Mesoscopic Physics with Seismic waves

« not nano but kilo »

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abstract

- Mesoscopic Physics
- Seismic Coda
- Equipartition
- Correlations
- Coherent backscattering
- Imaging in a disordered world
  - Relation with time-reversal
  - Relation with CBS

Matter Waves:

$$i\hbar\partial_t \Psi = H\Psi \;\Rightarrow\; \Psi(t) = \Psi(0)\exp\left(-\frac{i}{\hbar}Ht\right)$$

$$H = \underbrace{H_{\mathrm{sys}}}_{\text{The good...}} + \underbrace{H_{\mathrm{int}}}_{\text{the bad...}} + \underbrace{H_{\mathrm{disorder}}}_{\text{and the ugly}}$$

$\int d\mathbf{r}\, \lvert\Psi(\mathbf{r},t)\rvert^2 = 1$	probability
$\langle \Psi(\mathbf{r},t)\rangle_{\text{désordre}} \propto \exp(-t/\bar{\tau})$	dephasing
$\langle \lvert\Psi(\mathbf{r},t)\rvert^2 \rangle_{\text{désordre}} \propto \exp(-r^2/4Dt)$	diffusion
$\langle \Psi(\mathbf{r},0)\,\Psi^*(\mathbf{r}',t)\rangle_{\text{env}} \propto \exp(-t/\tau_\phi(\mathbf{r}-\mathbf{r}'))$	decoherence

$H = H_1 \otimes H_2$ ⟺ entangling ⟺ decoherence

Seismic Waves

$$\rho\, \partial_{t^2}^2 u_i = \partial^k \sigma_{ik} + \eta\, \Delta\, \partial_t u_i + f_i^{\text{source}}(\mathbf{r},t) + \delta f_i^{\text{noise}}(\mathbf{r},t)$$

$$\sigma_{ik} = \lambda(\nabla\cdot\mathbf{u})\delta_{ik} + \mu(\partial_i u_k + \partial_k u_i) \qquad f_i^{\text{source}}(\mathbf{r},\omega) = -M_i^j(\omega)\,\partial_j\,\delta(\mathbf{r}-\mathbf{r}_0)$$

medium $\lambda_0(\mathbf{r}), \rho_0(\mathbf{r})$ — viscosity η — fluctuations

source $\mathbf{f}_{\text{source}}(\mathrm{r},t)$ — noise $\delta\mathbf{f}_{\text{noise}}(\mathrm{r},t)$ — $\delta\lambda, \delta\mu, \delta\rho$

The good… **the bad…** **and the ugly**

$$\int d\mathbf{r}\, \Gamma^{ijkl}(\mathbf{r})\,\partial_i u_j\, \partial_k u_l^* = \exp(-t/\tau_{\text{abs}})$$

absorption

$$\langle \mathbf{u}(\mathbf{r},t)\rangle_{\text{disorder}} \propto \exp(-t/\bar{\tau})$$

dephasing

$$\langle u_i(\mathbf{r},t)\, u_j^*(\mathbf{r},t)\rangle_{\text{disorder}} \propto \exp(-r^2/Dt)$$

diffusion

$$\langle \mathbf{u}(\mathbf{r},t)\, \mathbf{u}^*(\mathbf{r}',t)\rangle_{\text{env}} \propto \exp(-t/2\tau_{\text{abs}}) + \text{noise}$$

$$\mathrm{H} = \mathrm{H}_1 \oplus \mathrm{H}_2$$ ⟺ No entangling

Mesoscopic criterion

$$\bar{\tau} \quad < \quad \Delta T(L) \quad < \quad \tau^{\max} = \begin{cases} \tau^{\text{abs}} \quad \log \dfrac{\text{source}}{\text{noise}} \\ \tau_{\phi} \end{cases}$$

$$\Delta T(L) \approx \frac{L^2}{2dD}:$$ Diffusion time $$= \frac{\hbar}{E_{\text{Thouless}}}$$

$$D \propto \frac{1}{d} \mathrm{v}\,\ell$$ Diffusion constant

$$\ell \equiv \bar{\tau}\,\mathrm{v}$$ Mean free path

$$\sqrt{D\tau^{\max}} \equiv L_{\max}$$ Absorption length /decoherence time

$$\ell \quad < \quad L \quad < \quad L_{\max}$$

L. Brillouin, 1960

Mesoscopic regime $\ell < L < L_{\phi,\max}$

1. Electrons: $\begin{cases} \ell \approx 1\,\text{nm} \\ L_\phi \approx 10\,\mu\text{m}\,(1\text{K}) \end{cases}$ **NANO**

2. Photons $\begin{cases} \ell \approx 300\,\text{nm} - 1\,\text{mm} \\ L_a \approx 100\,\mu\text{m-}1\text{cm} \end{cases}$ **MICRO-MILLI**

3. Micro waves $\begin{cases} \ell \approx 5\,\text{cm} \\ L_a \approx 50\,\text{cm} \end{cases}$ **CENTI**

4. Seismic waves $\begin{cases} \ell \approx 30\,\text{km} \\ L_a \approx 100\,\text{km}\ \ (1\,\text{Hz}) \end{cases}$ **KILO**

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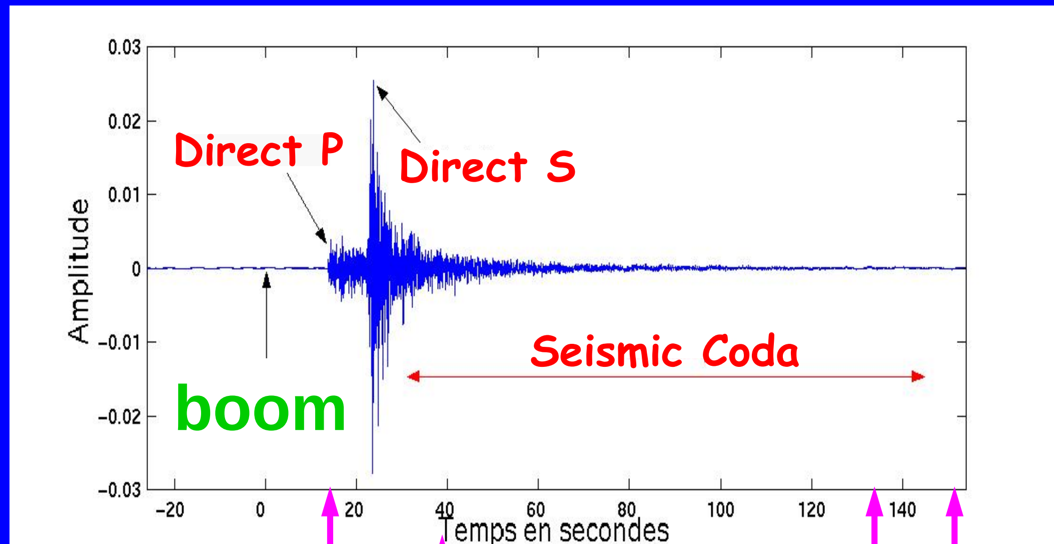
seismic coda



