S}_z = \left(\hat{n}_a - \hat{n}_b\right)/2 \propto \sin\theta$$

Evolution

Hamiltonian (**no tunneling**):

$$\begin{aligned}\hat{H} &= \frac{\chi_a}{2}\left(a^\dagger a^\dagger a a\right) + \chi_{ab}\left(a^\dagger a b^\dagger b\right) + \frac{\chi_b}{2}\left(b^\dagger b^\dagger b b\right) = \\ &= \chi \hat{S}_z^2 + f\left(\hat{N}\right)\end{aligned}$$

If $|\psi\rangle = \sum_{n=0}^{N} c_n |n, N-n\rangle$, then

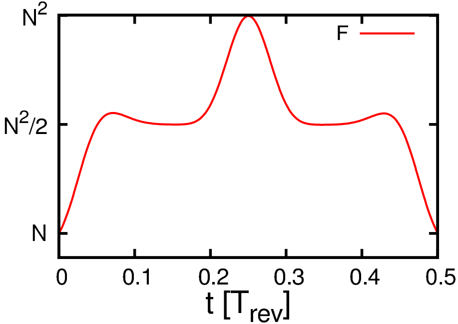
$$\begin{aligned}|\psi(t)\rangle &= \sum_{n=0}^{N} c_n e^{-i(\chi_a - \chi_{ab})\left(a^\dagger a^\dagger a a\right)t} \\ &\quad \times e^{-i(\chi_b - \chi_{ab})\left(b^\dagger b^\dagger b b\right)t} |n, N-n\rangle\end{aligned}$$

Evolution of a coherent state

Fisher information F

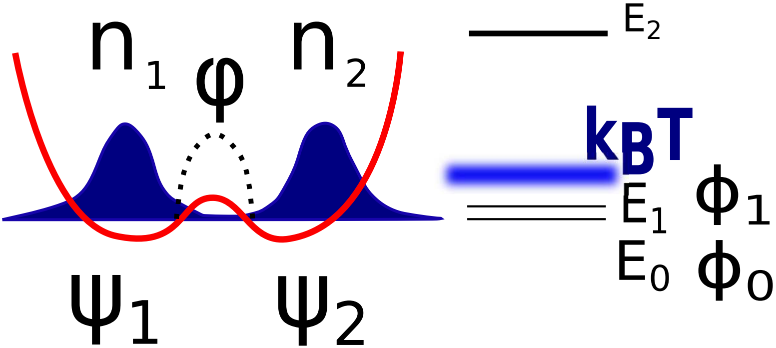
$F = F(\hat{\rho})$
$F > N$ - useful entangled state

$\max F = N^2$



P. Hyllus *et al.* Phys. Rev. A, **82** (2010) 012337

Experiments



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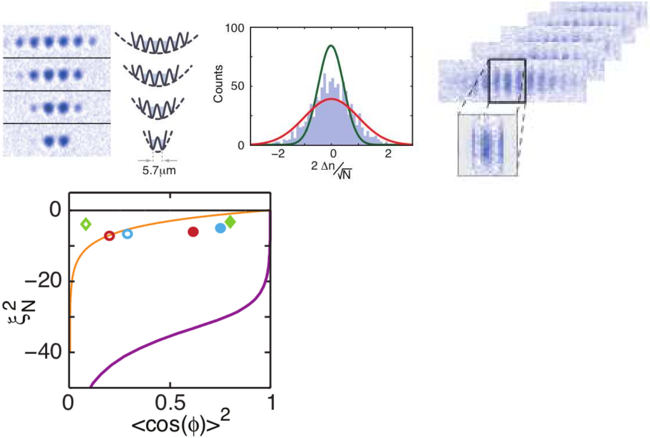

C. Gross *et all*, Nature **464**, (2010) 1165

Master equation

$$\underbrace{\partial_t\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]}_{\text{von Neuman equation}} + \underbrace{\mathcal{L}_1\hat{\rho} + \mathcal{L}_2\hat{\rho} + \mathcal{L}_3\hat{\rho}}_{\text{particle losses}},$$

where $\mathcal{L}_n = \mathcal{L}_n^{(a)} + \mathcal{L}_n^{(b)}$

$$\mathcal{L}_n^{(a)}\hat{\rho} = \gamma_n\left[\hat{a}^n, \hat{\rho}\left(\hat{a}^\dagger\right)^n\right] + \gamma_n\left[\hat{a}^n\hat{\rho}, \left(\hat{a}^\dagger\right)^n\right]$$

Exact solution

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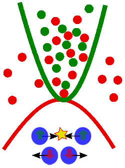
Exact solution

Here: only 2-body losses

γ_1, γ_2 - rates of 2−body losses in "a" and "b" mode



How does it work?

