Theory of Green function retrieval & smart imaging in a disordered world

towards applied mesoscopic wave physics

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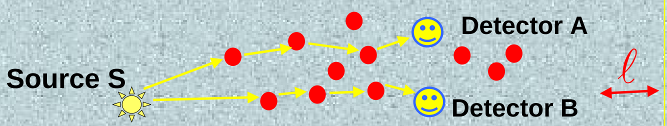
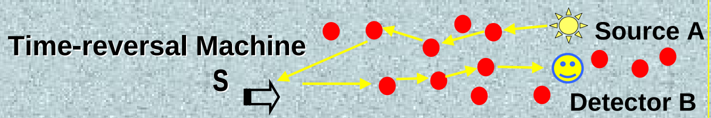
1. Scattered waves have random phase but are not phase-incoherent.
2. Multiple scattering leads to equipartition in phase-space.
3. All waves behave in a similar way.

1) Multiple scattering stabilizes the time-reversal of broad-band signals
2) Field Correlation = time-reversal.
   In fact, everything = time-reversal
3. All waves behave like acoustic waves.

correlation = time-reversal



$$\langle AB\rangle(\tau) = \int_0^\infty dt\,[S\to A](t-\tfrac{\tau}{2})[S\to B](t+\tfrac{\tau}{2}) = \int_0^\infty dt\, G(SA,t-\tfrac{\tau}{2})\,G(SB,t+\tfrac{\tau}{2})\otimes S(t)$$



$$B(\tau) = [A\to S](-t)\otimes(S\to B)(t+\tau) = \int_0^\infty dt\, G(AS,t-\tfrac{\tau}{2})\,G(SB,t+\tfrac{\tau}{2})\otimes A(t)$$

Stability & Symmetry

ω^2

$$\langle AB\rangle(\tau) = \int_{\Delta\omega}\frac{d\omega}{2\pi}\,\exp(-i\omega\tau)\,G(AS,\omega)\,G^*(BS,\omega)\,S(\omega)$$

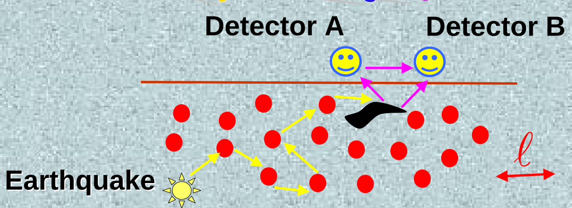
Ensemble-average — speckle

$$\frac{\text{broadband speckle}}{\text{ensemble average}} \approx \sqrt{\frac{v\ell}{r^2\Delta\omega}} << 1$$

$$\langle AB\rangle(\tau) = \rho(\mathbf{r})\frac{\partial}{\partial\tau}\left[G_{AB}(\mathbf{x},\tau)-G_{BA}(\mathbf{x},-\tau)\right] - v\ell\,\frac{\partial}{\partial\mathbf{r}}\rho(\mathbf{r})\bullet\frac{\partial}{\partial\mathbf{x}}\left[G_{AB}(\mathbf{x},\tau)-G_{BA}(\mathbf{x},-\tau)\right]$$

Symmetric in time (equipartition) + Anti-Symmetric in time (current from source)

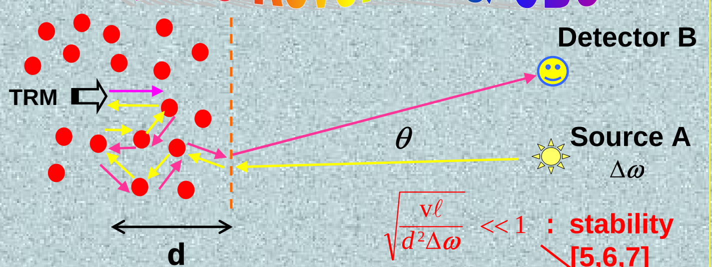
Mesoscopic Imaging



$$\langle AB\rangle(\tau) = \rho(\mathbf{r})\frac{\partial}{\partial\tau}\left[G_{AB}(\tau)-G_{BA}(\mathbf{x},-\tau)\right] + \text{speckle}$$

Direct Rayleigh wave [3]+ nearby ℓ) objects [5]

Time-Reversal & CBS



$\sqrt{\frac{v\ell}{d^2\Delta\omega}} << 1$: stability [5,6,7]

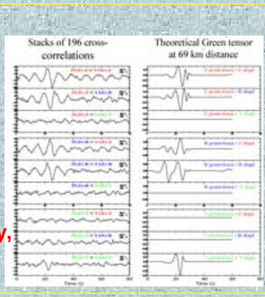
$$B(\tau) = A(\tau)\,\rho(B) + \text{speckle} = A(\tau)\,\frac{\ell}{d}\exp\left[-d\,k\,|\theta|\right] + \text{speckle}$$

Temporal Focalization [7] Spatial compression Cusp CBS ! 10]

Applications

In seismology [1,3]
and in ultrasound [2,8,9]

Role of source, ensemble-average, symmetry, different modes,....



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