Mean free time

Thouless time

Absorption time

noise

$$I(t) = \frac{1}{t^n} \exp\left(-\frac{\omega t}{Q}\right) \qquad n \approx 1$$

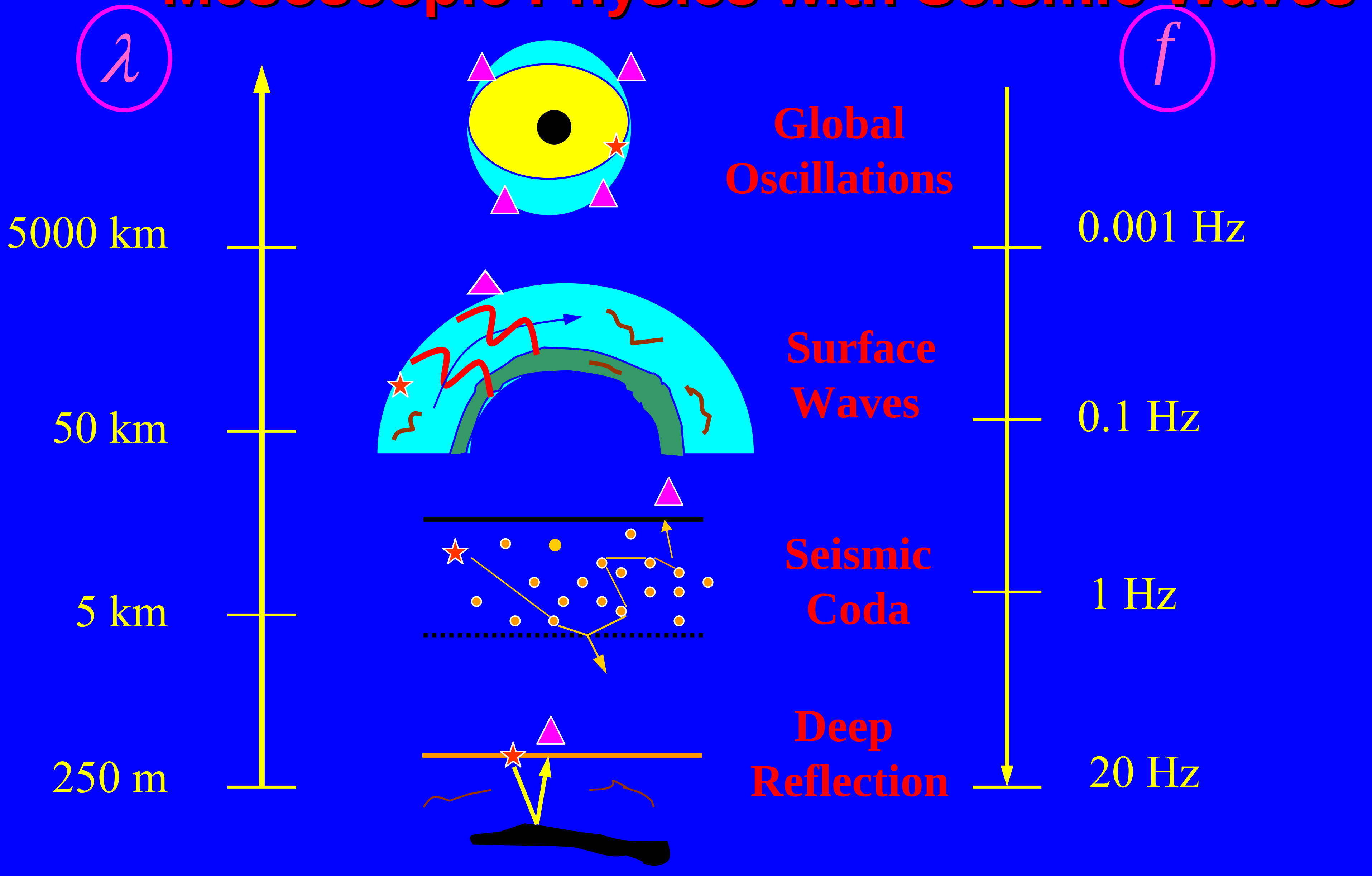
K. Aki, 1969
K. Aki & B. Chouet, 1975

What is it and what does it tell us?

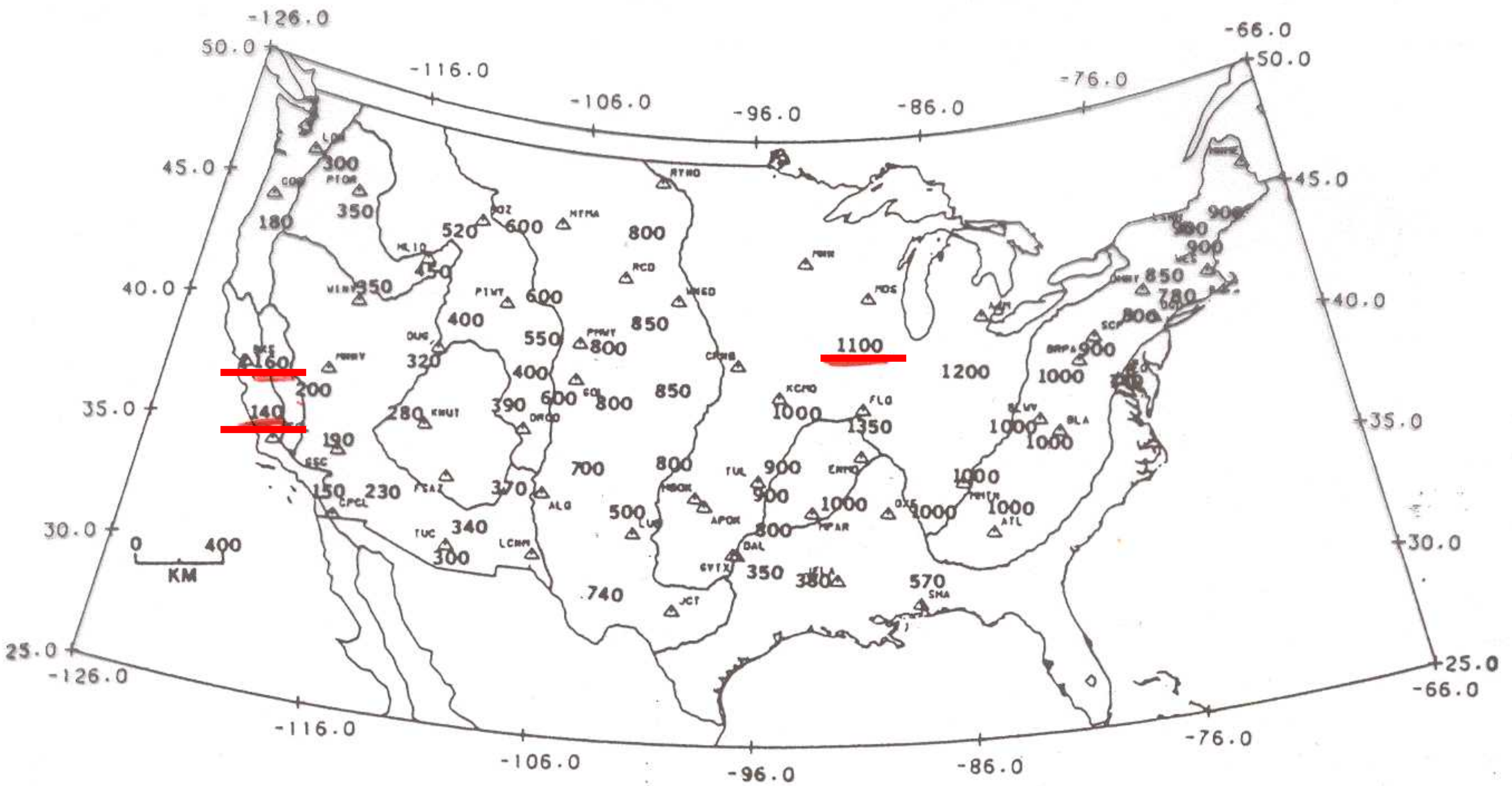
Is seismic Coda mesoscopic ?

Complex medium!

Mesoscopic Physics with Seismic Waves



seismic coda



seismic coda

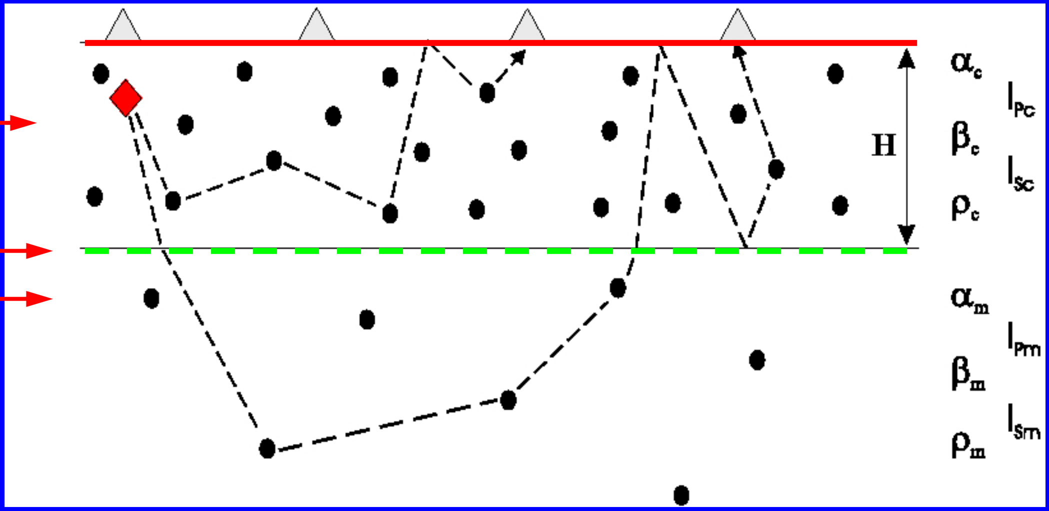
Numerical simulation of Seismic Coda

Margerin, Campillo, Van Tiggelen, Geophys. J. 134, 596 (1998)
Lacombe, Margerin, Trégourès, Campillo, Paul, Van Tiggelen, 2002