Example: atoms in only one mode, only 1-body losses, diagonal elements of the master equation $p_n = \langle n|\hat{\rho}|n\rangle$
$\hat{H}_0 = \chi_1 \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}$

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H}_0, \hat{\rho}\right] + \frac{\gamma}{2}\left[\hat{a}, \hat{\rho}\hat{a}^\dagger\right] + \frac{\gamma}{2}\left[\hat{a}\hat{\rho}, \hat{a}^\dagger\right]$$

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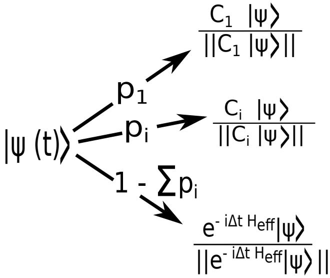
$\left\langle n\middle|\left[\hat{H}_0, \hat{\rho}\right]\middle|n\right\rangle = 0$
$\left\langle n\middle|2\hat{a}\hat{\rho}\hat{a}^\dagger\middle|n\right\rangle = 2(n+1)p_{n+1}$
$\left\langle n\middle|\hat{a}^\dagger\hat{a}\hat{\rho} + \rho\hat{a}^\dagger\hat{a}\middle|n\right\rangle = 2np_n$

$$\frac{dp_n}{dt} = \gamma(n+1)p_{n+1} - \gamma n p_n$$

Method 1: Quantum jumps approach

$$\partial_t \hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}_0, \hat{\rho}\right] + \frac{1}{2}\sum_i [\hat{C}_i\hat{\rho}, \hat{C}_i^\dagger] + [\hat{C}_i, \hat{\rho}\hat{C}_i^\dagger]$$



$p_i = \left\langle \hat{C}_i^\dagger \hat{C}_i \right\rangle \Delta t, \quad H_{eff} = H_0 - \frac{i}{2}\sum_i \hat{C}_i^\dagger \hat{C}_i \Delta t$ then

$$\lim_W \sum_{l=1}^{W} |\psi_l\rangle \langle\psi_l| \to \hat{\rho}(t)$$

Method 2: Exact diagonalization

Main idea:
(1) to solve one-mode problem
(2) use the solution (1) to recostruct solution for two modes

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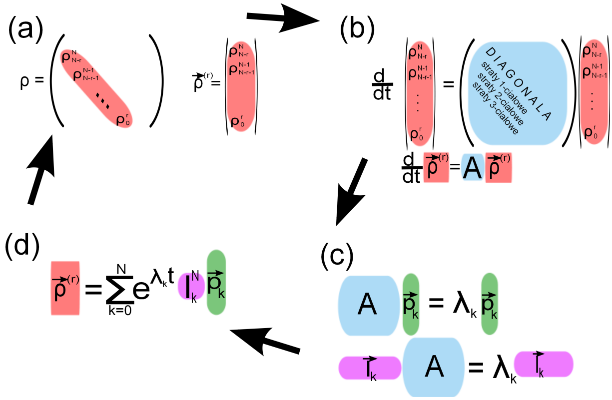
$a(k, r; n, t) = \langle k|\hat{\rho}(t)|k + r\rangle$ solution of the master equation in "a" mode with the initial condition $\langle k|\hat{\rho}(t = 0)|k + r\rangle = \delta_k^n$

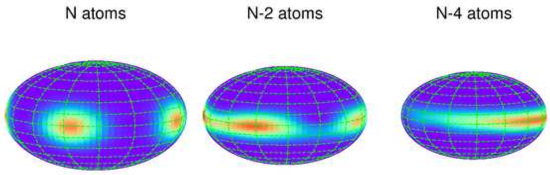
$$\rho_{k,l+r}^{k+r,l}(t) = \sum_{m=k}^{N_0-l-r} \rho_{m,N_0-m}^{m+r,N_0-m-r}(0)\; a(k, r; m, t)[b(l, r; N_0-m-r, t)]^*$$

