Dynamic correlations, interference and time-dependent speckles

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PhD:

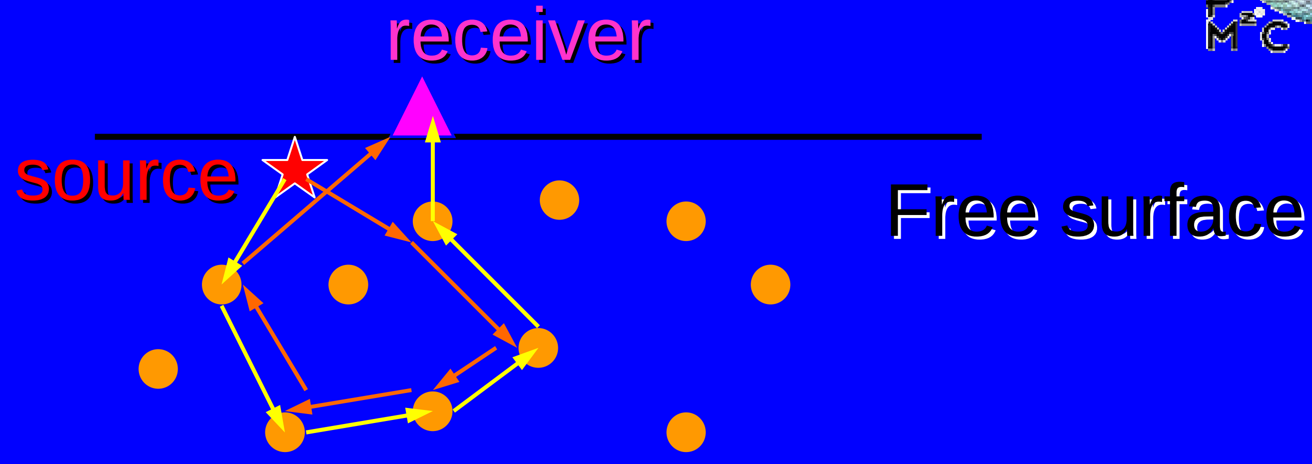
Eric Larose (LGIT)

Support: GDR PRIMA & IMCODE (CNRS), Ministère de la Recherche (ACI jeune chercheur), NSF (USA), ESA

abstract

- **Coherent Backscattering with Seismic Waves**
  Eric Larose, Ludovic Margerin, Michel Campillo, BavT
- **Phase Statistics**
  John Page, Micheal Cowan, BAvT, Azriel Genack, Patrick Sebbah
- **The Feigel process**
  Geert Rikken, BavT

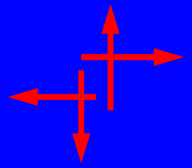
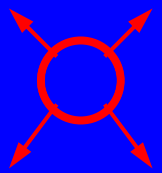
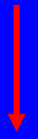
mesoscopic seismology



1. Distance source receiver < wavelength

$$CBS(r) \propto 1 + J_0^2\left(\frac{2\pi r}{\lambda}\right) \quad \mathbf{x} \quad 1 - e^{-t/\tau}$$

2. Symmetry source = symmetry receiver & magnitude

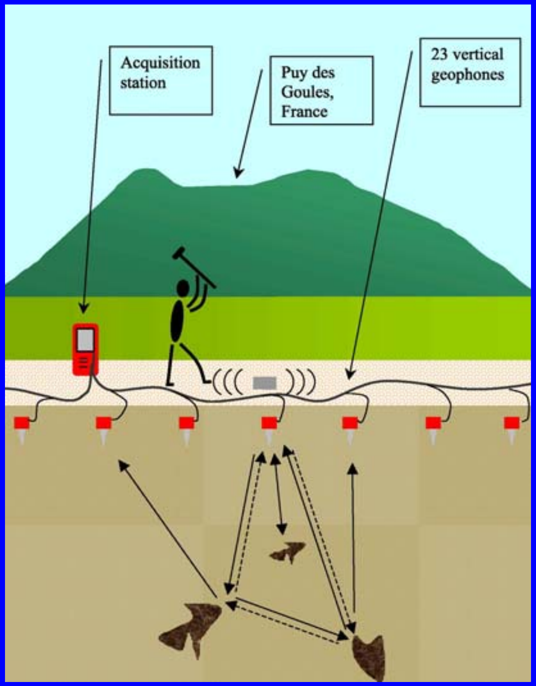
 measure $\left\|\partial_y u_x + \partial_x u_y\right\|^2$ **Earth quake**	 measure $\left\|\text{div}\,\mathbf{u}\right\|^2$ **Explosion**	 measure $\left\|u_z\right\|^2$ **Sledge hammer**