crust

Moho

mantle



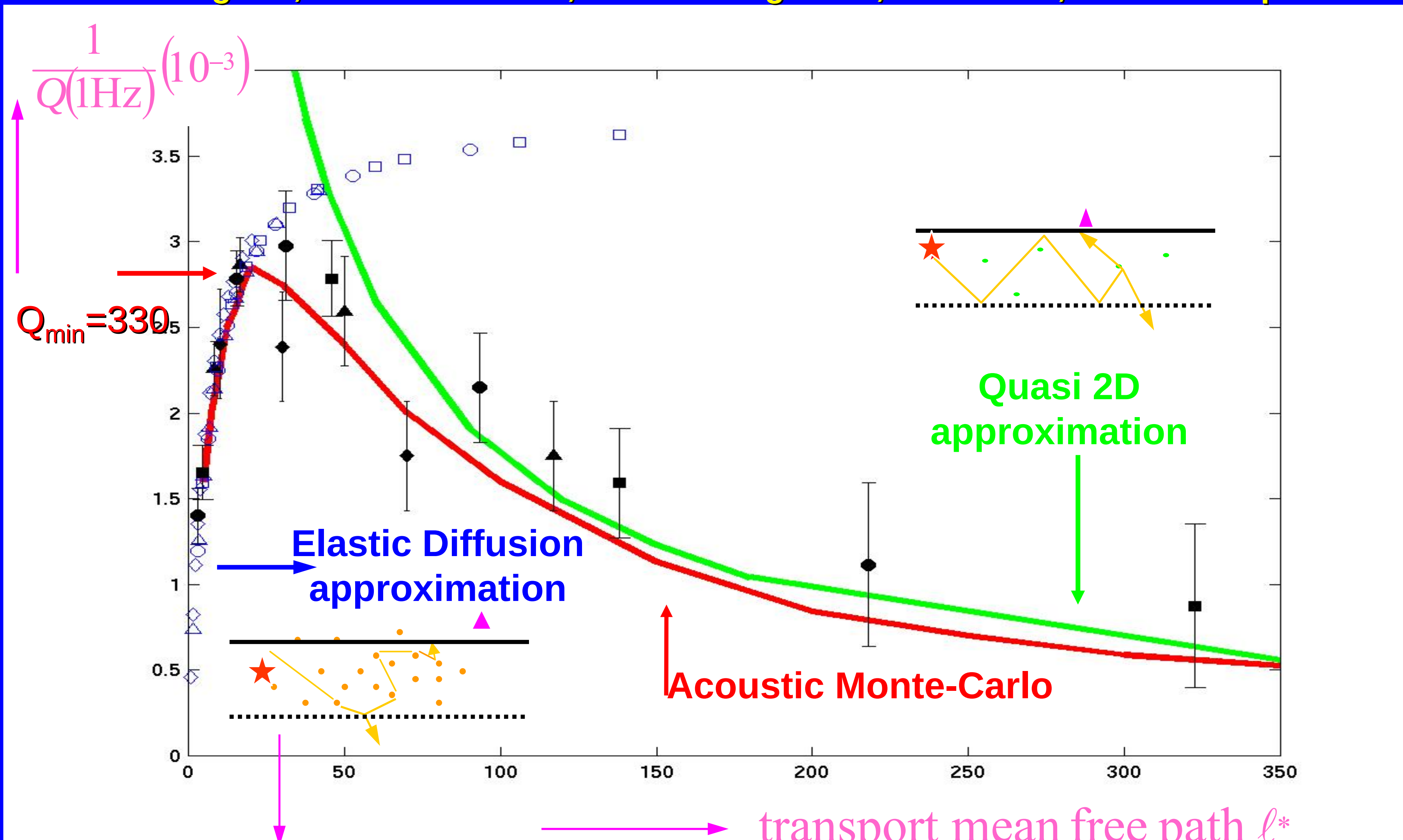
H = 30 km

Internal reflection! Mismatch = 1.3
Mantle = homogeneous

seismic coda

Transfert radiatif des ondes sismiques

ec Ludovic Margerin, Céline Lacombe, Nicolas Trégourès, Anne Paul, Michel Campillo



seismic coda

Equipartition of seismic waves

Multiple Scattering

↓

Uniform Micture of modes {i}
at given frequency

↓

Energy of (i) proportional to DOS (i)

↓

Plane waves in 3D Polarization i Velocity v_i

→

$$\mathrm{DOS}(i) = \frac{\omega^2}{2\pi^2 v_i^3}$$

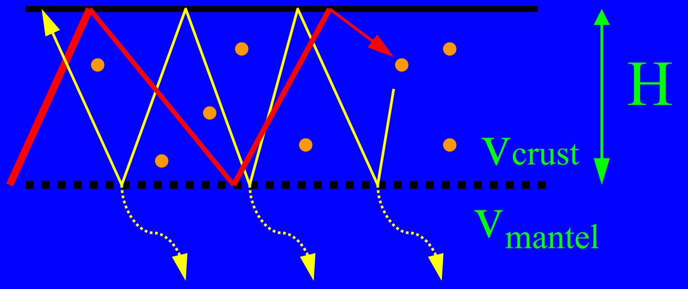
→

$$\frac{\text{Energie } S}{\text{Energie } P} = \frac{2 \times v_S^3}{v_P^3} = 2(\sqrt{3})^3 = 10.4$$

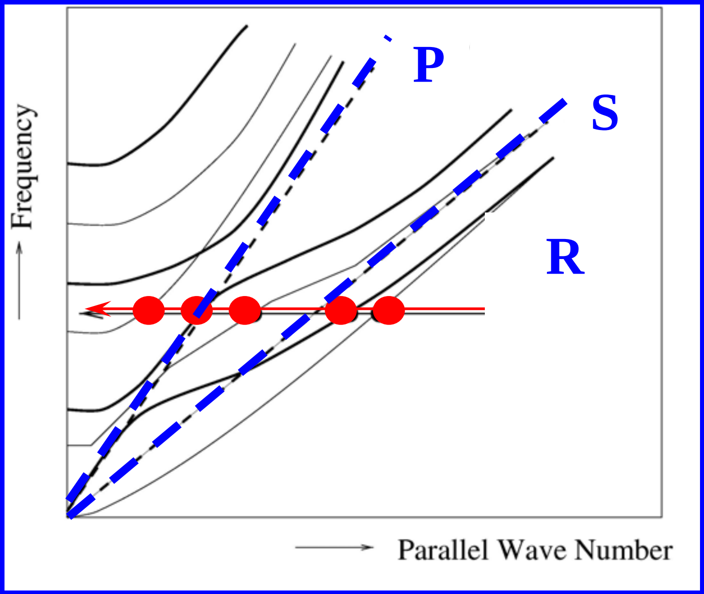
(Weaver, 1982)

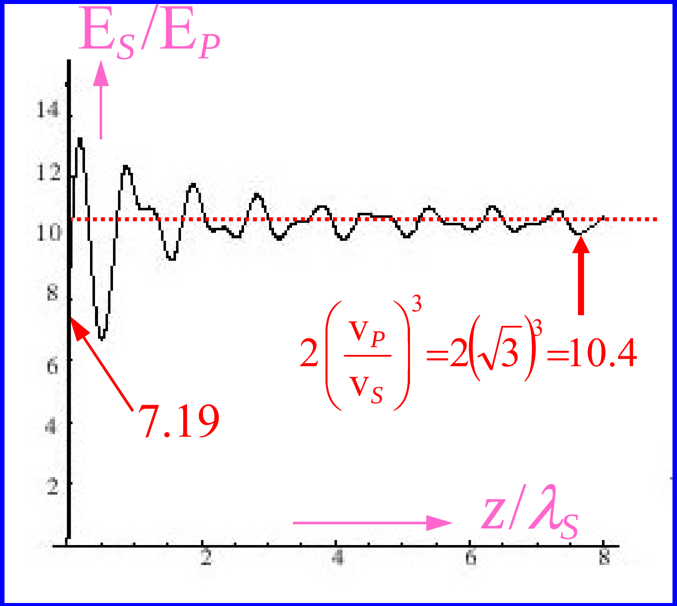
Equipartition of seismic waves

But the real modes, …



… have dispersion….