Method 2: Exact diagonalization



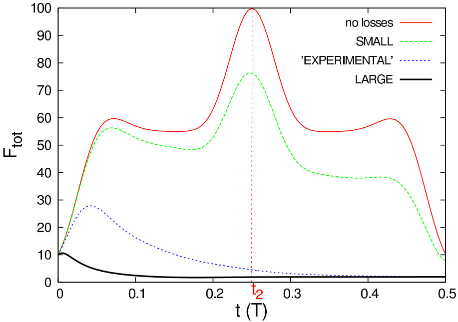
Particle losses

Bloch sphere?



$$\hat{\rho} = p_N\,\hat{\rho}_N \quad + \quad p_{N-2}\,\hat{\rho}_{N-2} + p_{N-4}\,\hat{\rho}_{N-4} + \ \ldots + p_0\,\hat{\rho}_0$$

Fisher information with losses



From macroscopic to mesoscopic!

C. Gross *et all*, Nature **464**, (2010) 1165, M.F. Riedel *et all*, Nature **464**, (2010) 1170

Fisher information with losses

$$\hat{H} = \frac{\chi_1}{2}\left(a^\dagger a^\dagger a a\right) + \frac{\chi_2}{2}\left(b^\dagger b^\dagger b b\right) \qquad \partial_t \hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}\right] + \mathcal{L}_2^{(1)}\hat{\rho} + \mathcal{L}_2^{(2)}\hat{\rho}$$

number of lost atoms $\sim 30\% - 40\%$

γ_1, γ_2 - loss rates in the first and the second mode

χ_1, χ_2 - effective interaction energy of a pair of atoms in the first and the second mode

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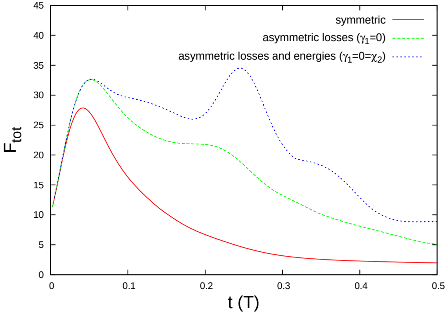


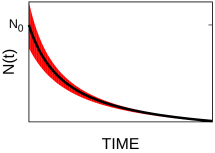
Subspace with N atoms

Case, when no losses occurred
(although they were possible)

$p(N,t)$ - probability of an loss event at time t in the cloud with initially N atoms
2-body losses

IF $N_1 > N_2$ THEN
$p(N_1,t) > p(N_2,t)$

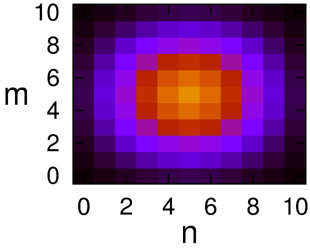
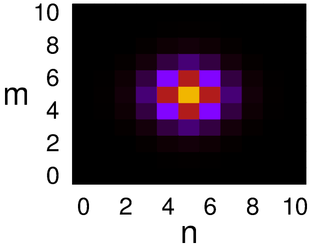
losses more probable in $|0,N\rangle$ than in $|N/2,\ N/2\rangle$



S. Whitlock *et al.* Phys. Rev. Let. **104** (2010) 120402

Do nothing and gain !

$$\langle n, N-n|\hat{\rho}_N|m, N-m\rangle$$

no losses	with 2-body losses
$\hat{\rho}_N^{(0)}$	$\hat{\rho}_N^{(0)} e^{-\gamma_2 t\left(\left(n-\frac{N}{2}\right)^2+\left(m-\frac{N}{2}\right)^2\right)}$
	

Final state $= \left|\frac{N}{2}, \frac{N}{2}\right\rangle$ (but with probability $p_N \ll 1$)

process nr 1: 'Gaussian shrinking'

