Synthesis of Green functions from Coda Correlation
from ultrasonics to seimology.

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Abstract : We present a passive imaging technique based on correlation of coda waves. The aim of this technique is to measure the impulse response between two points with passive sensors (no source neither on one point nor on the other). We will take advantage of the scattered waves contained in the Coda with no need of controlling the source.

Introduction : Most conventional imaging process's are based on the direct pulse/echo measurement. In various domains, especially for seismic waves in the crust or in volcanoes, this measurement is prevented by scattering attenuation. Anyway, in seismology it is very hard to handle high energy sources, that is why the Synthesis of Green functions from Coda Correlation we are presenting here is of high interest.

In their pioneering experiments, Weaver and Lobkis retrieved the elastic Green Function in a chaotic cavity at ultrasonic frequencies. The mathematical demonstration was based on discrete modal expansion. We show that this technique remains valid in open scattering medium where modal expansion is not valid, and should apply to seismic imaging. Recent works in seismology show that cross-correlation functions recorded at two points and averaged for several earthquakes contain deterministic arrivals. To go further, we have designed ultrasonic laboratory experiments to test the feasibility of imaging from correlations of coda waves. We show that our passive imaging technique developed here allows the precise measure of velocity between two sensors in a medium with velocity changes. The reflection from a strong interface can be detected as well as the diffraction from a single scatterer.

In practice the quality of the reconstruction is controlled by the duration of the time windows, by the number of sources but also by the spatial distribution of sources and scaterers relative to the two sensor locations. This is made easily understandable by the analogy between the empirical synthesis of Green function from field correlation and time-reversal experiments (Derode et al [98 99]). This analogy gives also a physical interpretation for the loss of time symmetry of the cross-correlations observed both with seismological data and in the laboratory.

These ultrasonic experiments were devoted to seismology, but apply to all domains of wave physics.

Imaging of a velocity contrast : Experiment 1

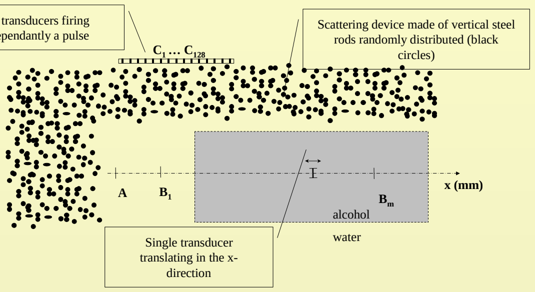

Figure 1: Laboratory configuration of the first experiment. Behind the scattering slab is the medium to image: a plastic bag filled with alcohol. The whole setup is immersed in a water tank. Sources C are analogue to earthquakes. They fire a 1 µs long pulse the one after the other. The field is recorded each time in A and we get $h_{CA}(t)$. Then the receiving transducer is moved to B_1 and we get the 128 $h_{CB_1}(t)$ impulse responses, and so on for all the B_i positions.

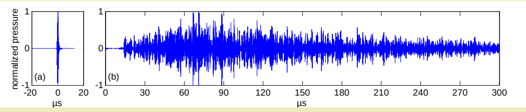

Figure 2: (a) Sources C are emitting a 1 µs long pulse at 3 MHz central frequency. (b) Example of one impulse response $h_{CA}(t)$ obtained through the scattering medium. The long lasting signal following ballistic waves is called « CODA » in seismology. It is essentially made of multiply reflected/refracted waves. In such media, conventional imaging process almost fail. The data processing is as follow :

1) For each earthquake C, we compute the cross-correlation of the field in the coda :

$$C_C(t) = \int h_{CA}(t+\tau)h_{CB}(t)dt$$

2) We compute an average over the sources: $C_{AB}(t) = \sum_c C_{ABC}(t)$

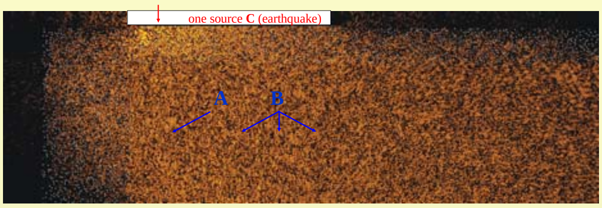

Figure 3: Snapshot of the field amplitude through the scattering medium after 200 µs of propagation. This image is obtained using a 2D numerical simulation of ultrasonic wave propagation. White points are rigid scatterers. The source was placed in C (red arrow). As in the laboratory experiment, the field is sensed in A and all other aligned B points. The field (in logarithmic scale) is diffused, with no direct wave front between A and B.