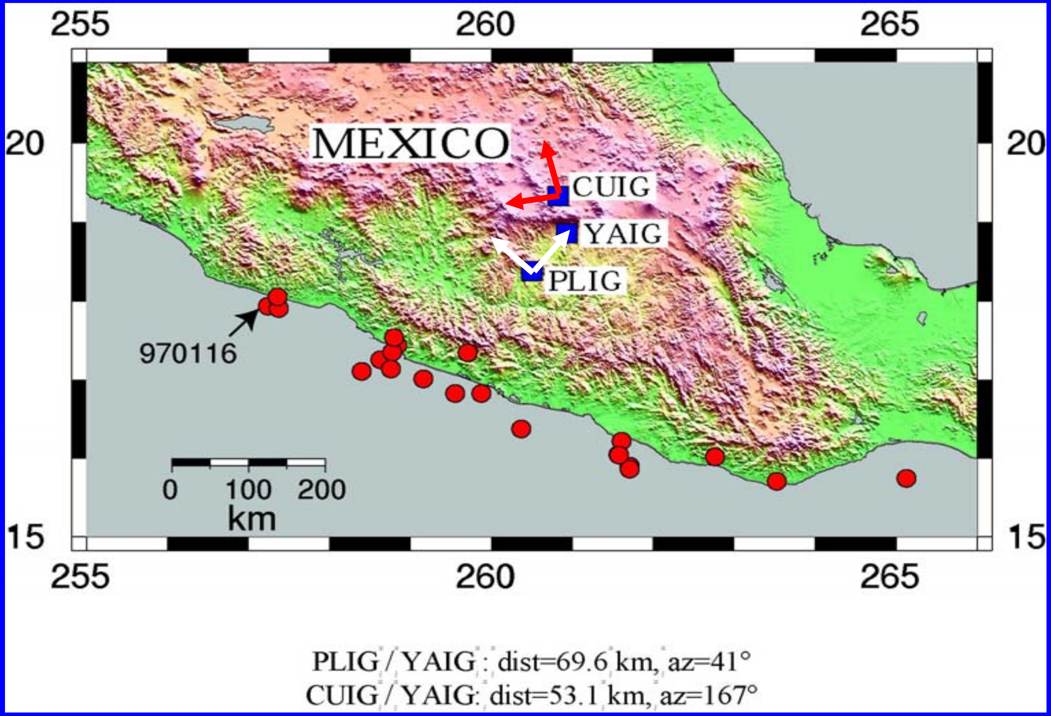


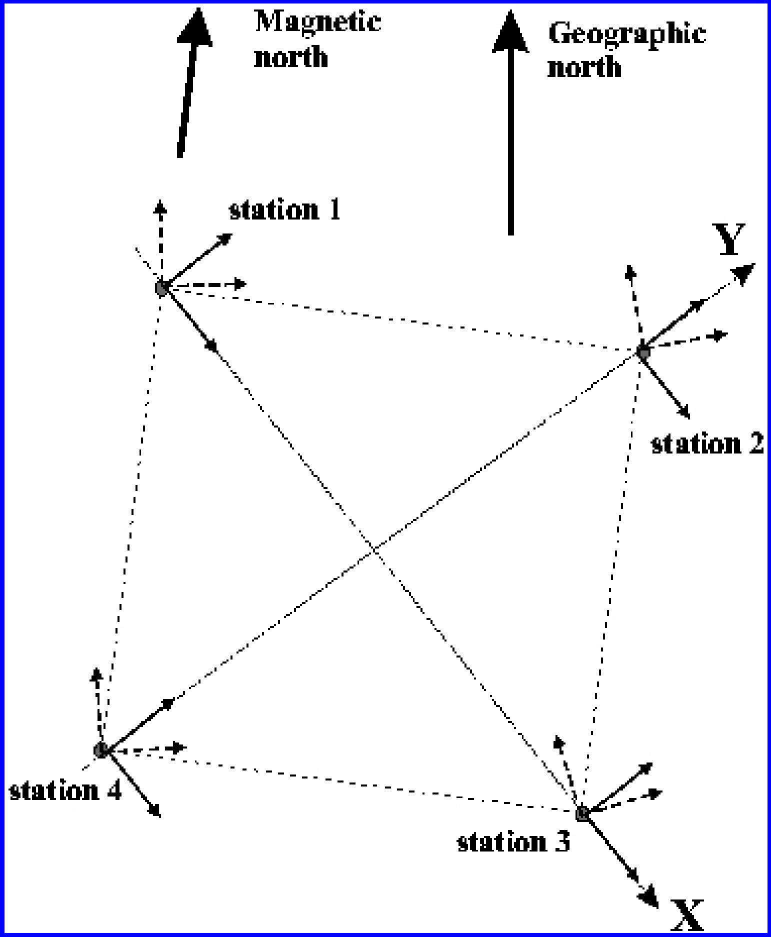


…that affects the ratio S/P(z)…..

seismic coda

Equipartition of seismic waves





Seismic watchers expensive!

A smart collaborator with no support

seismic coda

$$\text{Elastic Energy} = S + P + \underbrace{K}_{H+V} + I$$

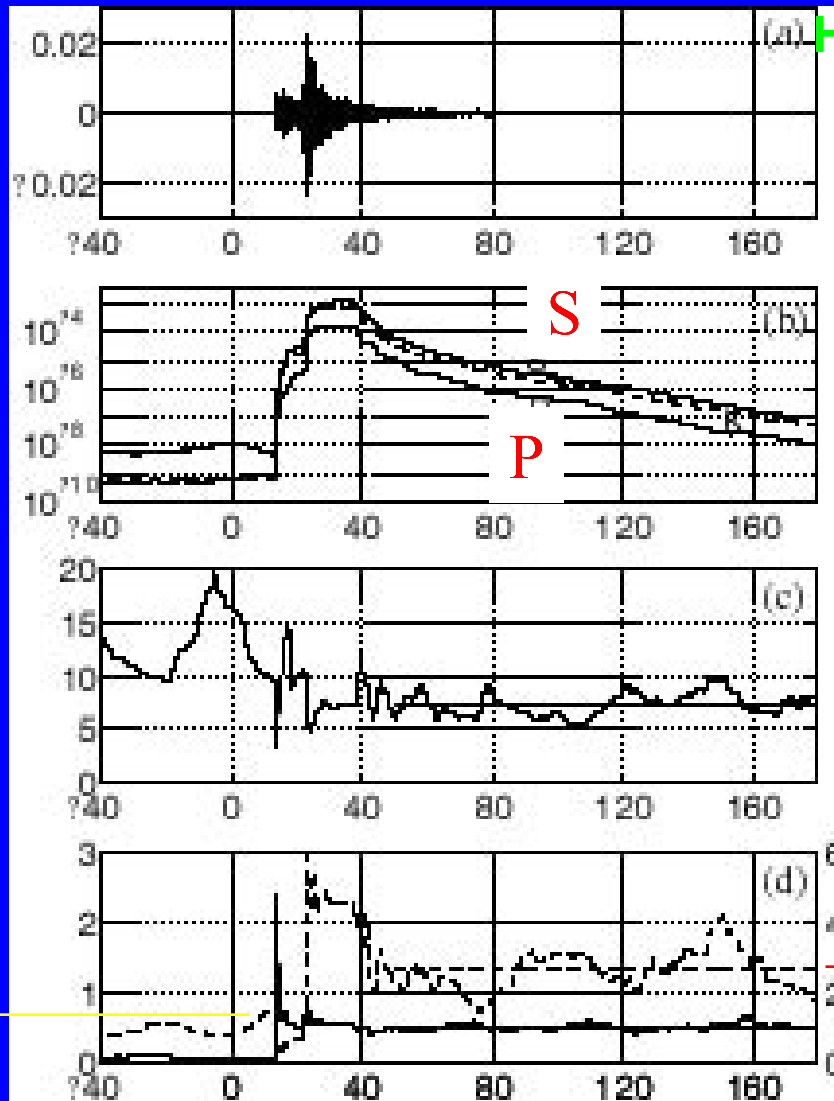
Hennino, Trégourès, Shapiro, Margerin, Campillo, Van Tiggelen, Weaver, PRL. 86, 3447 (2000)

