# Estimating Scattering Strength from the Transition to Equipartitioning S31B-1053


Alison E. Malcolm*, John A. Scales**, and Bart A. van Tiggelen†

*Center for Wave Phenomena, Colorado School of Mines, Golden CO.
**Physical Acoustics Laboratory, Colorado School of Mines, Golden CO.
†Laboratoire de Physique et Modélisation des Milieux Condensés CNRS Maison des Magistères
Université Joseph Fourier. BP 166 38042 Grenoble Cedex 9 France.

**Electronic address: amalcolm@dix.mines.edu Web: http://acoustics.mines.edu**


## Introduction

It has long been known (e.g, Lee (1960)) that driving a linear system with random noise and correlating the output with the random input yields the Green function (GF) for the system; this is a special case of a matched filter. Surprisingly, it is also true that under certain circumstances, cross-correlating the output recorded at two locations yields the Green function between locations, as if one or the other of the outputs was actually an impulsive input. In other words, the cross-correlation is able to synthesize the result of an experiment not performed. For this to occur, the wave field must be equipartitioned; that is, there must be no preferred wave vector. In fact, the basic result is that the cross-correlation of the equipartitioned field at two detectors gives the sum of the advanced and retarded GFs (Lobkis & Weaver (2001)). This idea has been exploited recently in several fields (Campillo & Paul (2003); Duvall *et al.* (1993); Rickett & Claerbout (1996); Lobkis & Weaver (2001); Weaver & Lobkis (2001)) to retrieve the GF from a randomized field. Here we use this idea to extract the GF between two detectors in a rock sample and to study the emergence of equipartitioning.

Imagine observing an ultrasonic pulse some distance away from the source position. If intrinsic attenuation is sufficiently small relative to scattering the wavefield will go through three distinct regimes before relaxing to the ambient level. First, all of the source energy will travel together as a packet, in the ballistic regime. In this regime the correlation gives the advanced GF only. Second, some energy will begin to travel away from the packet back toward the source while the net energy still travels away from the source; this is the diffusive regime. In the diffusion regime, both the advanced and the retarded GFs are present in the correlation but the retarded GF has much lower amplitude. Thirdly, scattering will dominate to the point that an equal amount of energy travels in all directions; this is the equipartitioned regime. In this regime, the advanced and retarded GFs have equal amplitude in the correlation.

## Experiment

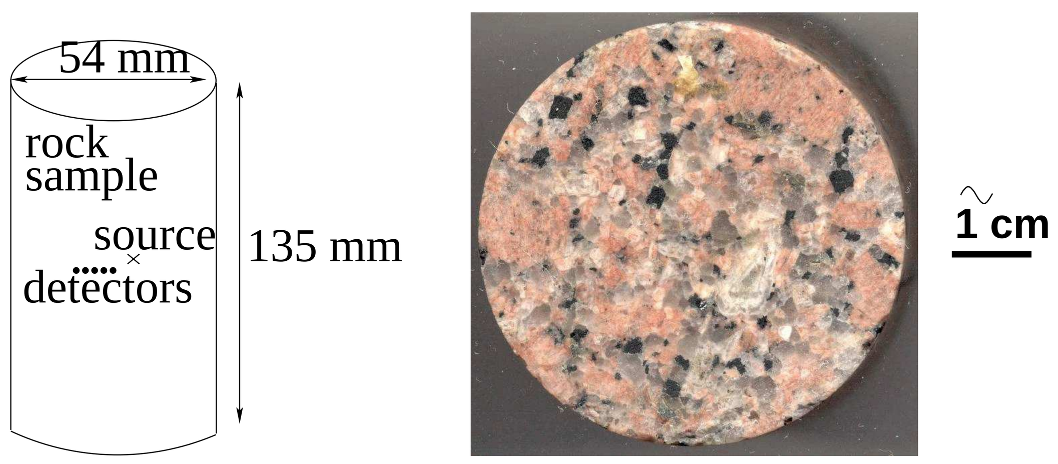


Figure 1. The experimental setup, shown here on the left was repeated every 5° on nine different horizontal lines separated by about 5 mm vertically. Each angle on each line represents an independent member of the ensemble. On the right is a picture of the Llano granite sample used in this experiment. This sample is strongly scattering because the average grain size is larger than the wavelength, shown beside the picture.