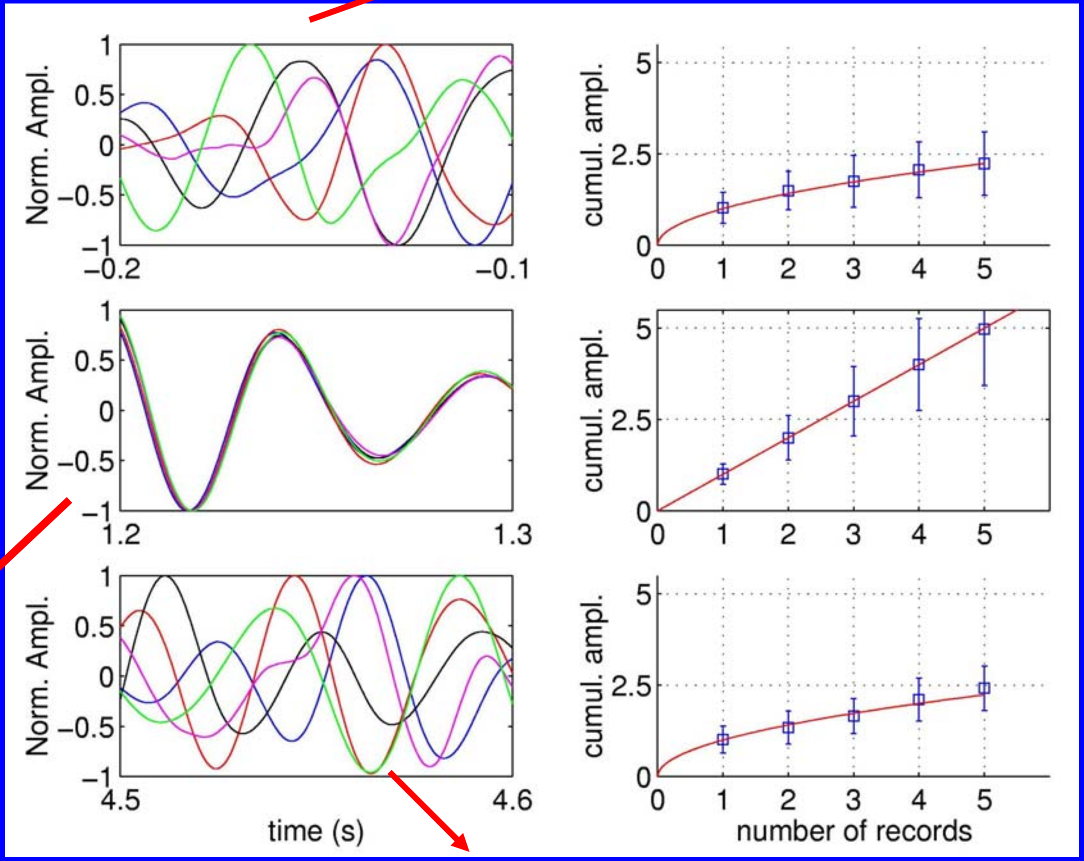
← magnitude

Seismic waves in the French Auvergne

Eric Larose, Ludovic Margerin, Michel Campillo et Bart van Tiggelen , PRL, July 2004

Operator noise



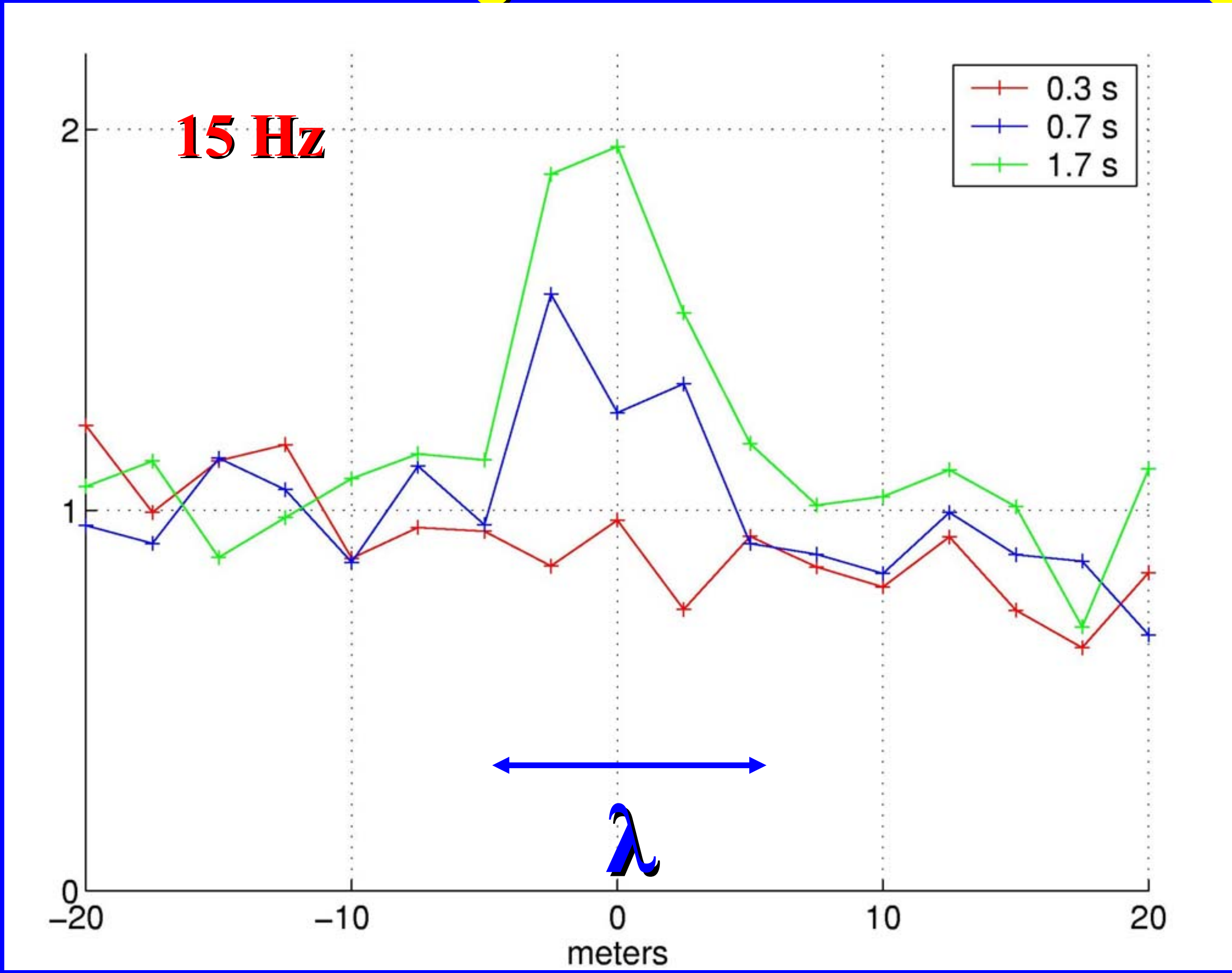


Mesoscopic signal

Background noise

mesoscopic seismology

Coherent Backscattering in the French Auvergne

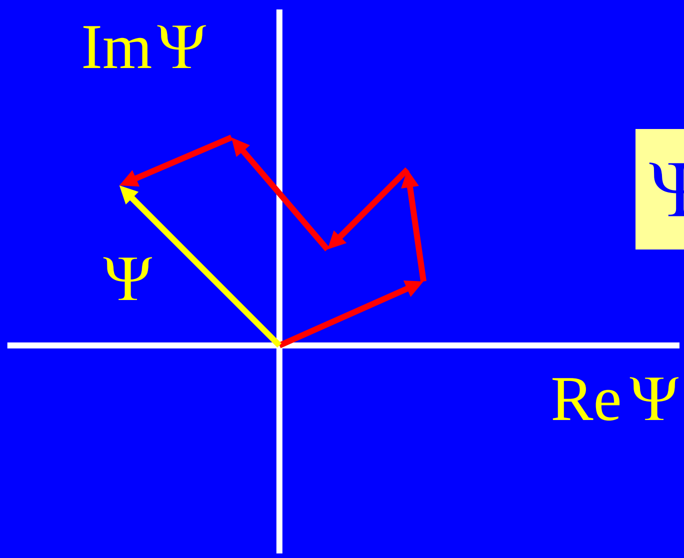


Mean free time=0.7 seconds

Wavelength= 20 meter $c_{Rayleigh} = 300$ m/s

Mean free path = 210 m

Phase Speckle



$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 + \ldots$$

probability distribution

$$P(\Psi_1, \Psi_2, \ldots \Psi_N) = \frac{1}{\pi^N \det \mathbf{C}} \exp\left(-\mathbf{\Psi}^* \cdot \mathbf{C}^{-1} \cdot \mathbf{\Psi}\right) \qquad C_{ij} \equiv \langle \Psi_i \Psi_j^* \rangle$$