$u_z(t)$

$Energie(t)$

$\frac{S(t)}{P(t)}$

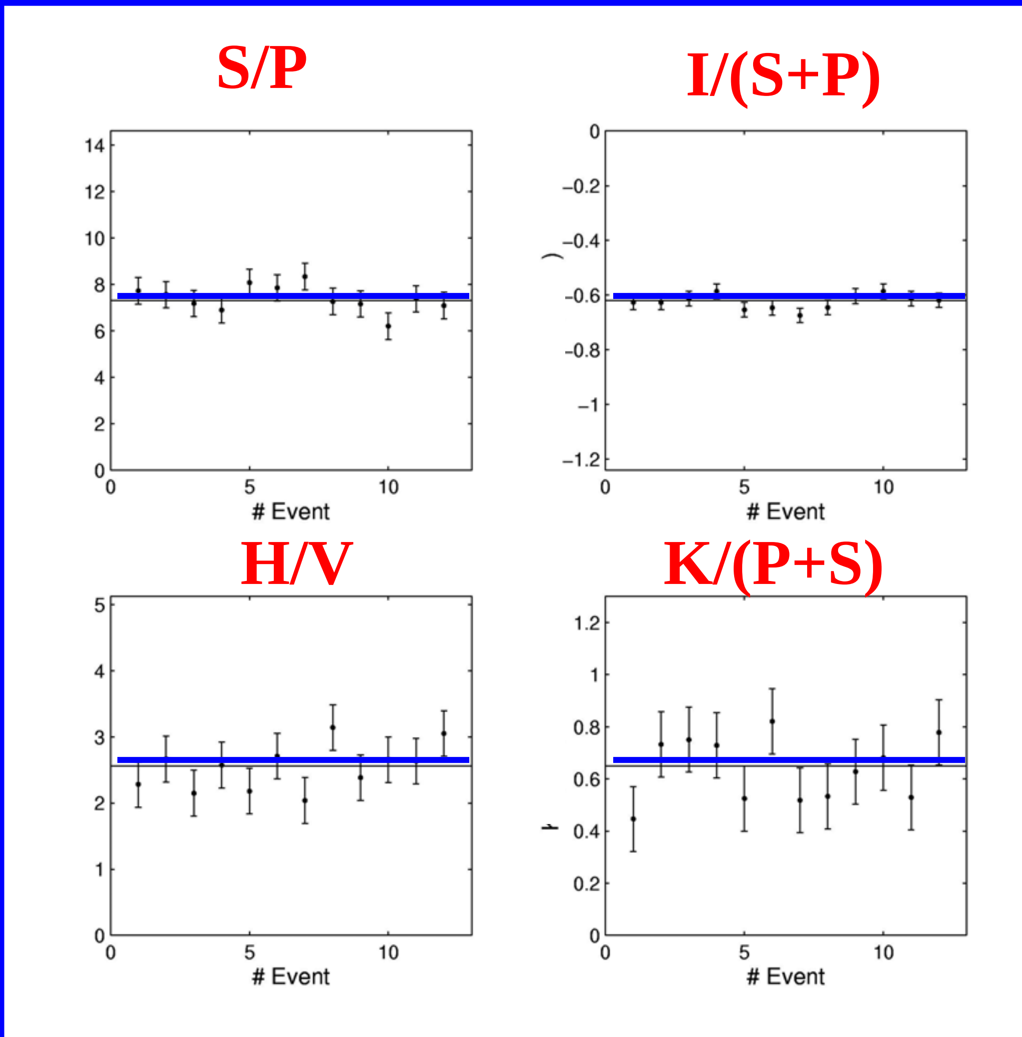
$\frac{Kin(t)}{Pot(t)}$



$\frac{hor(t)}{ver(t)}$

seismic coda

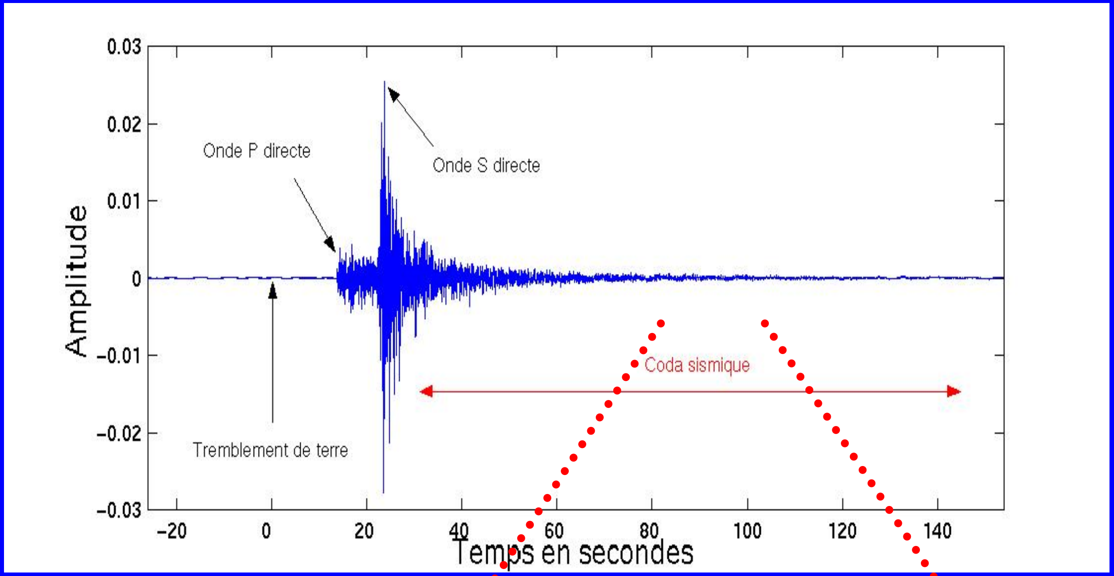
$$\text{Energie Elastique} = S + P + \underbrace{K}_{H+V} + I$$



	obs	théo z=0	théo z=∞	théo R
$\frac{S}{P}$	7.3±0.7	7.2	10.4	6.46
$\frac{K}{S+P}$	0.65±0.1	0.53	1	0.27
$\frac{I}{S+P}$	–0.62 ±0.03	–0.62	0	–1.45
$\frac{H}{V}$	2.56 ±0.36	1.77	1	0.46

Dream 1: Imaging without a source

Equipartition → **Correlation = Green function**



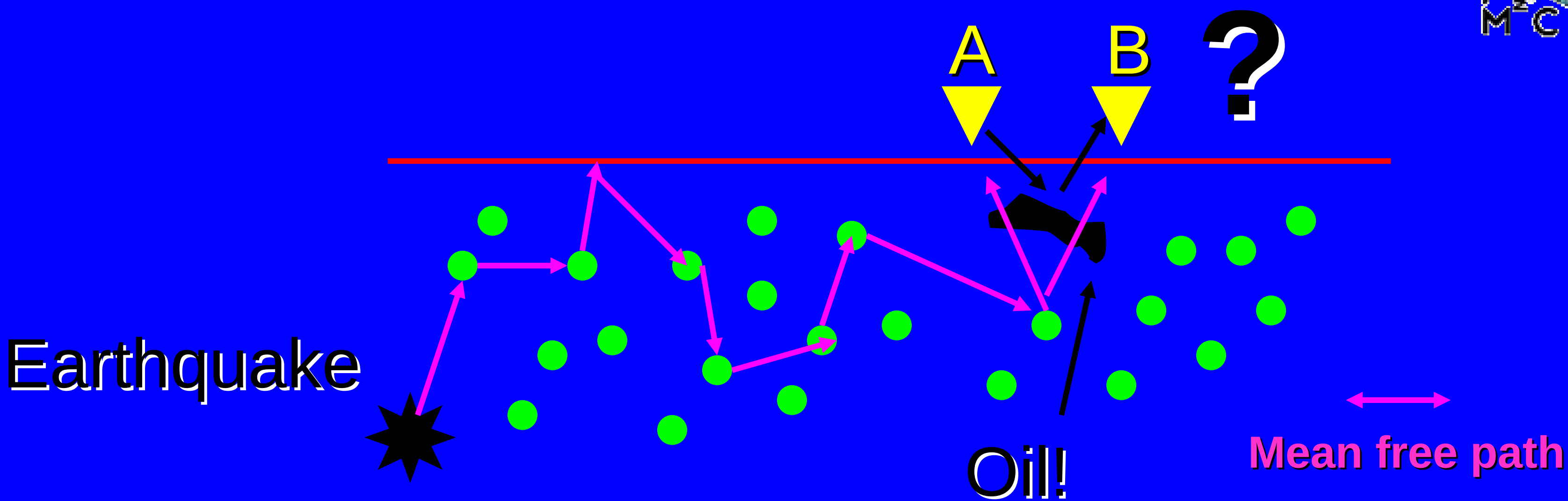
$$\left\langle u\left(\mathbf{r}=A, t-\frac{1}{2}\tau\right) u\left(\mathbf{r}=B, t+\frac{1}{2}\tau\right)\right\rangle$$

$$\propto$$

$$G(A\rightarrow B,\tau)+G(A\rightarrow B,-\tau)$$

Helio-seismology
Duval, Nature 1993
Thermal phonons
Weaver & Lobkis, PRL 2001

and the dream goes on...

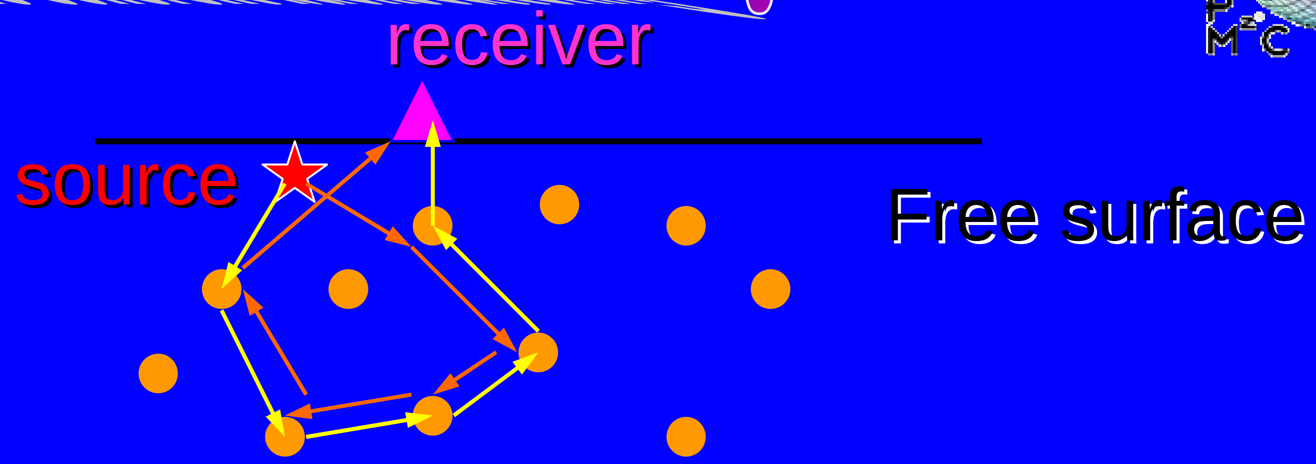


$$\int dt\, A(t-\tau)\, B(t+\tau) = \frac{\partial}{\partial \tau}\left[G_{AB}(\tau) - G_{BA}(-\tau)\right] + \text{speckle}$$

$$\frac{\text{speckle}}{\text{signal}} \approx \sqrt{\frac{\text{Thouless frequency}}{\text{bandwidth}}} \approx \sqrt{\frac{D}{W\, r^2_{\text{source}}}} << 1$$