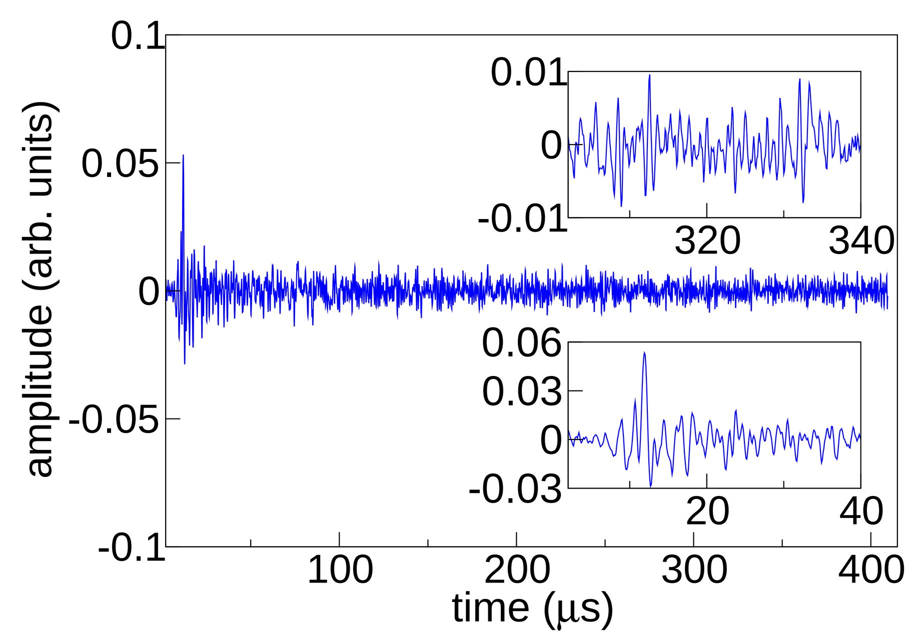


Figure 2. This is an example of data collected, at early times distinct arrivals can be seen whereas at later times, the wavefield appears random although it is still above the background noise level.

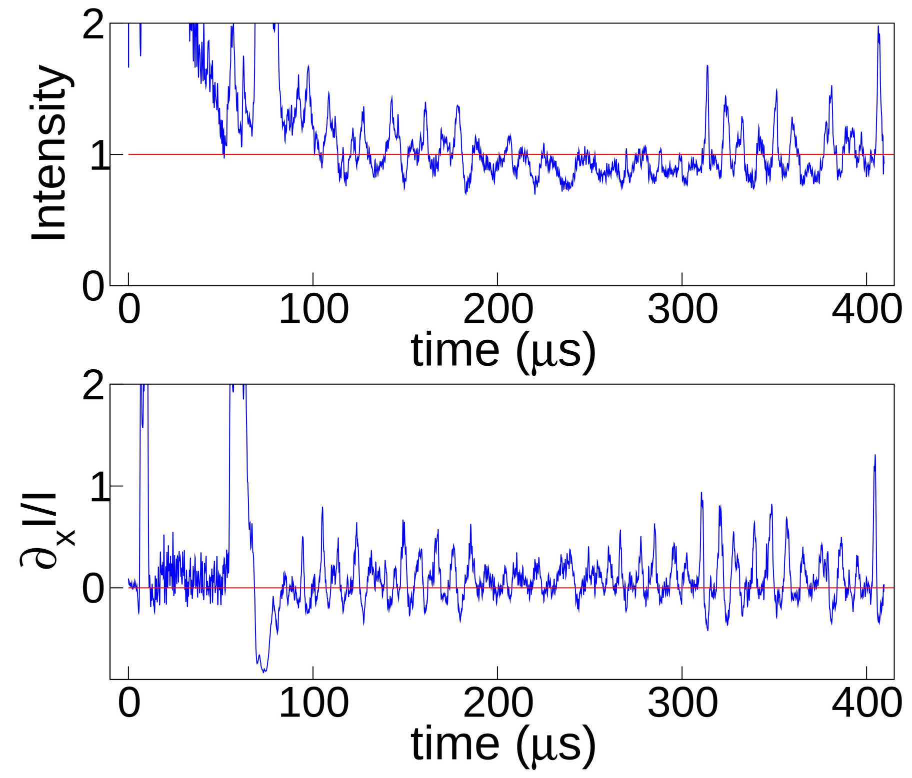


Figure 3. To estimate when the wavefield becomes equipartitioned, we computed the the 1-D spatial gradient of the intensity shown above; when this quantity has decayed to zero, the wavefield is equipartitioned. The intensity itself, shown in the top plot, decays to zero after approximately 100 $\mu s$, which gives an estimate of when the signal has decayed to the ambient level.