diffusion equation

Phase Speckle

Gaussian Speckles

$$\Psi = \sqrt{I}\ \mathrm{e}^{\,i\phi}$$

→ intensity

→ phase

1. **Stationary**: Distribution of speckle intensity

$$P(I,\phi) = \frac{1}{<I>} \exp\left(-I/<I>\right)$$

2. **Dynamics** :Distribution of « Wigner delay » time $\frac{\mathrm{d}\phi}{\mathrm{d}\omega}$

$$P\left[\Psi\left(\omega-\frac{\Omega}{2}\right),\ \Psi\left(\omega+\frac{\Omega}{2}\right)\right] = \frac{1}{\pi^2 \det \mathbf{C}} \exp\left(-\mathbf{\Psi}^* \cdot \mathbf{C}(\Omega)^{-1} \cdot \mathbf{\Psi}\right)$$

$$\Rightarrow\ P\left(\frac{\mathrm{d}\phi}{\mathrm{d}\omega} = \phi'\right) = \frac{Q}{2} \frac{1}{\left[Q + \left(\hat{\phi}' - 1\right)^2\right]^{3/2}}$$

Speckle

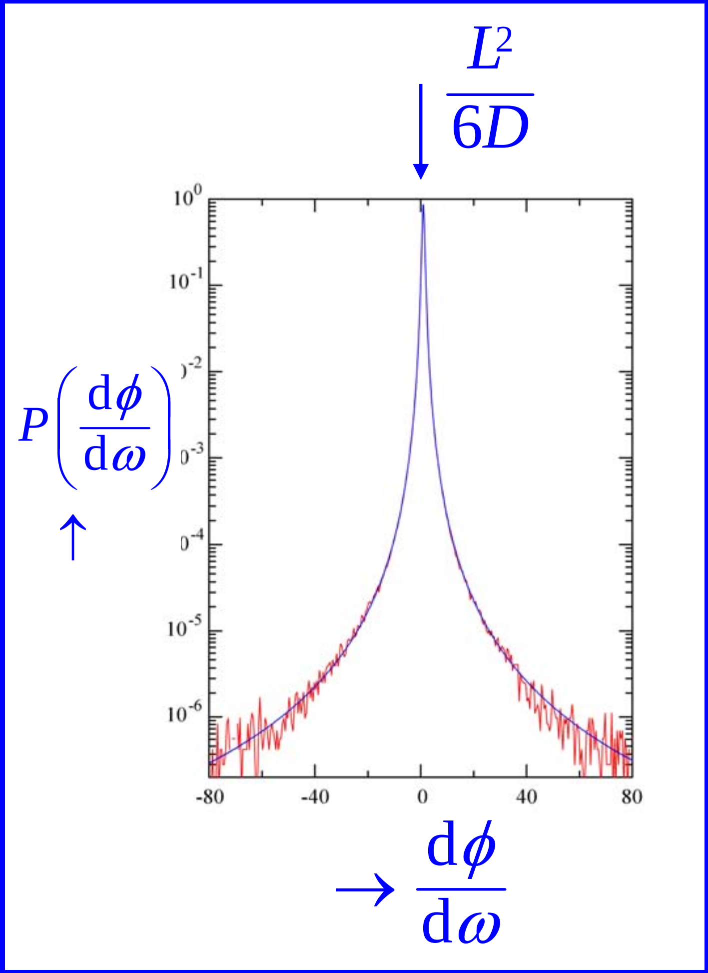
Speckles of Micro-waves in Quasi 1D media

Distribution of delay time in transmission

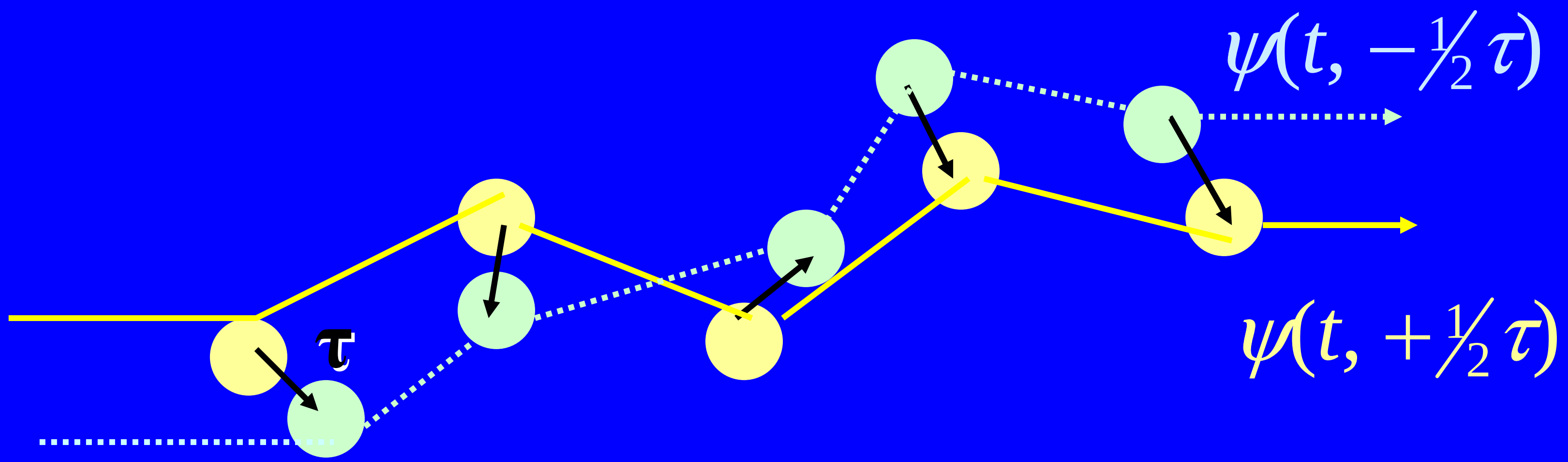
$$P\left(\frac{\mathrm{d}\phi}{\mathrm{d}\omega}=\phi'\right)=\frac{Q}{2}\quad\frac{1}{\left[Q+\left(\hat{\phi}'-1\right)^{2}\right]^{3/2}}$$