(Van Tiggelen, 2003)

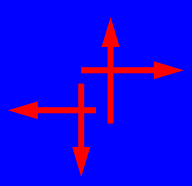
Dream 2: seismic coherent backscattering



1. Distance source receiver < wavelength

$$CBS(r) \propto 1 + \mathrm{sinc}^2\left(\frac{2\pi r}{\lambda}\right)$$

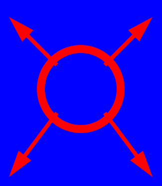
2. Symmetry source = symmetry receiver & magnitude



measure

$$\left|\partial_y u_x + \partial_x u_y\right|^2$$

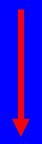
Earth quake



measure

$$\left|\mathrm{div}\,\mathbf{u}\right|^2$$

Explosion



measure

$$\left|u_z\right|^2$$

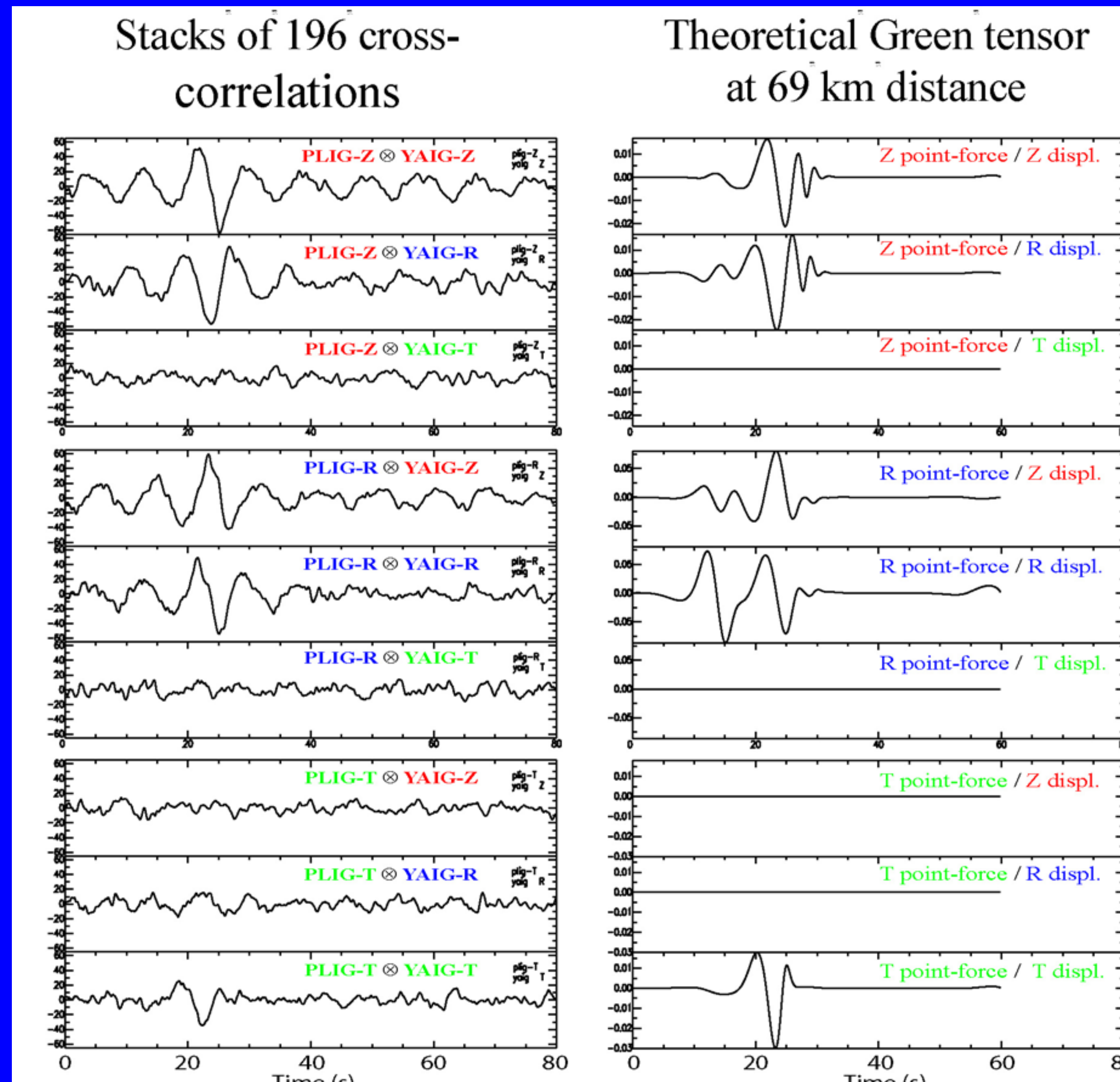
Sledge hammer

← magnitude

but the first is no longer a dream!