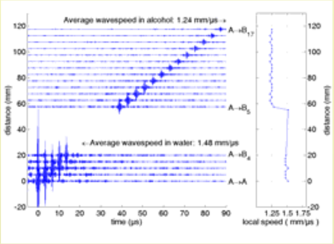

Figure 4: On the left, we plot reconstructed impulse responses $h_{AB_i}(t)$ obtained by CCI principle for different positions of B_i. The lowest signal is corresponding to the impulse response $h_{AA}(t)$ as if A were a source. The wave paquet then diverges from A and propagates at a velocity corresponding to the local medium : first water (1.48 mm/µs) then alcohol (1.24 mm/µs). Local velocities are plotted on the right. They are calculated using arrival times for each couple of neighboring points. There was not enough sources C to reconstruct the rest of the impulse response (especially reflections on the first steel rods). Because the moving transducer was 40 mm thick, it was also not possible to record the field for positions ranging from x=25 mm to x=55 mm. Results are corresponding to expected wave speeds.

Imaging of a fluid/solid Interface and localization of an isolated scaterer : Experiment 2 and 3

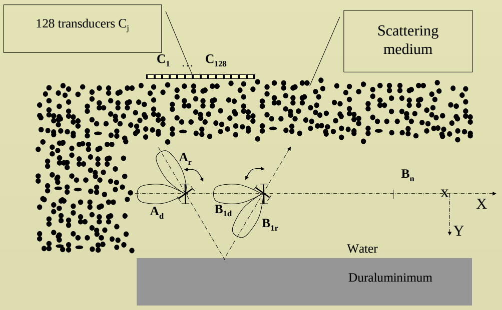

Figure 5 : Laboratory configuration of the second and third experiments. The plastic bag is removed. In the second experiment we place a thick aluminum plate. Because the moving receiver has a narrow directivity (+/- 20° at –6 dB), we can not obtain the direct and reflected wave front in the same time. To compensate this anisotropy we rotated the sensor and acquired the impulse response twice each time : $h_{CAr}(t)$ and $h_{CAd}(t)$, $h_{CBr}(t)$ and $h_{CBd}(t)$ (the subscript r stands for reflected, and d for direct).

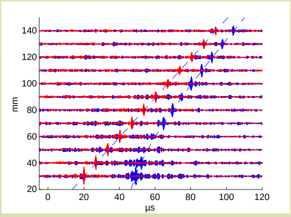

Figure 6: Wave fronts reconstructed by averaged correlations. Each line corresponds to the correlation of coda received in A (x=0 mm) and another B point (from x=30 mm to x=140 mm). The abscissa of B is in y-coordinate. The reconstructed direct wave fronts (red) are superimposed with the reconstructed reflected wave fronts (blue). The sound speed calculated in this experiment is 1.49~mm/µs (variations are due to temperature). The position of the interface obtained by least square value method is y=33.2 mm. Theoretical arrival times for the direct and reflected wave fronts are plotted in dotted lines.

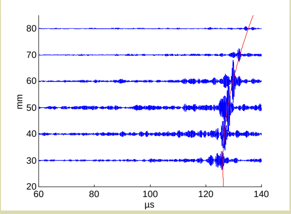

Figure 7 : Results of the third experiment : the aluminum plate is removed and we place a strong scaterer instead (a 1 cm thick vertical steel bar). Each line corresponds to the correlation of codas received in A and another B point. The abscissa of B is in y-coordinate. The theoretical arrival times are plotted in red solid line. They are calculated for a position of the scaterer obtained by a least square value method: x=28.3 mm, y=91.5 mm

Conclusion :

References :

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