diffusion equation : $Q=\frac{2}{5}$

Genack, Sebbah, Stoytchev & Van Tiggelen
PRL, 1999

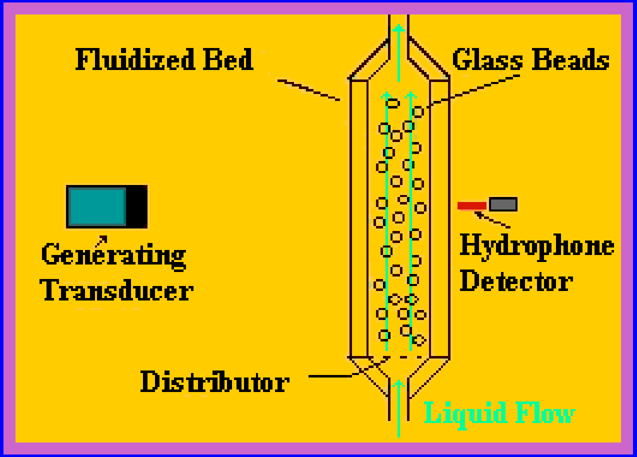


Diffuse Acoustic Wave Spectroscopy



$$\frac{\langle \psi(t, -\frac{1}{2}\tau), \psi(t, +\frac{1}{2}\tau) \rangle}{\langle \psi(t)^2 \rangle} = g(\tau) = \exp\left(-\tfrac{1}{6} k^2 n \langle \Delta \mathbf{r}^2(\tau) \rangle \right)$$

$$g(\tau) \approx \exp\left(-\frac{1}{6} \frac{\tau^2}{t_{\mathrm{DAWS}}^2} \right)$$



Diffuse Acoustic wave Spectroscopy

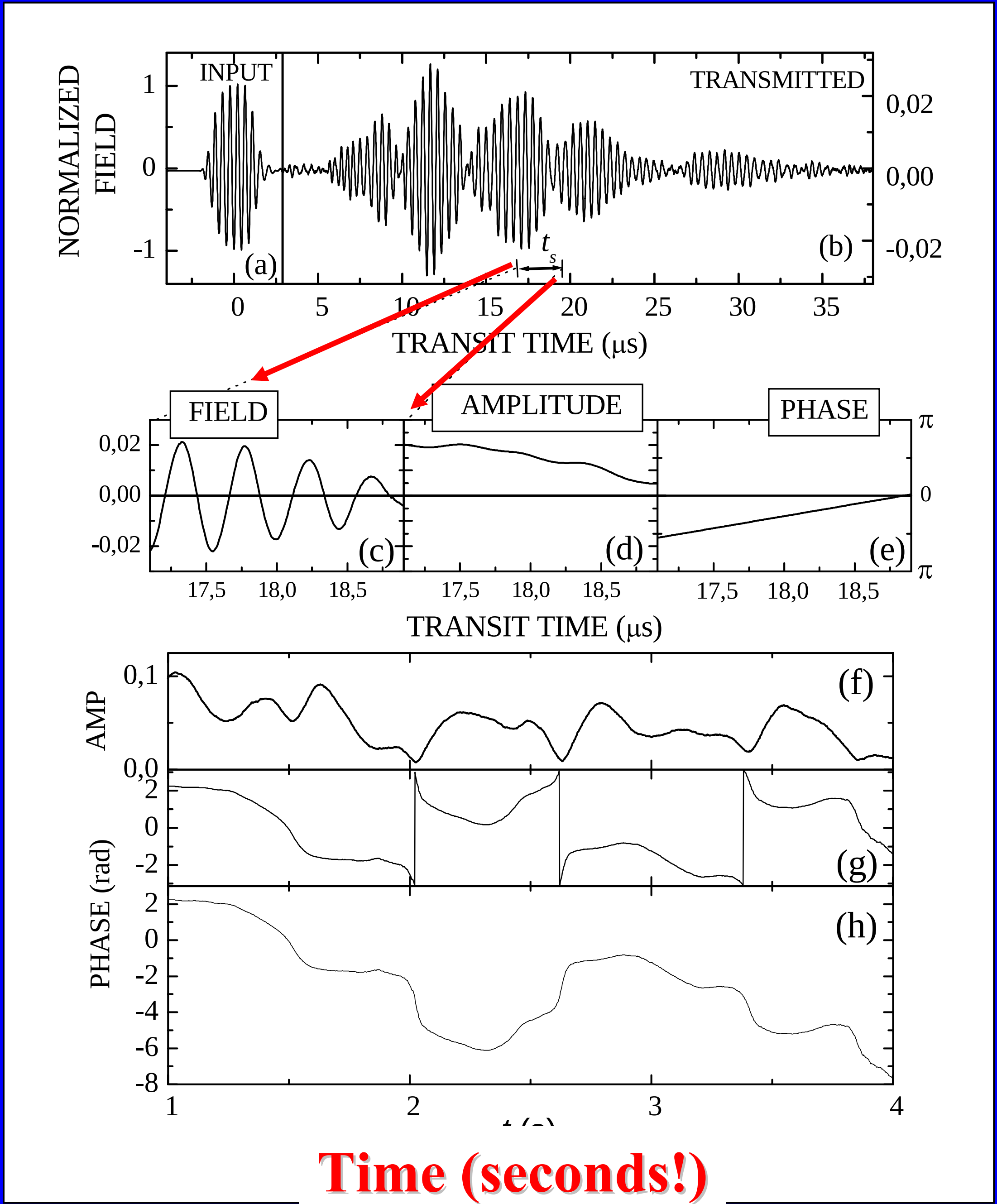
John Page, Dave Weitz, Michael Cowan

amplitude →

Wrapped phase →

unwrapped phase →

$\ell^* = 1.5\,mm; \;\; \tau^* = 1\,\mu s$



Time (seconds!)