Seismic Coda in Mexico

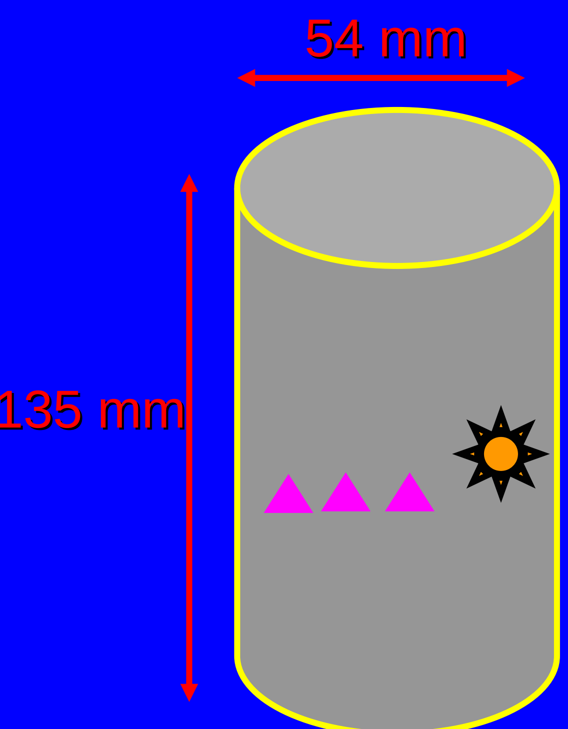
Campillo & Paul
Science, Janvier 2003



and excites new people

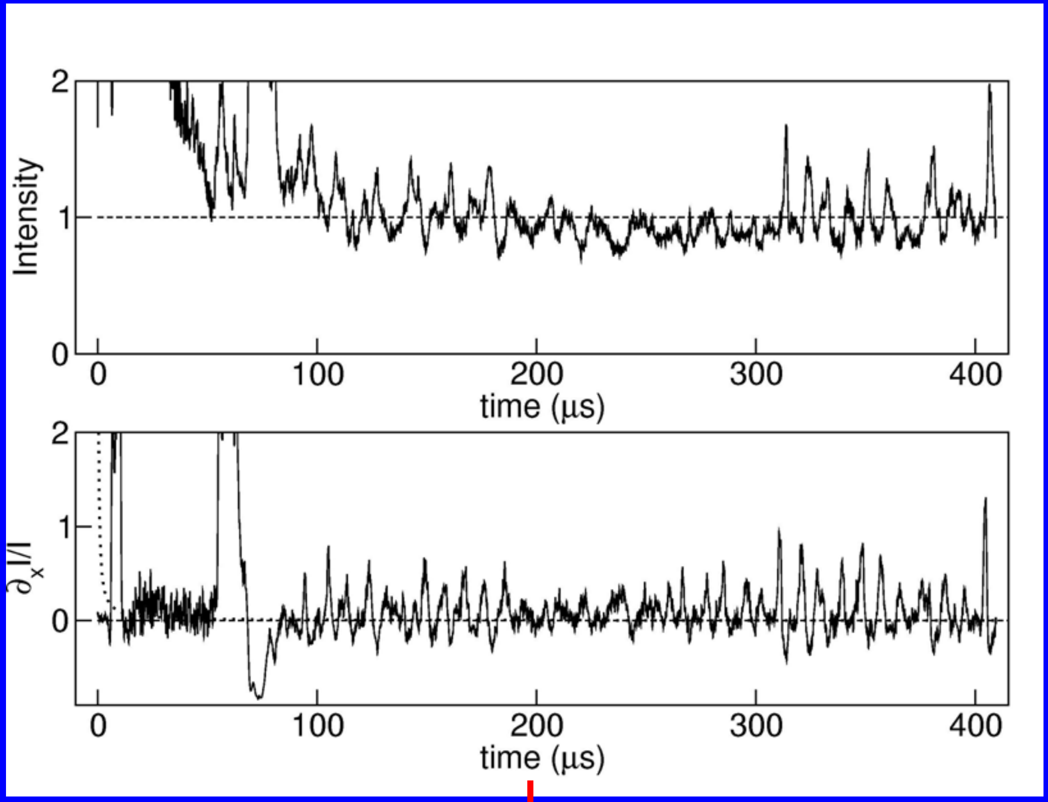
Elastic waves in granite

A. Malcolm, J. Scales & B. van Tiggelen
2004



energy

flux



Mean free time = 3 µs
Mean free path = 10 mm

ballistic **mesoscopic** **noise**

$\sqrt{Dt} \approx 40\,\mathrm{mm}$

and excites new people

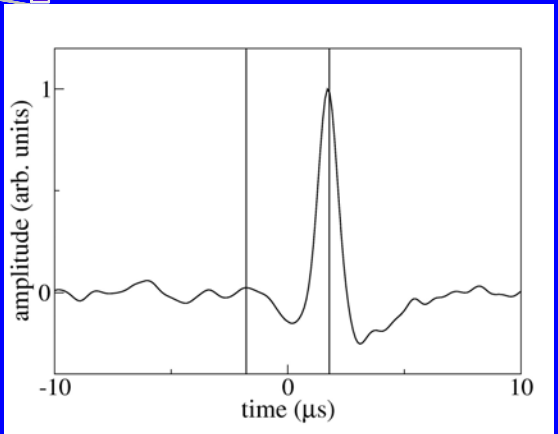
Elastic waves in granite

Time-correlation of elastic motion

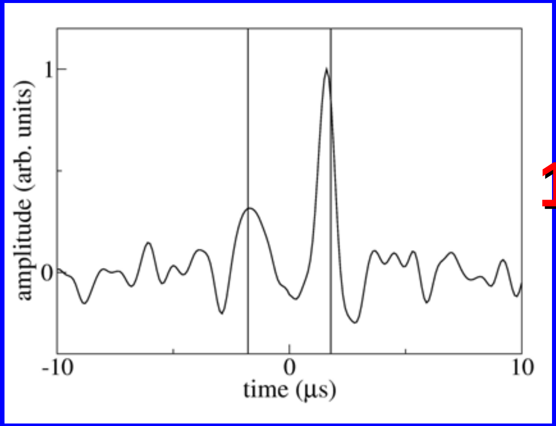
$$\left\langle v\left(\mathbf{r}=A, t-\frac{1}{2}\tau\right) v\left(\mathbf{r}=B, t+\frac{1}{2}\tau\right)\right\rangle$$
$$\propto$$
$$G(A\rightarrow B,\tau) + G(A\rightarrow B,-\tau)$$
$$+$$
$$R(t)\times \left[G(A\rightarrow B,\tau) - G(A\rightarrow B,-\tau)\right]$$

Mean free time = 3 µs

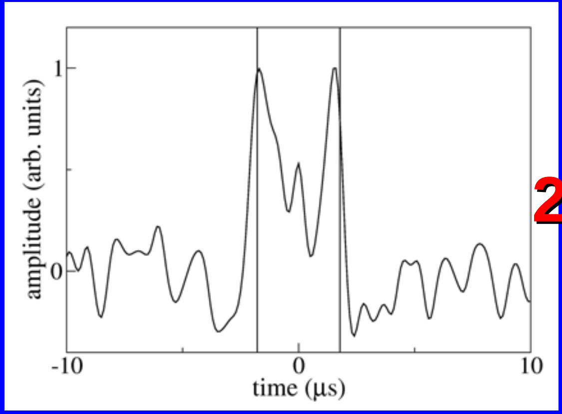
Mean free path = 10 mm



Coda time t < 20 µs



Coda time 12< t < 40 µ



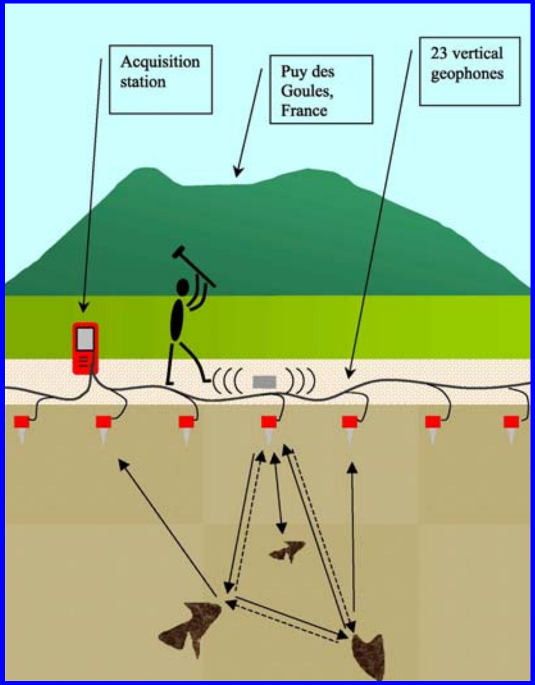
Coda time 25< t < 60 µ

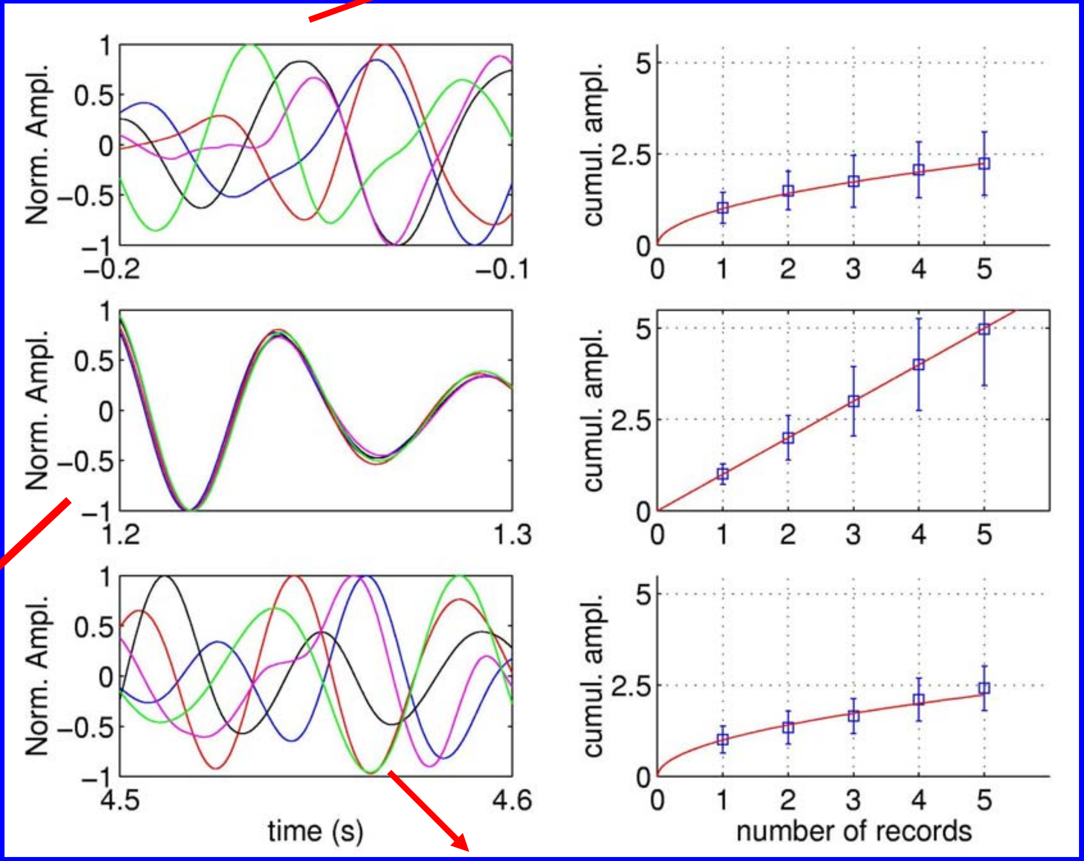
and neither is the second!