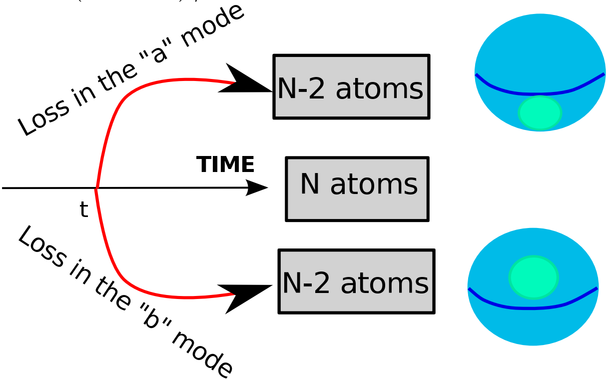
Subspace with $N - 2$ atoms

Two body losses only

What happens when **SINGLE** lost event occurred

Subspace with $N - 2$ atoms - after one loss event

$\hat{S}_z = (\hat{n}_a - \hat{n}_b)/2$

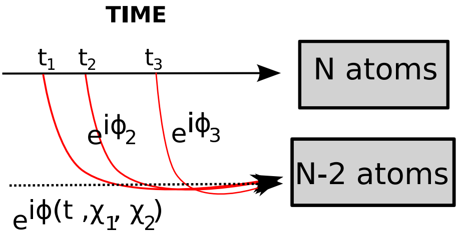


$\Rightarrow$**Mixture**

process nr 2: Channeling (with very strong destructive interference for the cat state)

to avoid it: $\gamma_1 \gg \gamma_2$

Subspace with $N-2$ atoms - after one loss event

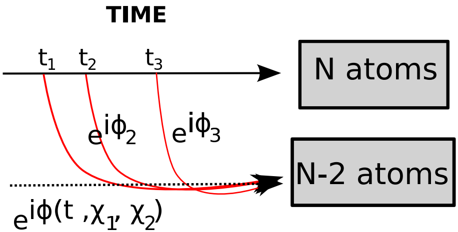


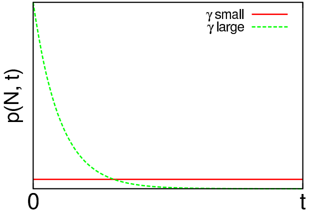
state = incoherent mixture of $|\theta_i, \phi_i\rangle$ $\quad \Delta\phi \propto \Delta t^2 \propto \frac{\chi}{\gamma N}$

(estimated for small losses)

process nr 3: phase noise

Subspace with $N-2$ atoms - after one loss event





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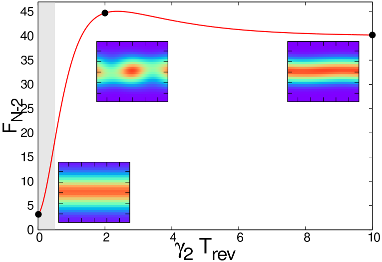
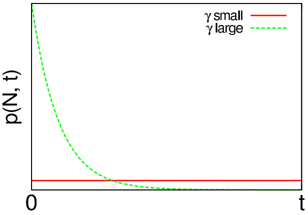
process nr 3: phase noise

Subspace with $N-2$ atoms - after one loss event

Fixed time ($T_{rev}/2$) but different loss rates

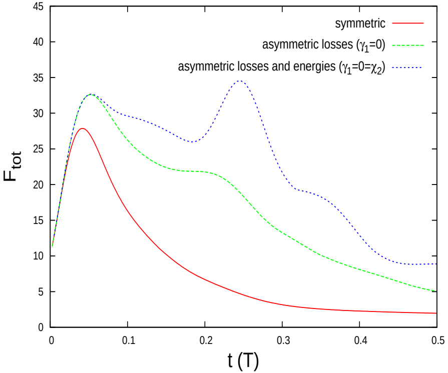




state = incoherent mixture of $|\theta_i, \phi_i\rangle$ $\qquad \Delta\phi \propto \frac{\chi}{2\gamma N}$

to avoid it: increase γ or suppress χ

Fisher information with losses



$\gamma_1 = 0$ no channeling
$\gamma_1 = 0$ and $\chi_2 = 0$ no phase noise

subspaces with smaller number of atoms - similar to $N-2$ subspace

Summary

1. Exact solution of the master equation
2. Decoherence in subspaces with different number of atoms
   - 'Gaussian shrinking'
   - Phase noise
   - Destructive interference (atoms lost from a or b?)
3. Huge advantage for highly asymmetric losses, using Feshbach resonances
4. Gain via post-selection (?)

TODO-list

- scaling with N
- beyond B-H model
- experimental conditions (phase noise, finite temperature)

Thank you for your attention!!