Probability distribution $P(\Delta\Phi)$

for phase shift $\Delta\Phi(\tau)$

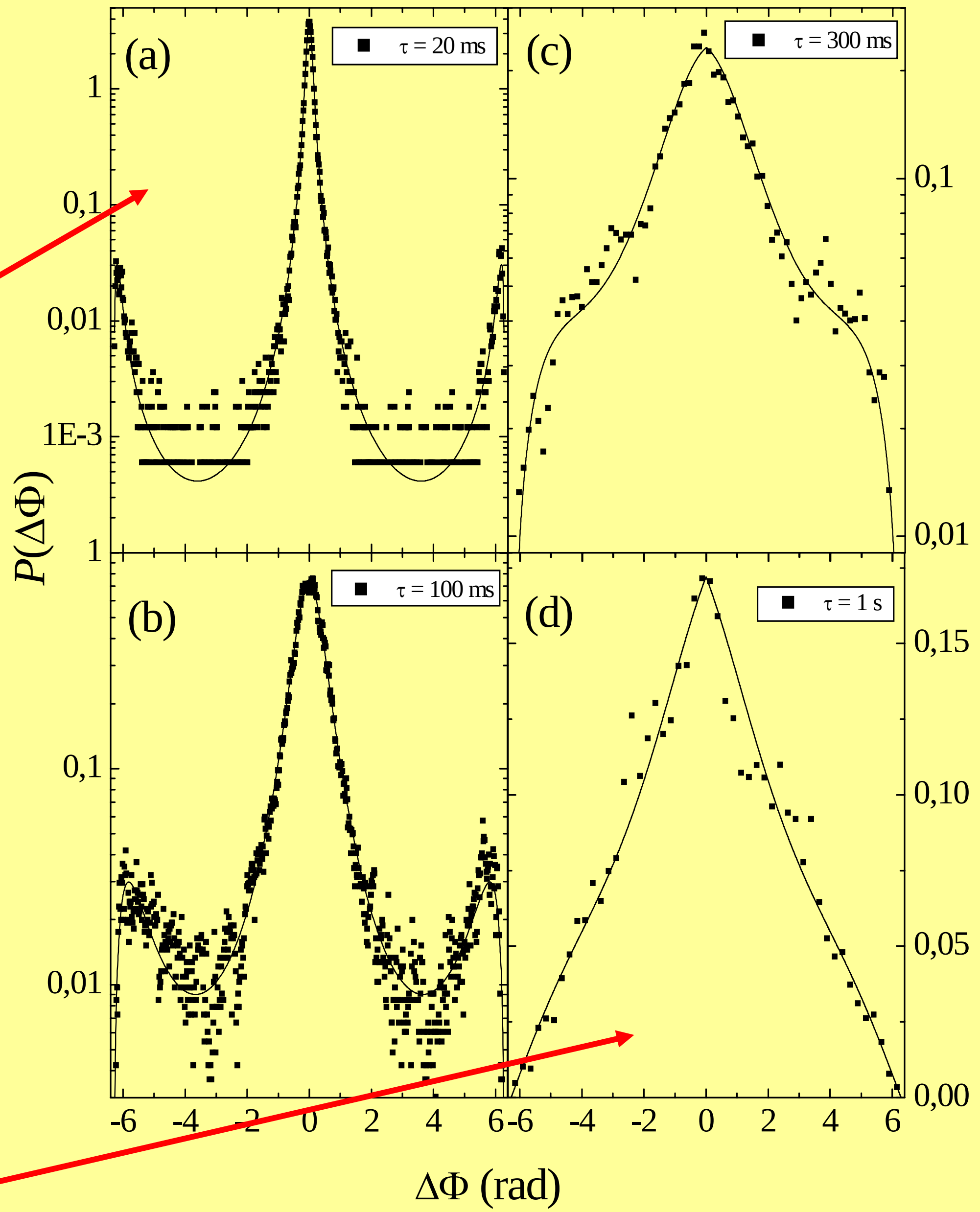
after time τ

$$P\left(\frac{d\phi}{d\tau}\right) = \frac{Q}{\left[2Q + \left(\frac{d\phi}{d\tau}\right)^2\right]^{3/2}}$$

$$Q = \frac{1}{6t_{\mathrm{DAWS}}^2}$$

$t_{\mathrm{DAWS}} = 100\,\mathrm{ms}$

$$P(\Delta\phi) = \frac{1}{2\pi^2}\left(2\pi - |\Delta\phi|\right)$$



Probability distribution of SECOND derivative

$$P\left[\psi(t_1),\psi(t_2),\psi(t_3),\psi(t_4)\right]$$

$$\int\int dA_1\, dA_2\, dA_3\, dA_4\, d\phi_4$$

$$t_4 \rightarrow t_1 \qquad t_3 \rightarrow t_2$$

$$P\left[\phi(t_2)-\phi(t_1),\phi'(t_1),\phi'(t_2)\right]$$

$$\phi(t)=\phi_0+\phi'\Delta t+\frac{1}{2}\phi''(\Delta t)^2+\frac{1}{6}\phi'''(\Delta t)^3$$

?

$$P\left[\phi'(t),\phi''(t),\phi'''(t)\right]$$

Probability distribution of SECOND derivative

$$P\left[\psi(t_1),\psi(t_2),\psi(t_3),\psi(t_4)\right]$$

$$\int \int dA_1\, dA_2\, dA_3\, dA_4\, d\phi_4$$

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$$P\left[\phi(t_2)-\phi(t_1),\phi'(t_1),\phi'(t_2)\right]$$

$$\phi(t)=\phi_0+\phi'\Delta t+\frac{1}{2}\phi_{\pm}''(\Delta t)^2$$

$$P\left[\phi'(t),\phi_{+}''(t),\phi''_{-}(t)\right]$$

Phase is not an analytic function

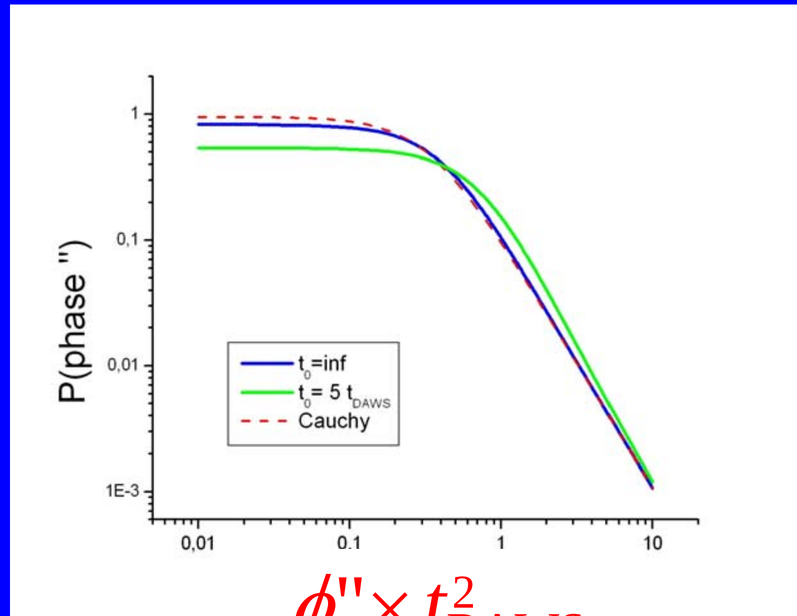
Probability distribution of SECOND derivative

$$P\left[\bar{\phi}''\right]=\frac{1}{\pi}\int_0^\infty dx\,\frac{\left(4x^2+R\right)^{3/2}}{\left[\left(\bar{\phi}''\right)^2+\left(x^2+\frac{1}{2}\right)\left(4x^2+R\right)\right]^2}$$