Seismic waves in the French Auvergne

Eric Larose, Ludovic Margerin, Michel Campillo et Bart van Tiggelen , 2004

Operator noise

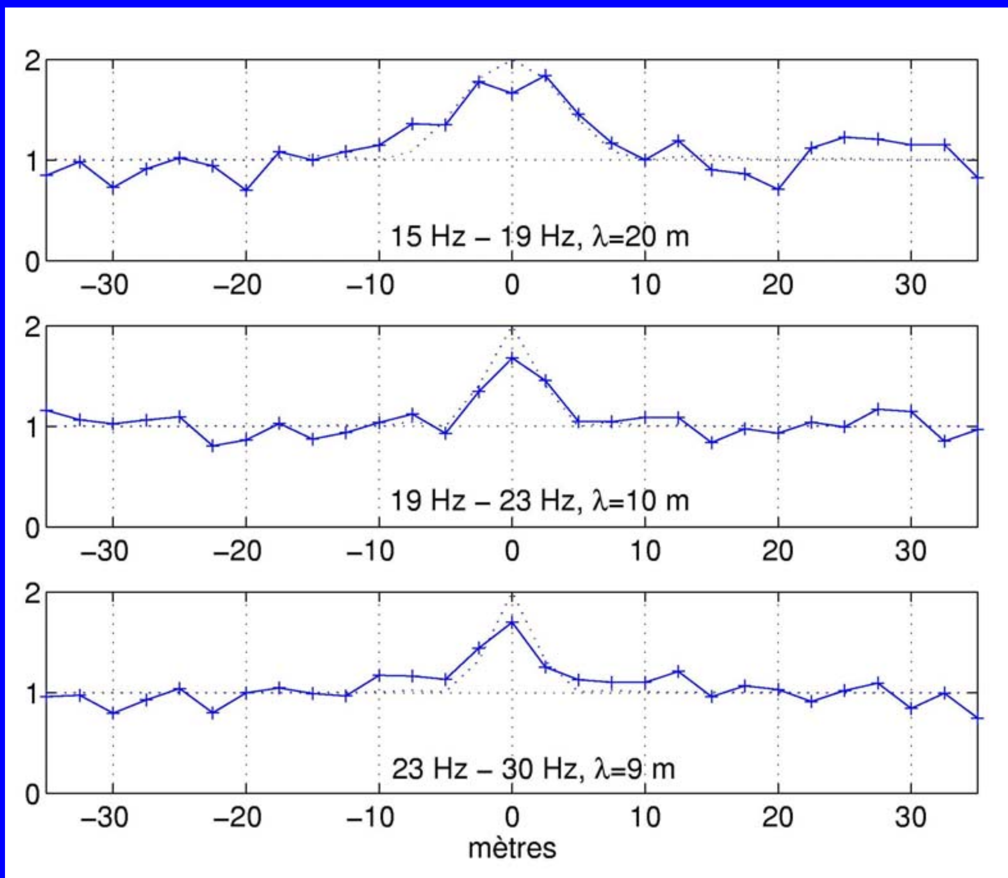




Mesoscopic signal

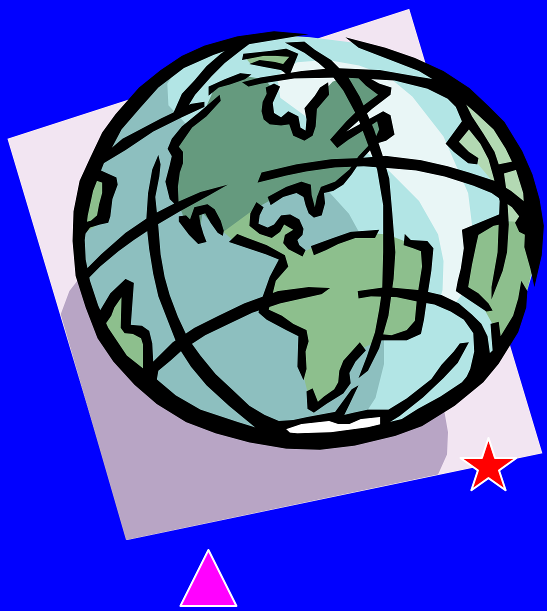
Background noise

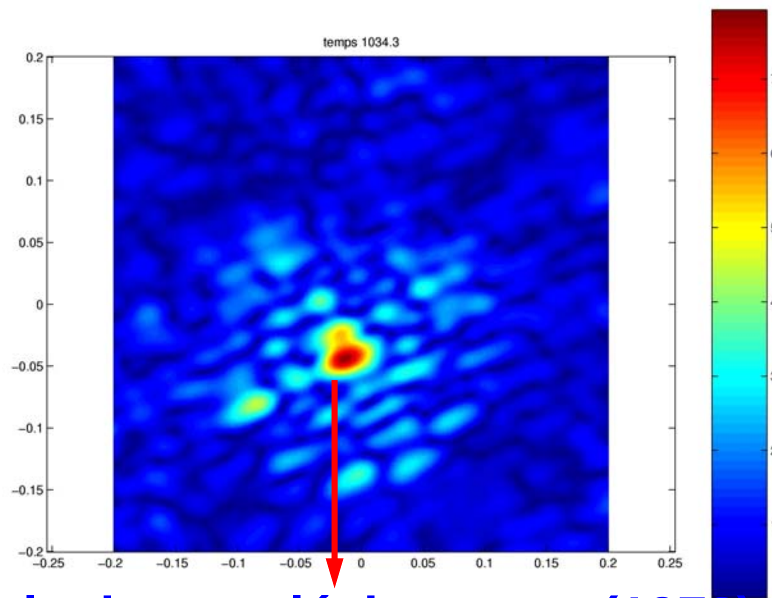
Coherent Backscattering in the French Auvergne



$$\frac{E(r)}{E(r \geq 15m)} = 1 + J_0^2\left(\frac{2\pi r}{\lambda}\right)$$

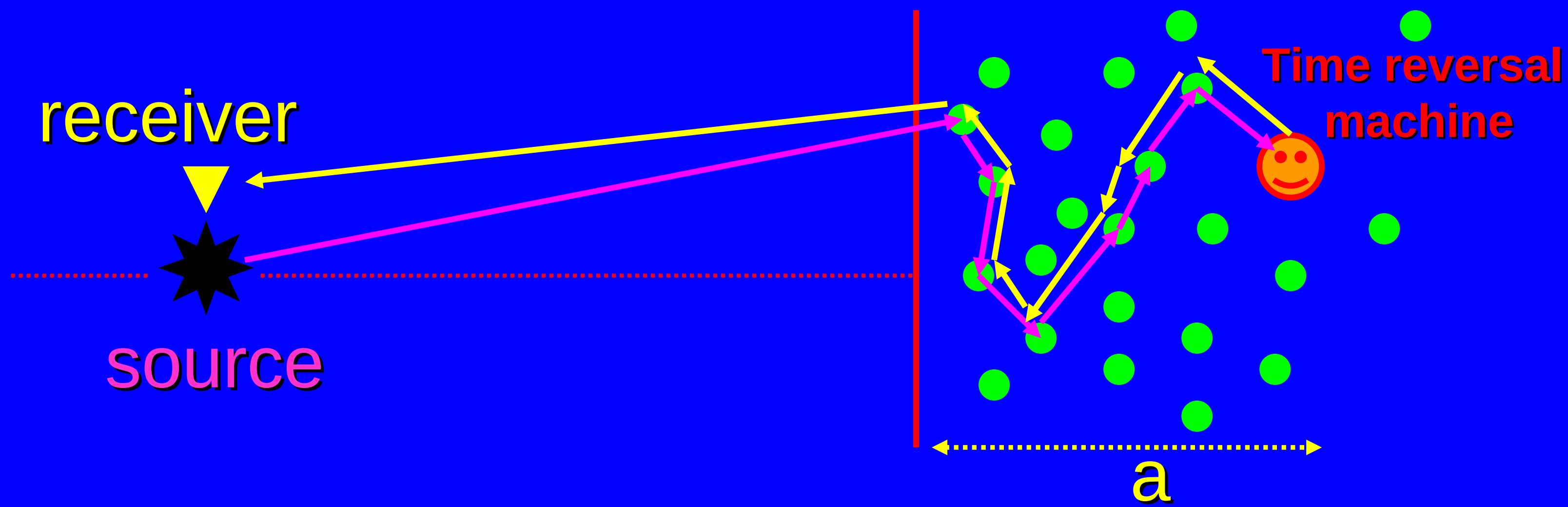
Mesoscopic physics at kilo scale





explosion nucléaire russe (1972) enrégistrée aux US

Relation with Time-Reversal and Coherent backscattering



$$\left[S \to TRM \to R\right](\tau) = \int dt\, \left[TRM \to S\right](t-\tau)\, \left[TRM \to R\right](t+\tau)$$

Time-reversal ⟷ **correlation method**

$$R(z,\tau) = S(\tau) \times \mathrm{CBS}\left(\theta \frac{\ell}{\lambda} \to \theta \frac{a}{\lambda}\right) + \text{speckle}$$

$$\approx \sqrt{\frac{D}{Wa^2}} << 1$$

Stable time-reversal at source……

….. with CBS cusp !!

Comparison to Codas observed in Mexico

argerin, Campillo, Shapiro, Van Tiggelen, Geophys. J. Int. 138, 343 (1999)

$$\frac{1}{Q(f)} = \frac{1}{Q_{scat}(1\,\mathrm{Hz})}\,\frac{1}{f(\mathrm{Hz})} + \frac{1}{Q_{abs}}$$

$$\frac{1}{Q_{scat}(1\,\mathrm{Hz})} \leq 3 \cdot 10^{-3}\left(\frac{30\ \mathrm{km}}{H}\right)$$

$$H \approx 20\mathrm{km} \Rightarrow \begin{cases} \ell^* \approx 10\mathrm{km} \\ \ell^* \approx 70\mathrm{km} \end{cases}$$

$$Q_i \approx 1000 \Rightarrow \tau_{abs} \approx 150\mathrm{sec}$$