## Symmetry

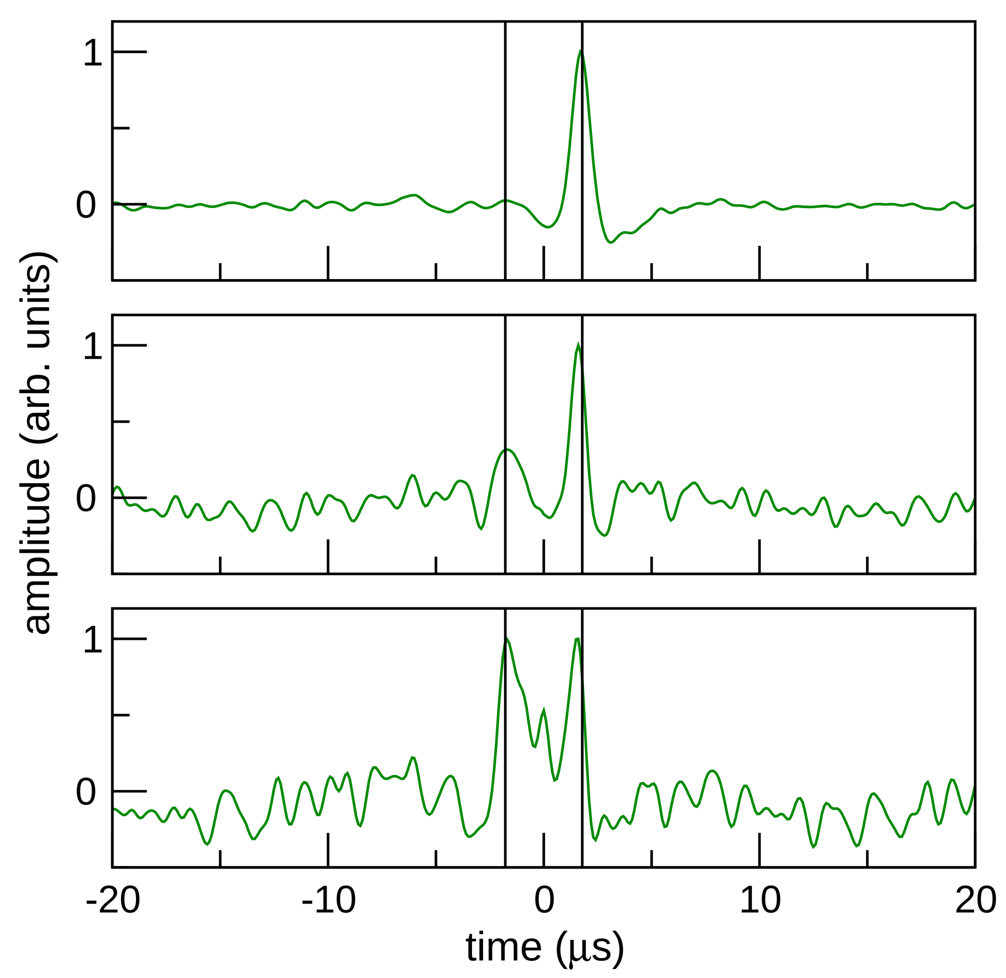


Figure 4. Each plot above shows the cross-correlation of the wavefield at two detectors, 5 mm apart, using data in different time windows. In the top plot, which uses data from the earliest time window, the data are in the ballistic regime, so all of the energy is traveling away from the source resulting in a large spike at positive time. At later times, energy is scattered back toward the source, resulting in a spike at negative time as seen in the second panel. Eventually, in the EP regime, equal amounts of energy are propagating in all directions and the correlation becomes symmetric as in the third plot. The vertical lines are at the expected arrival time of the Rayleigh wave. Theoretically, the correlation should be a weighted sum of the advanced and retarded Green functions (Malcolm *et al.* (2004)), with the weights becoming equal once the field is EP.