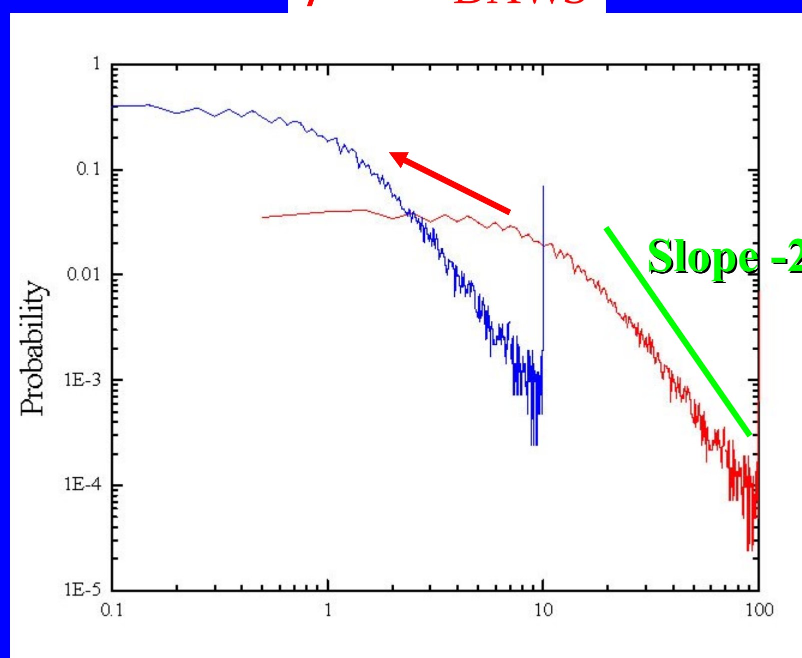
$$P\left[\Delta\phi''\equiv\phi_+''-\phi_-''\right]=\frac{1}{4T}\,\frac{1}{\left[\frac{\left(\Delta\phi''\right)^2}{T}+\frac{1}{2}\right]^{3/2}}$$

$$R=\frac{1}{2}\left[\frac{g^{(4)}(0)}{\left(g''(0)\right)^2}-1\right] \qquad T=\frac{4}{3}\,\frac{g^{(4)}(0)}{\left(g''(0)\right)^2}$$

Probability distribution of SECOND derivative

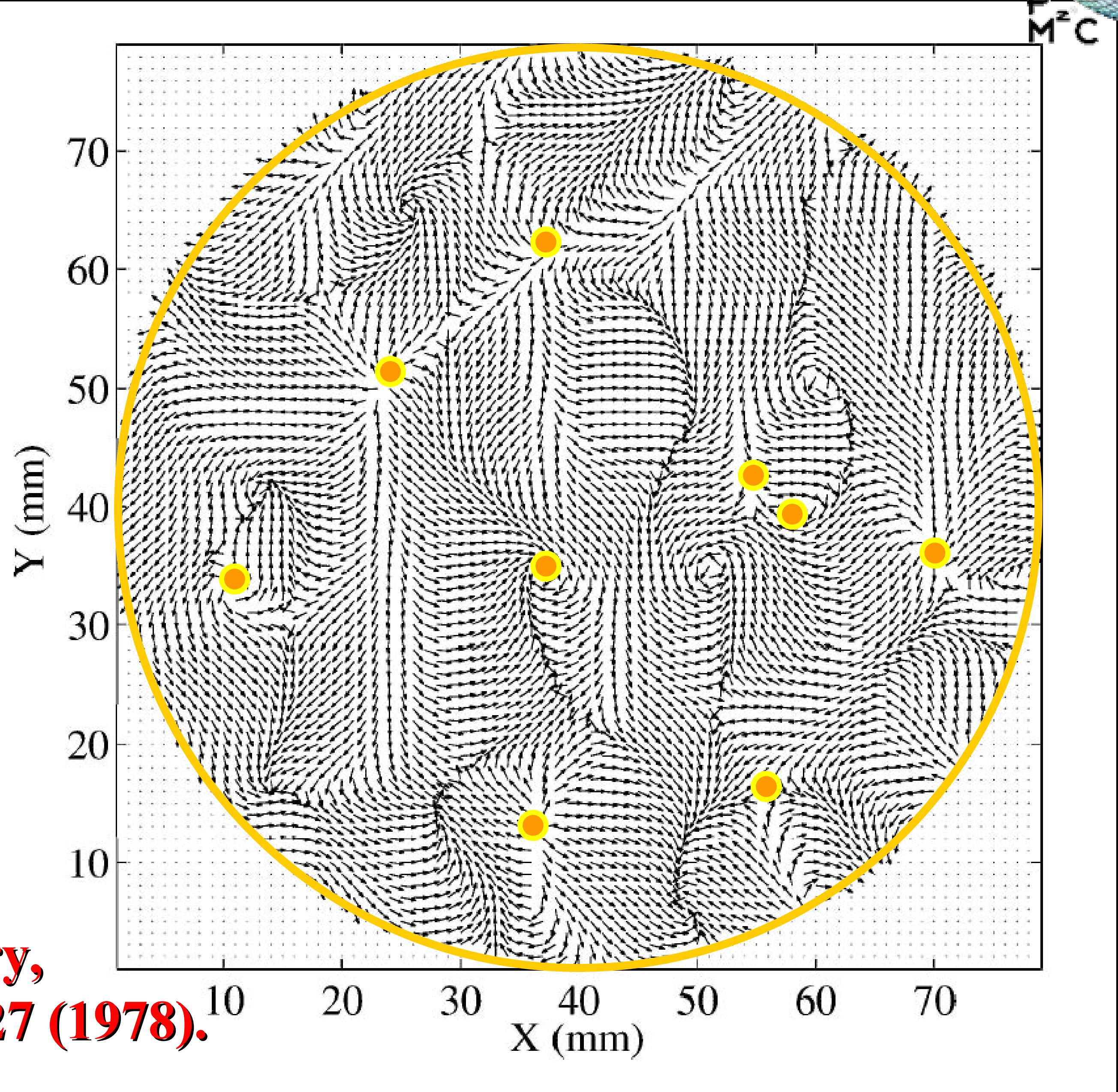


$$\phi'' \times t^2_{\mathrm{DAWS}}$$



DAWS signal
or dynamic noise ?
Noise is interesting

Spatial Phase



Patrick Sebbah
Azriel Genack

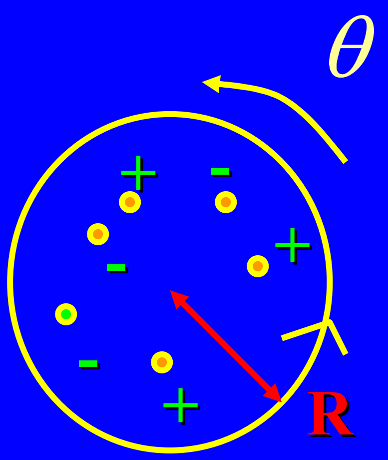
M. Berry,
J. Phys.A. 11, 27 (1978).

Spatial Phase

theorem

$$\oint d\mathbf{l}\cdot\nabla\phi(\mathbf{r}) = 2\pi Q$$

$$Q = \sum_{zero\ i} q_i$$



$$\langle Q \rangle = 0$$

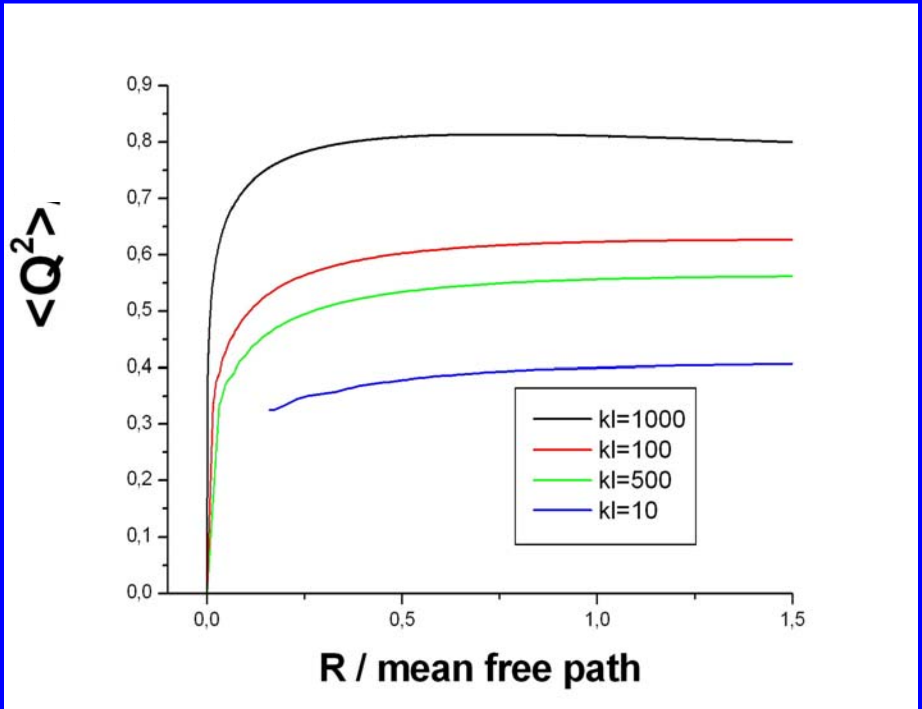
$$\langle Q^2(\text{circle}) \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

Count the mean free path?

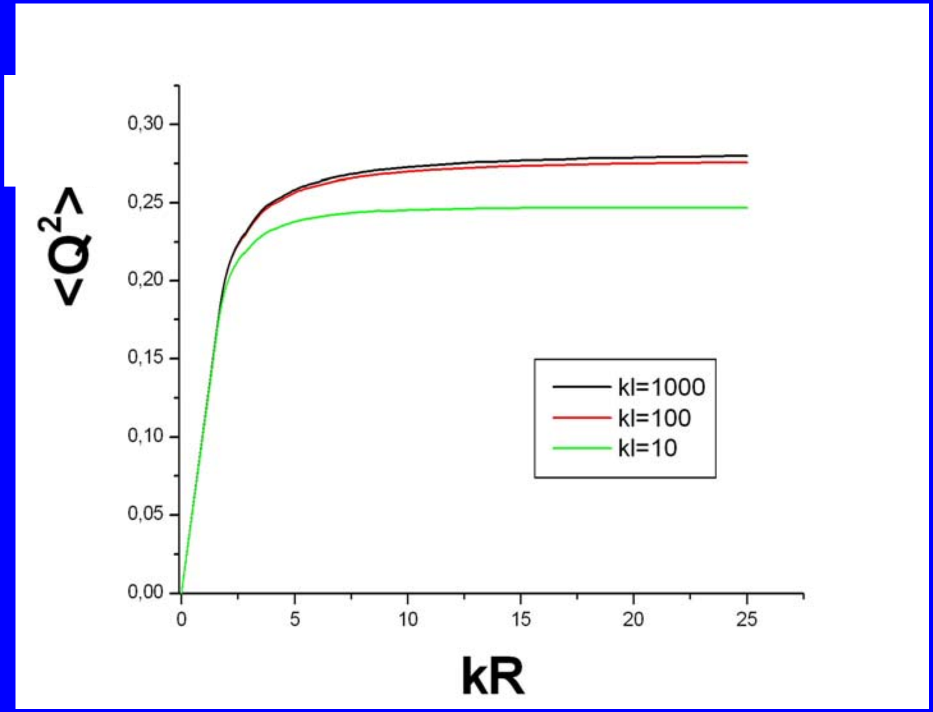
$$\langle Q^2(\text{circle})\rangle = \frac{1}{2\pi}\int_0^{2\pi} d\Delta\theta \left\langle \frac{d\phi}{d\theta}\left(-\frac{\Delta\theta}{2}\right)\frac{d\phi}{d\theta}\left(\frac{\Delta\theta}{2}\right)\right\rangle$$