## The Green Function

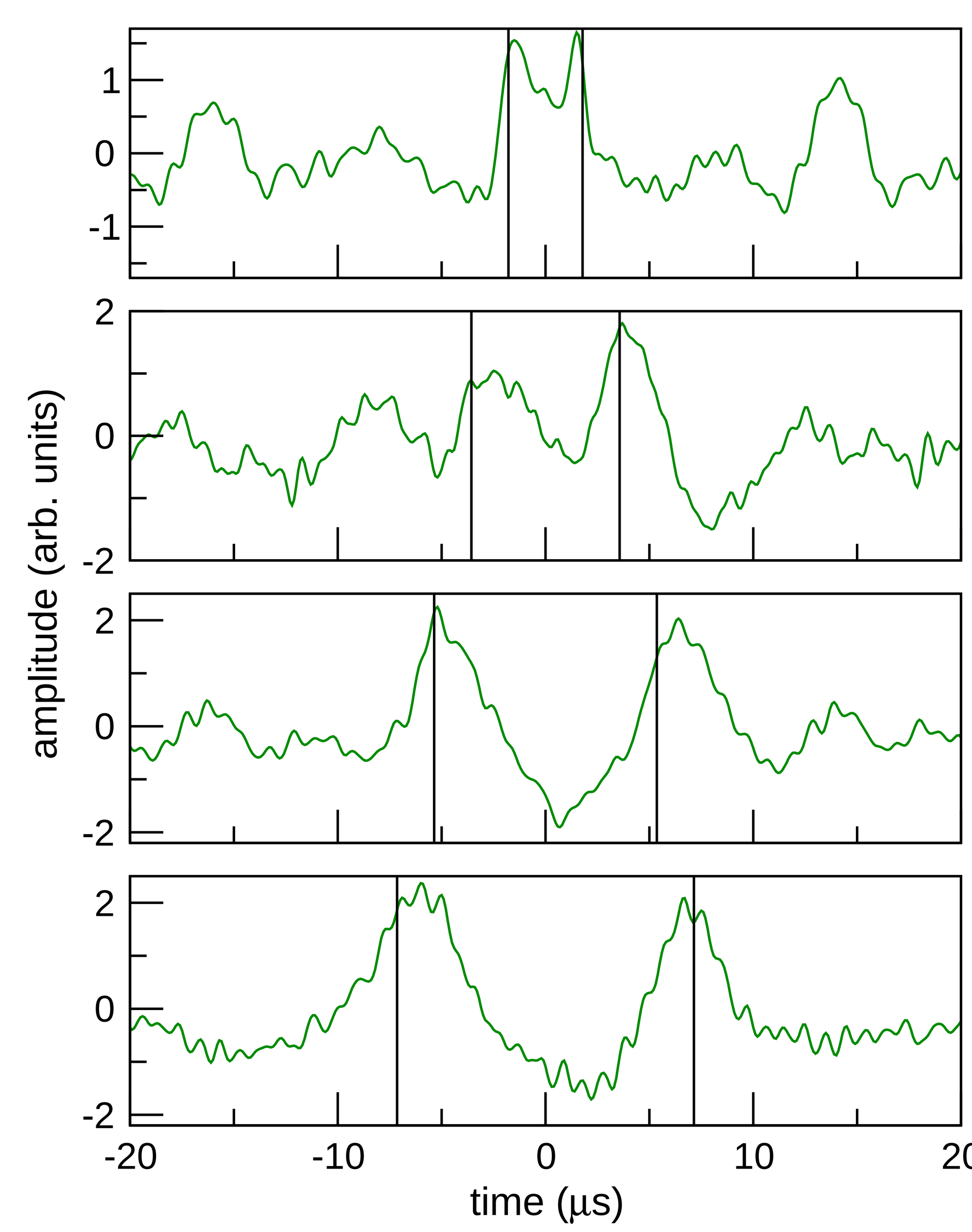


Figure 5. Beginning at the top, the four plots above show the correlation of the signal with detector separations of 5, 10, 15 and 20 mm. The linear change in the arrival time of the largest spike is at the same time as the estimated Rayleigh wave arrival time, shown with the vertical lines. This indicates that the correlation is equal to the GF, as predicted by theory (Lobkis & Weaver (2001)).

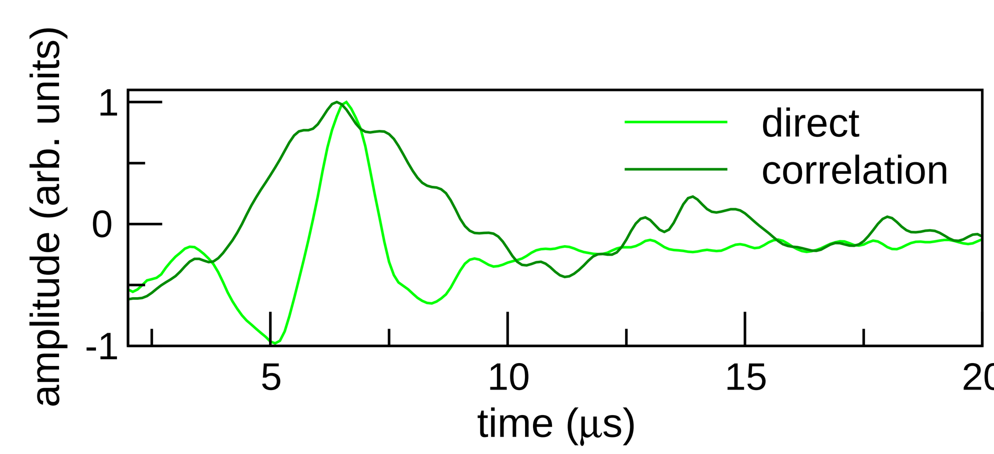


Figure 6. To confirm that we indeed retrieve the GF through the correlation of the field, we measured the GF directly using the laser source. (See Scales & Malcolm (2003) for an overview of the experimental setup.) We see that there is a good kinematic agreement between the two. The difference in frequency content is due to the frequency spectrum of the source which is squared in the retrieved GF.