$P[\psi(\mathbf{r}_1),\psi(\mathbf{r}_2),\psi(\mathbf{r}_3),\psi(\mathbf{r}_4)]$

$\langle\psi(\mathbf{r})\psi^*(\mathbf{r}')\rangle = J_0(k\Delta r)\exp(-\Delta r/2\ell)$



2 dimensions



3 dimensions

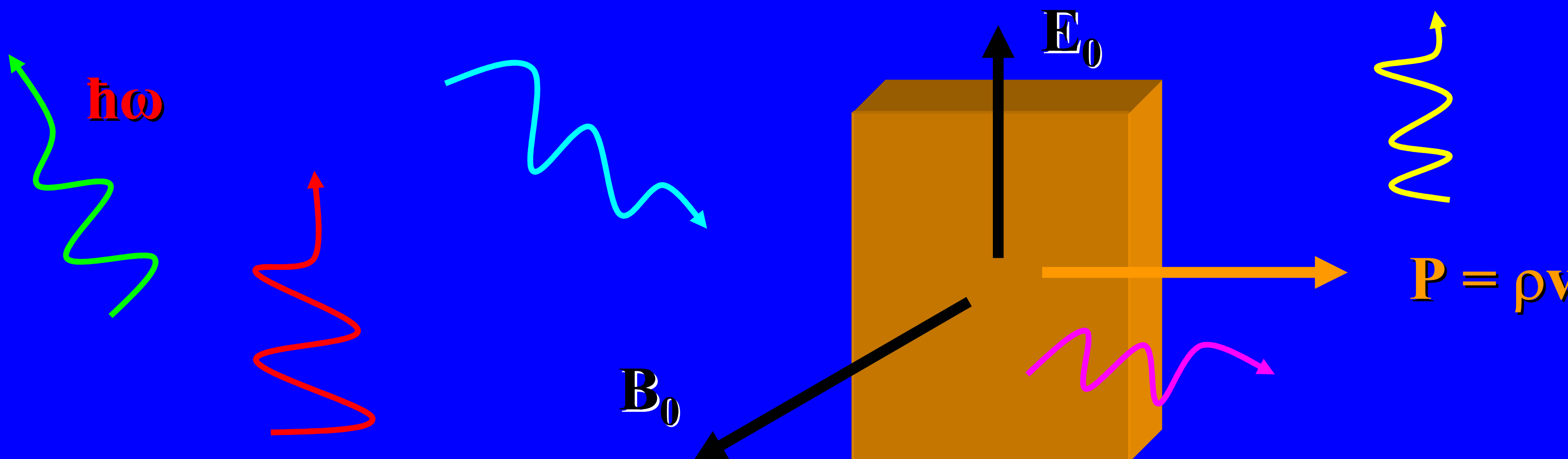
The Feigel process: Momentum from nothing ?

A. Feigel, Phys. Rev. Lett. **92**, 020404 (2004)

BaVT & G. Rikken, PRL Comment 2004

bi-anisotropic media:

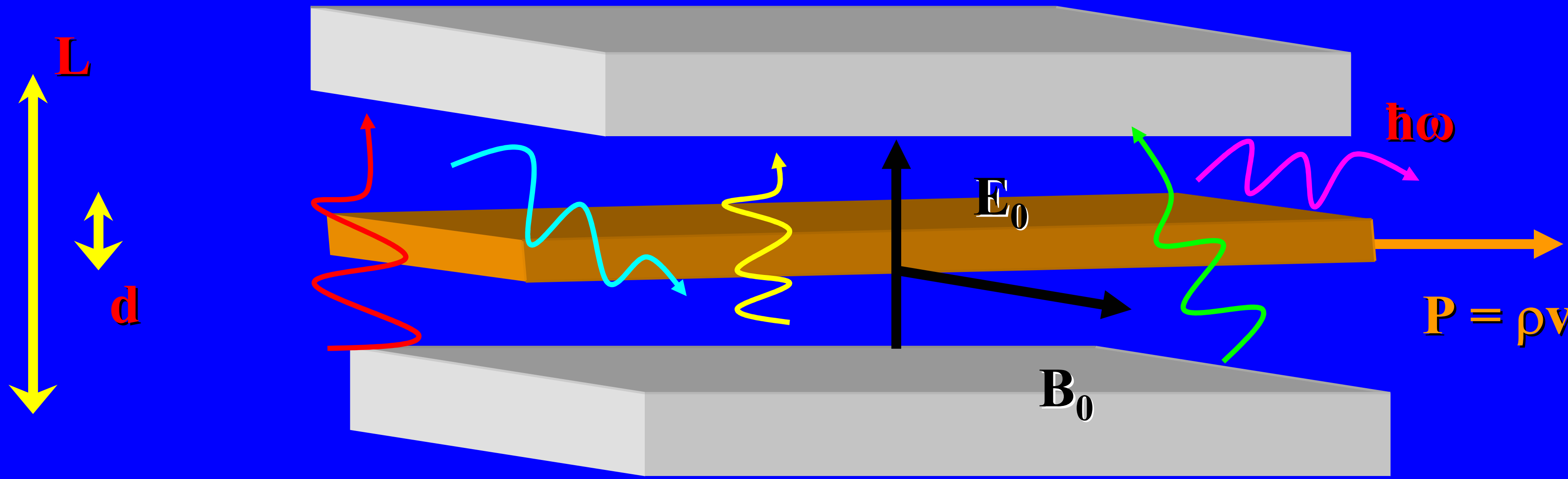
$$\begin{cases} \mathbf{D} = \varepsilon \cdot \mathbf{E} + \chi \cdot \mathbf{B} \\ \mathbf{H} = \mathbf{B} - \chi \cdot \mathbf{E} \end{cases}$$



$$\langle 0 | \rho \, \mathrm{v}_n | 0 \rangle = \frac{2}{3} \frac{\hbar \omega_c^4}{\pi^3 \, c^4} (1+\varepsilon) \, \varepsilon_{nkl} \chi^{kl} \propto \hbar \omega_c^4 \; \mathbf{E}_0 \times \mathbf{B}_0$$

Lorentz invariance? divergence….?

The Feigel process: Momentum from nothing ?



BAvT & G. Rikken
En préparation

$$\langle 0|\rho\,\mathbf{v}|0\rangle = -\frac{\pi^3}{L^4}\;\hbar c_0\;\chi\,\mathbf{E}_0\times\mathbf{B}_0\left(1-\frac{30\,L}{\pi d}\,\frac{\sin\frac{\pi d}{2L}}{\cos^3\frac{\pi d}{2L}}\right)$$