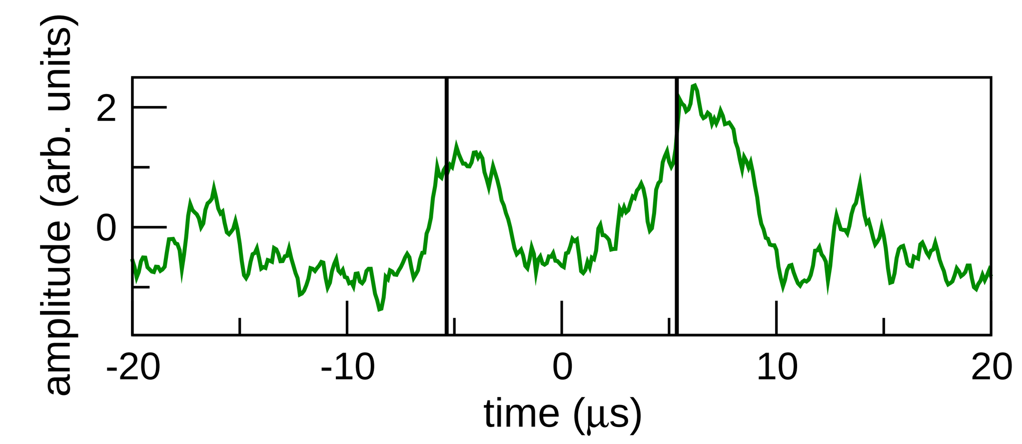


Figure 7. Theoretically it is only the phase that plays a role in extracting the GF. To test this, we resampled the data to single bit precision and again computed the correlation, for a detector separation of 15 mm. The GF shown here should be compared to the 15 mm GF shown in Figure 5.

## Conclusions

- The symmetry of the correlation function of the signal at two detectors is an indicator of equipartitioning.
- The ensemble-averaged cross-correlation of the signal at two detectors is equal to a weighted sum of the advanced and retarded Green functions.
- Correlating data with only single-bit precision gives the same result as higher precision data.

## Acknowledgments


We acknowledge many useful discussion with Roel Snieder, Kasper van Wijk, Alex Grêt, Matt Haney, Huub Douma, and Carlos Pacheco of CSM and Michel Campillo, Eric Larose and Ludovic Margerin of LGIT. This work was partially supported by the National Science Foundation (EAR-0111804), the US Army Research Office (DAAG55-98-1-0070), the Center for Wave Phenomena, and ACI 2066 of the French Department for Research.


### REFERENCES


Campillo, M., & Paul, A. 2003. Long-Range Correlations in the Diffuse Seismic Coda. *Science*, **299**, 547–549.

Duvall, T. L., Jefferies, S. M., Harvey, J. W., & Pomerantz, M. A. 1993. Time-distance helioseismology. *Nature*, **362**, 430.

Lee, Y.W. 1960. *Statistical theory of communication.* John Wiley.

Lobkis, O. I., & Weaver, R. L. 2001. On the emergence of the Green's function in the correlations of a diffuse field. *J. Acoust. Soc. Am.*, **110**(6).

Malcolm, A. E., Scales, J. A., & van Tiggelen, B. A. 2004. Extracting the Green function from diffuse, equipartitioned waves. *Physical Review E*, **70**(1), 015601.

Rickett, J., & Claerbout, J. 1996. Passive seismic imaging applied to synthetic data. *SEP*-**92**, 83.

Scales, J. A., & Malcolm, A. E. 2003. Laser characterization of ultrasonic wave propagation in random media. *Phys. Rev. E*, **67**, 046618.

Weaver, R. L., & Lobkis, O. I. 2001. Ultrasonics without a Source: Thermal Fluctuation Correlations at MHz Frequencies. *Phys. Rev. Lett.*, **87**(13).


This poster was prepared with Brian Wolven's Poster LaTeX macros v2.1.

Fig. 1.—

Fig. 2.—

Fig. 3.—

Fig. 4.—

Fig. 5.—

Fig. 6.—

Fig. 7.—